PERPINDAHAN KALOR RADIASI

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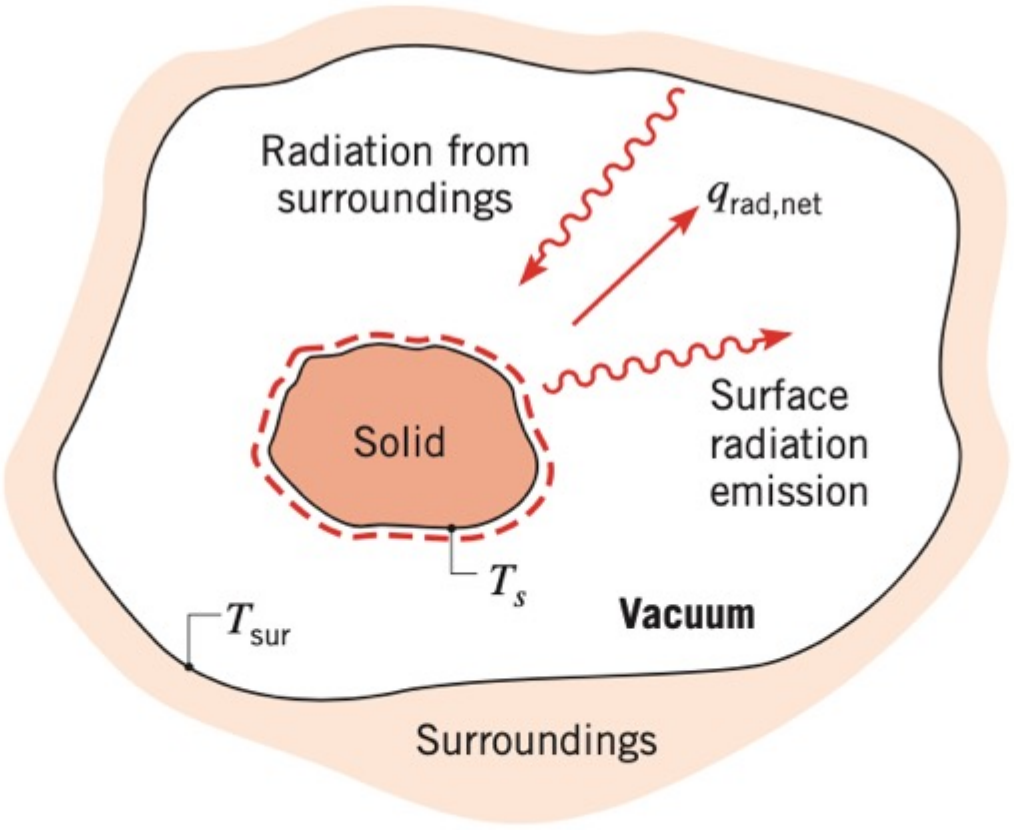
Pendahuluan

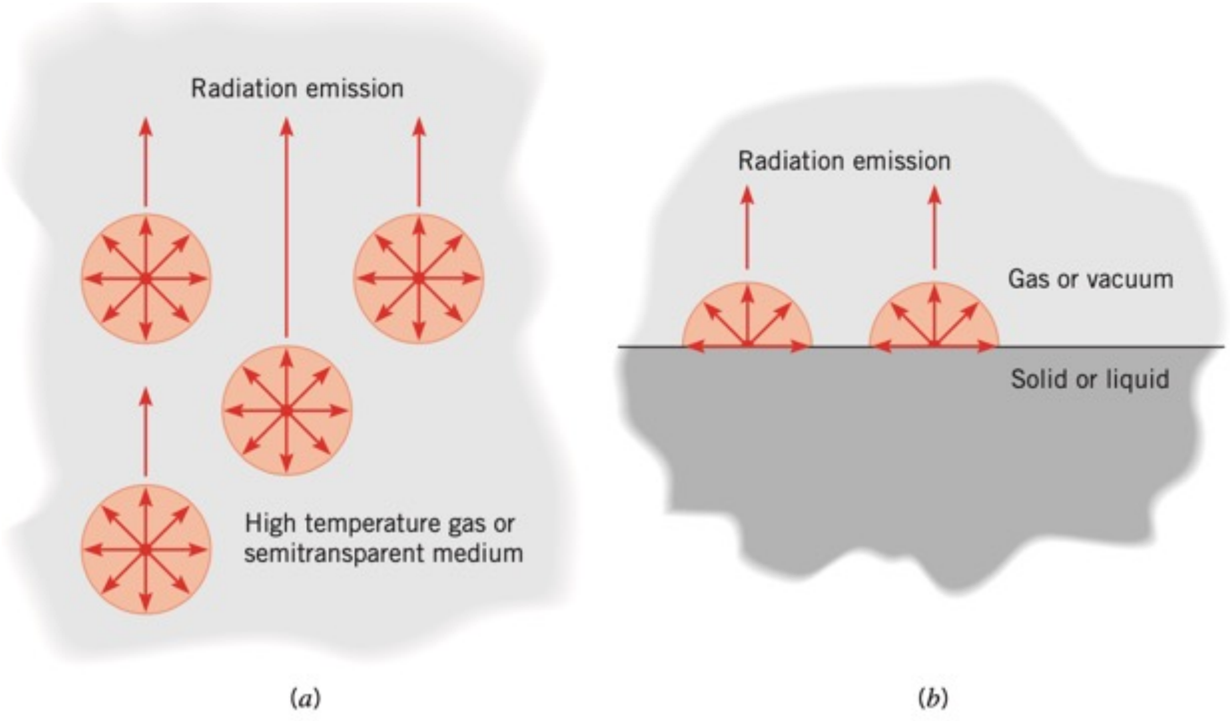
- Selain konduksi dan konveksi, panas dapat ditransfer melalui radiasi.
- Perpindahan panas secara konduksi dan konveksi keduanya membutuhkan kehadiran molekul untuk "membawa" atau meneruskan energi.
- Tidak seperti konduksi atau konveksi, radiasi tidak memerlukan keberadaan media apapun antara sumber panas dan *heat sink* karena energi panas bergerak sebagai gelombang elektromagnetik.
- Fenomena energi radiasi (radiasi termal) ini dipancarkan oleh setiap benda yang memiliki suhu lebih besar dari nol mutlak.
- Kuantitas radiasi yang dipancarkan oleh suatu benda merupakan fungsi dari suhu dan keadaan permukaan.
- Aplikasi radiasi termal termasuk pemanasan industri, pengeringan, konversi energi, radiasi matahari, dan pembakaran.

Konsep Dasar

- Temperatur $T_s > T_{sur}$, tetapi di sekitarnya vakum (Gambar 1).
- Adanya vakum menghalangi hilangnya energi dari permukaan solid secara konduksi atau konveksi.
- Solid akan mendingin dan akhirnya mencapai kesetimbangan termal dengan lingkungannya.
- Pendinginan ini dikaitkan dengan pengurangan energi internal yang disimpan oleh solid dan merupakan konsekuensi langsung dari emisi radiasi termal dari permukaan.
- Jika $T_s > T_{sur}$, laju perpindahan panas bersih oleh radiasi $q_{qrad,\ neto}$ berasal dari permukaan, dan permukaan akan mendingin hingga T_s mencapai T_{sur}.



Gambar 1. Radiation cooling of a hot solid.



GAMBAR 2. THE EMISSION PROCESS. (*a*) AS A VOLUMETRIC PHENOMENON. (*b*) AS A SURFACE PHENOMENON.

- Semua bentuk materi memancarkan radiasi. Untuk gas dan solid semitransparan, seperti kaca dan kristal garam pada suhu tinggi, emisi adalah **fenomena volumetric**. Artinya, radiasi yang timbul dari volume suatu materi yang terbatas merupakan pengaruh yang terintegrasi dari pancaran lokal di seluruh volume.
- Radiasi dapat diperlakukan sebagai fenomena permukaan.
- Pada sebagian besar solid dan liquid, radiasi yang dipancarkan dari suatu molekul diserap dengan kuat oleh molekul yang berdekatan. Dengan demikian, radiasi yang dipancarkan dari solid dan liquid berasal dari molekul yang berada dalam jarak sekitar 1 μm dari permukaan yang terpapar.
- Karena alasan inilah pancaran/emisi dari benda padat atau cairan ke gas atau ruang hampa yang berdekatan dapat dilihat sebagai fenomena permukaan, kecuali pada keadaan untuk perangkat berskala nano atau mikro.

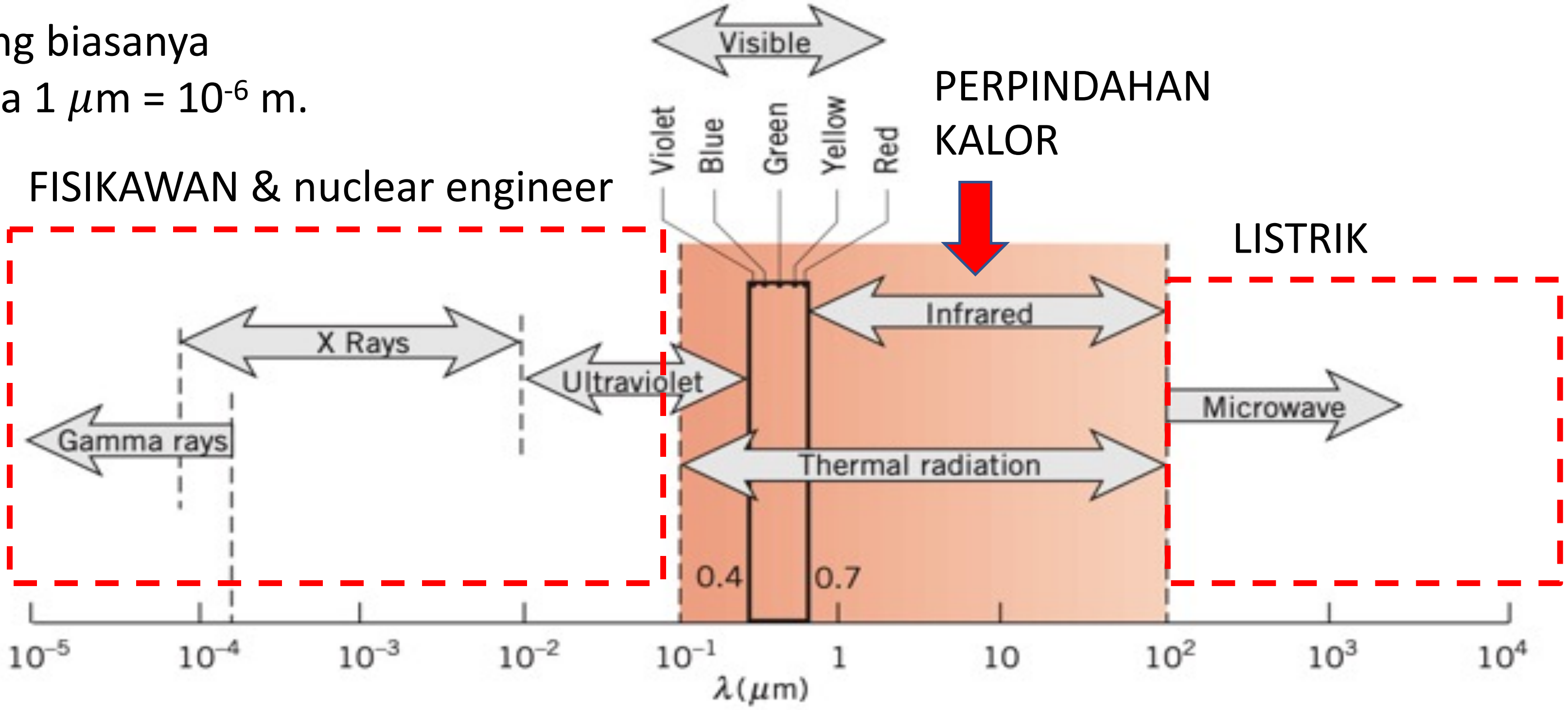
$$Radiasi\ dinyatakan\ dalam : \lambda = \frac{c}{\nu}$$

$\lambda = panjang\ gelombang$

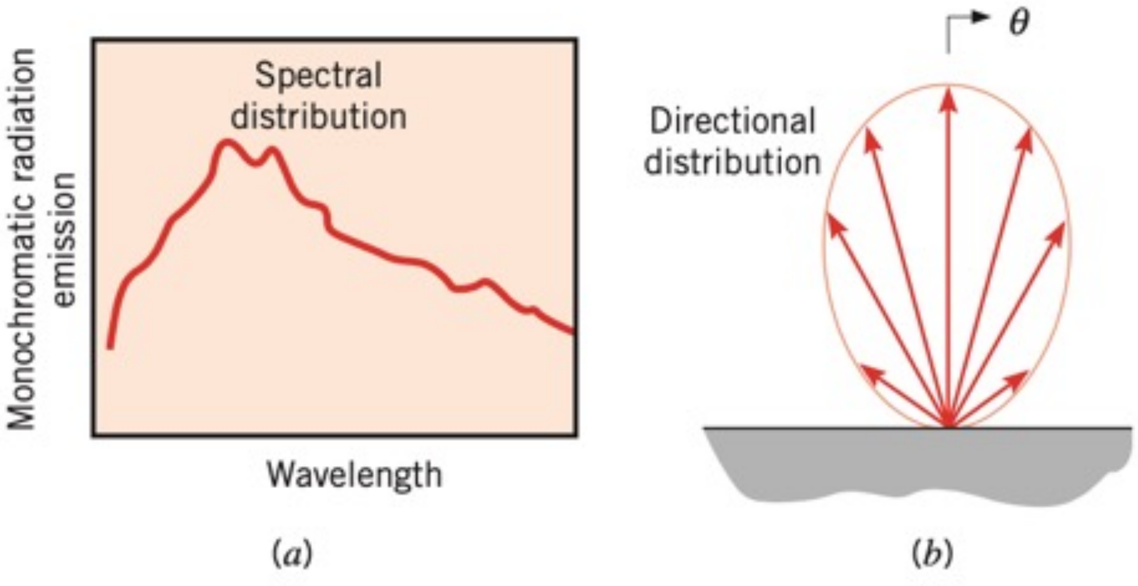
$\nu = frekuensi$

$c = kecepatan\ cahaya\ dalam\ medium, untuk\ di\ dalam\ vacum,$ c_o = 2,998 x 10^8 m/s.

Satuan panjang gelombang biasanya mikrometer (μm), di mana 1 μm = 10^{-6} m.



GAMBAR 3. SPECTRUM OF ELECTROMAGNETIC RADIATION.



GAMBAR 4. RADIATION EMITTED BY A SURFACE. (A) SPECTRAL DISTRIBUTION. (B) DIRECTIONAL DISTRIBUTION

*Cahaya polikromatik adalah cahaya dengan beberapa gelombang yang berbeda-beda frekuensi dan panjang gelombangnya. berarti **memiliki beberapa warna.** Cahaya polikromatik akan mengalami penguraian atau dispersi, yaitu pemisahan cahaya-cahaya berwarna berbeda penyusunnya. Contoh **sinar matahari**.*

Cahaya monokromatik adalah cahaya yang hanya terdiri dari satu jenis frekuensi dan panjang gelombang yang seragam atau cahaya satu warna maka cahaya monokromatik tak dapat diurai.
*Contoh cahaya monokromatik adalah **cahaya pada lampu LED**, yang memiliki warna tertentu saja.*

- Seperti yang ditunjukkan pada Gambar 4a, besarnya radiasi bervariasi dengan panjang gelombang,
- Besaran radiasi pada setiap panjang gelombang dan distribusi spektral bervariasi dengan sifat dan suhu permukaan pancaran.
- Seperti yang ditunjukkan pada Gambar 4b, permukaan dapat memancarkan secara istimewa ke arah tertentu, menciptakan distribusi terarah dari radiasi yang dipancarkan.
- Untuk mengukur konsep emisi, absorpsi, refleksi, dan transmisi perlu mengetahui efek spektral dan arah.

TABEL 1. RADIATIVE FLUXES (OVER ALL WAVELENGTHS AND IN ALL DIRECTIONS)

Flux (W/m^2)	Description	Comment
Emissive power, E	Rate at which radiation is emitted from a surface per unit area	$E = \varepsilon \sigma T_s^4$
Irradiation, G	Rate at which radiation is incident upon a surface per unit area	Irradiation can be reflected, absorbed, or transmitted
Radiosity, J	Rate at which radiation leaves a surface per unit area	For an opaque surface $J = E + \rho G$
Net radiative flux, $q''_{rad} = J - G$	Net rate of radiation leaving a surface per unit area	For an opaque surface $q''_{rad} = \varepsilon \sigma T_s^4 - \alpha G$

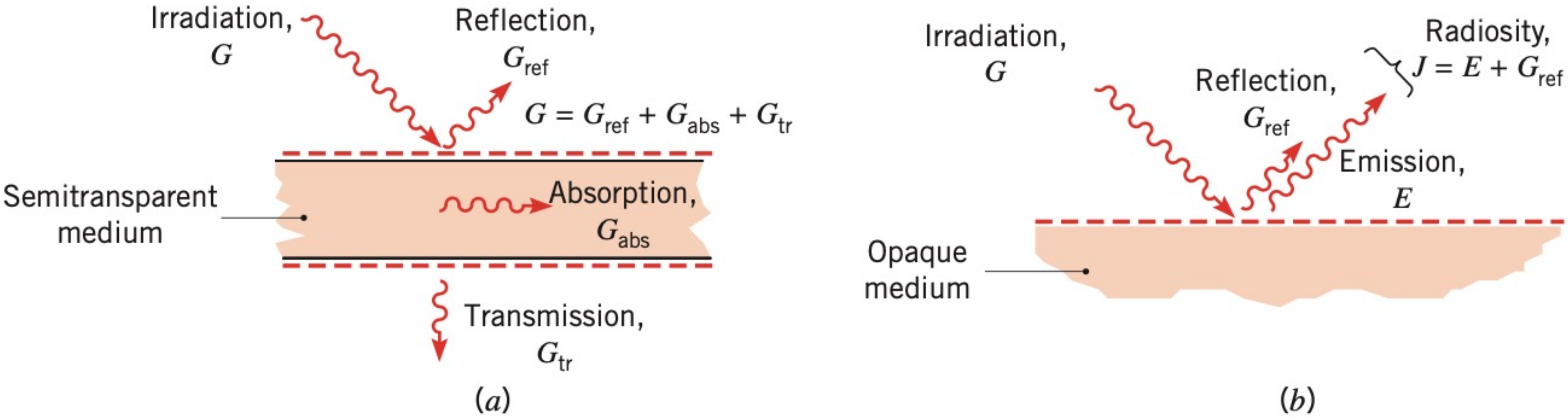
Daya pancaran/emisi *E (W/m^2)* terkait dengan perilaku benda hitam *(black body)* dinyatakan dengan:

$$E = \varepsilon \sigma T_s^4$$

$\varepsilon = sifat\ permukaan\ yang\ disebut\ emisivitas$

T_s = temperatur absolut (K) dari suatu permukaan.

σ = konstanta *Stefan Boltzmann* (σ = 5,67 x 10^{-8} W/m^2K^4)



Radiasi pada suatu permukaan:

a. Refleksi, absorbsi dan transmisi dari irradiation pada medium semitransparan;
b. Radiasitas untuk media yang buram (opaque medium)

Sifat-sifat Emisi atau Pancaran

The rate of emission of radiation by a body depends upon the following factors:

(*i*) The temperature of the surface,

(*ii*) The nature of the surface, and

(*iii*) The wavelength or frequency of radiation.

The *parameters* which deal with the surface emission properties are given below :

(*i*) **Total emissive power (E).** The "*emissive power*" is defined as the *total amount of radiation emitted by a body per unit area and time*. It is expressed in W/m^2. The *emissive power of a black body*, according to Stefan-Boltzmann, is *proportional to absolute temperature to the fourth power.*

$$E_b = \sigma T^4 \text{ W/m}^2 \quad \text{...(11.2)}$$

$$E_b = \sigma A\, T^4 \text{ W} \quad \text{...(11.2 } a\text{)}$$

where, σ = Stefan-Boltzmann constant = 5.67×10^{-8} W/m^2 K^4

(*ii*) **Monochromatic (spectral) emissive power (E_λ).** It is often necessary to determine the spectral distribution of the energy radiated by a surface. At any given temperature the amount of radiation emitted per unit wavelength varies at different wavelengths. For this purpose the *monochromatic emissive power* E_λ of the surface is used. It is defined as *the rate of energy radiated per unit area of the surface per unit wavelength.*

The total emissive power is given by,

$$E = \int_0^\infty E_\lambda \, d\lambda \text{ W/m}^2 \quad \text{...(11.3)}$$

(*iii*) **Emission from real surface-emissivity.** The emissive power from a real surface is given by

$$E = \varepsilon \, \sigma \, AT^4 \text{ W} \qquad \text{...(11.4)}$$

where, ε = Emissivity of the material.

***Emissivity* (ε).** It is defined as the *ability of the surface of a body to radiate heat*. It is also defined as the *ratio of the emissive power of any body to the emissive power of a black body of equal temperature* (*i.e.*, $\varepsilon = \frac{E}{E_b}$). Its values varies for different substances ranging from 0 to 1. For a black body ε = 1, for a white body surface ε = 0 and for gray bodies it lies between 0 and 1. It may vary with temperature or wavelength.

(*iv*) **Intensity of radiation.**

(*v*) **Radiation density and pressure.**

(*vi*) **Radiosity (J).** It refers to all of the radiant energy leaving a surface.

(*vii*) **Interrelationship between surface emission and irradiation properties.**

Contoh 1:

Suatu permukaan mempunyai luas 1,5 m^2 dan dipertahankan pada temperature 300 ^oC dimana terjadi perpindahan panas secara radiasi dengan permukaan lain pada 40 ^oC. Faktor bentuk geometri dan emisivitas 0,52, tentukanlah:

(i) Kerugian panas akibat radiasi

(ii)Tahanan termal

(iii)Koefisien konveksi

Penyelesaian:

$A = 1,5\ m^2$, $T_1 = t_1 + 273 = 573$ K, $T_2 = t_2 + 273 = 313$ K, $F = 0,52$

(*i*) Heat lost by radiation, Q :

$$Q = F \sigma A (T_1^4 - T_2^4)$$

(where $\sigma = 5.67 \times 10^{-8}$ W/m² K⁴)

or, $$Q = 0.52 \times 5.67 \times 10^{-8} \times 1.5 [(573)^4 - (313)^4]$$

$$= 0.52 \times 5.67 \times 1.5 \left[\left(\frac{573}{100}\right)^4 - \left(\frac{313}{100}\right)^4\right]$$

(Please note this step)

or, $$Q = \mathbf{4343\ W}$$

(*ii*) The value of thermal resistance, (Rth)rad :

We know that, $$Q = \frac{(T_1 - T_2)}{(R_{th})_{rad}}$$

$$\therefore \quad (R_{th})_{rad.} = \frac{(T_1 - T_2)}{Q} = \frac{(573 - 313)}{4343} = \mathbf{0.0598\ °C/W}$$

(*iii*) The value of equivalent convection coefficient, hr :

$$Q = h_r A (t_1 - t_2)$$

or, $$h_r = \frac{Q}{A (t_1 - t_2)} = \frac{4343}{1.5 (300 - 40)} = \mathbf{11.13\ W/m^2\ °C}$$

[Alternatively, $$h_r = F \sigma (T_1 + T_2)(T_1^2 + T_2^2) \quad \text{...From eqn. (1.10)}$$

$$= 0.52 \times 5.67 \times 10^{-8} (573 + 313)(573^2 + 313^2)$$

$$= \mathbf{11.13\ W/m^2\ °C}$$]

Contoh soal 2:

Diasumsikan matahari merupakan benda hitam memancarkan radiasi dengan intensivitas maksimum pada $\lambda = 0{,}49\ \mu m$, hitunglah:

(i) Temperatur permukaan matahari

(ii) Heat fluks pada permukaan matahari

Solution. *Given*: $l_{max} = 0.49\ \mu m$

(*i*) **The surface temperature of the sun, *T*:**

According to Wien's displacement law,

$$\lambda_{max} T = 2898\ \mu mK$$

$$\therefore \quad T = \frac{2898}{\lambda_{max}} = \frac{2898}{0.48} = \mathbf{5914\ K}\ \textbf{(Ans.)}$$

(*ii*) **The heat flux at the surface of the sun, $(E)_{sun}$:**

$$(E)_{sun} = \sigma T^4 = 5.67 \times 10^{-8}\ T^4 = 5.67\left(\frac{T}{100}\right)^4$$

$$= 5.67 \times \left(\frac{5914}{100}\right)^4 = \mathbf{6.936 \times 10^7\ W/m^2}\ \textbf{(Ans.)}$$

Contoh soal 3:

Temperatur efektif suatu benda yang mempunyai luas 0,12 m^2 adalah 527 ^{o}C.

Hitunglah:

a. Jumlah total emisi energi.

b. Intensitas radiasi normal

c. Panjang gelombang daya emisi kromatik maksimum

Solution. *Given*: $A = 0.12\ m^2$; $T = 527 + 273 = 800\ K$

(*i*) **The total rate of energy emission, E_b:**

$$E_b = \sigma A T^4 \text{ W (watts)} \qquad \text{...[Eqn. (11.2a)]}$$

$$= 5.67 \times 10^{-8} \times 0.12 \times (800)^4 = 5.67 \times 0.12 \times \left(\frac{800}{100}\right)^4 = \mathbf{2786.9\ W\ (Ans.)}$$

(*ii*) **The intensity of normal radiation, I_{bn} :**

$$I_{bn} = \frac{E_b}{\pi}, \quad \text{where } E_b \text{ is in W/m}^2\ K^4 \qquad \text{...(Eqn. 11.23)}$$

$$= \frac{\sigma T^4}{\pi} = \frac{5.67 \times \left(\frac{800}{100}\right)^4}{\pi} = \mathbf{7392.5\ W/m^2 .sr\ \ (Ans.)}$$

(*iii*) **The wavelength of maximum monochromatic emissive power, l_{max} :**

From Wien's displacement law,

$$\lambda_{max} T = 2898\ \mu mK \qquad \text{...[Eqn. (11.18)]}$$

or,

$$\lambda_{max} = \frac{2898}{T} = \frac{2898}{800} = \mathbf{3.622\,\mu m\ \ (Ans.)}$$

Example 1.7. *A carbon steel plate (thermal conductivity = 45 W/m°C) 600 mm × 900 mm × 25 mm is maintained at 310°C. Air at 15°C blows over the hot plate. If convection heat transfer coefficient is 22 W/m² °C and 250 W is lost from the plate surface by radiation, calculate the inside plate temperature.*

Solution. Area of the plate exposed to heat transfer,

$$A = 600 \text{ mm} \times 900 \text{ mm} = 0.6 \times 0.9 = 0.54 \text{ m}^2$$

Thickness of the plate, $L = 25 \text{ mm} = 0.025 \text{ m}$

Surface temperature of the plate, $t_s = 310°\text{C}$

Temperature of air (fluid), $t_f = 15°\text{C}$

Convective heat transfer coefficient,

$$h = 22 \text{ W/m}^2°\text{C}$$

Heat lost from the plate surface by radiation,

$$Q_{rad.} = 250\text{W}$$

Thermal conductivity, $k = 45 \text{ W/m °C}$

Inside plate temperature, t_i :

In this case the heat conducted through the plate is removed from the plate surface by a *combination of convection and radiation.*

Heat conducted through the plate = Convection heat losses + radiation heat losses.

or, $$Q_{cond.} = Q_{conv.} + Q_{rad.}$$

$$-kA\frac{dt}{dx} = hA(t_s - t_f) + F\sigma A\,(T_s^4 - T_f^4)$$

or, $$-45 \times 0.54 \times \frac{(t_s - t_i)}{L} = 22 \times 0.54\,(310 - 15) + 250 \text{ (given)}$$

or, $$-45 \times 0.54 \times \frac{(310 - t_i)}{0.025} = 22 \times 0.54 \times 295 + 250$$

or, $$972\,(t_i - 310) = 3754.6$$

or, $$t_i = \frac{3754.6}{972} + 310 = \mathbf{313.86°C}$$

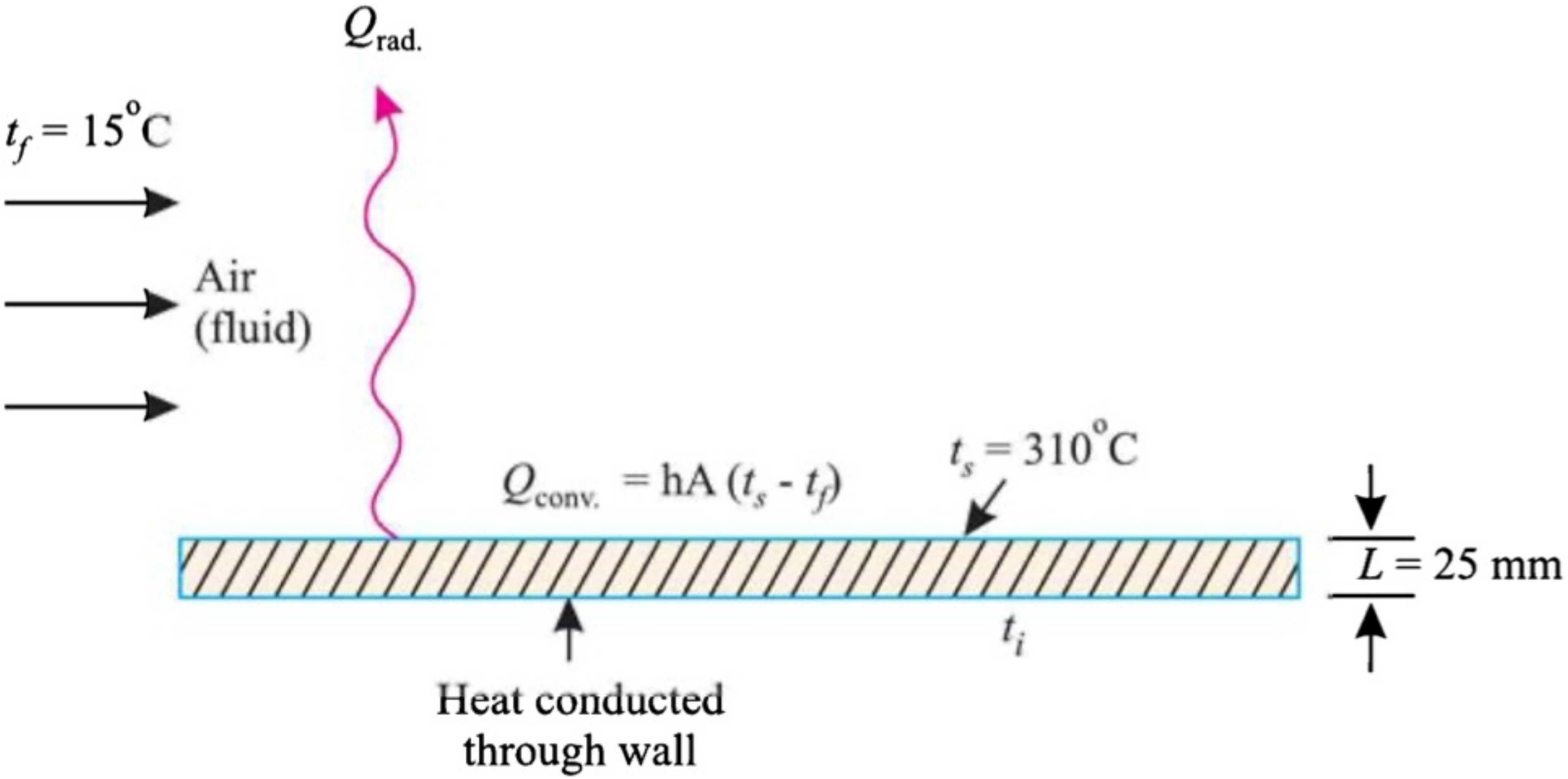


Fig. 1.11. Combination of conduction, convection and radiation heat transfer.

Example 1.8. *A surface at 250°C exposed to the surroundings at 110°C convects and radiates heat to the surroundings. The convection coefficient and radiation factor are 75W/m²°C and unity respectively. If the heat is conducted to the surface through a solid of conductivity 10W/m°C, what is the temperature gradient at the surface in the solid ?*

Solution. Temperature of the surface, $t_s = 250°C$

Temperature of the surroundings, $t_{sur} = 110°C$

The convection co-efficient, $h = 5 W/m^2 °C$

Radiation factor, $F = 1$

Boltzmann constant, $\sigma = 5.67 \times 10^{-8} W/m^2K^4$

Conductivity of the solid, $k = 10 W/m°C$

Temperature gradient, $\frac{dt}{dx}$:

Heat conducted through the plate = Convection heat losses + radiation heat losses

i.e., $$Q_{cond.} = Q_{conv.} + Q_{rad.} \quad - kA\,\frac{dt}{dx} = hA\,(t_s - t_{sur}) + F\sigma A\,(T_s^4 - T_{sur}{}^4)$$

Substituting the values, we have

$$-\,10 \times \frac{dt}{dx} = 75\,(250 - 110) + 1 \times 5.67 \times 10^{-8}\,[(250 + 273)^4 - (110 + 273)^4]$$

$$-\,10 \times \frac{dt}{dx} = 10500 + 5.67\left[\left(\frac{523}{100}\right)^4 - \left(\frac{383}{100}\right)^4\right]$$

$$= 10500 + 3022.1 = 13522.1$$

$$\therefore \qquad \frac{dt}{dx} = -\,\frac{13522.1}{10} = \mathbf{-\,1352.21\ ^\circ C/m}$$

Absorbsi, Refleksi dan Transmisi

When incident radiation (G) also called **irradiation** (defined as the *total incident radiation on a surface from all directions per unit time and per unit area of surface; expressed in* W/m^2 and denoted by (G)) impinges on a surface, three things happens; a part is *reflected* back (G_r), a part is *transmitted* through (G_t) and the remainder is *absorbed* (G_a), depending upon the characteristics of the body, as shown in Fig. 11.2.

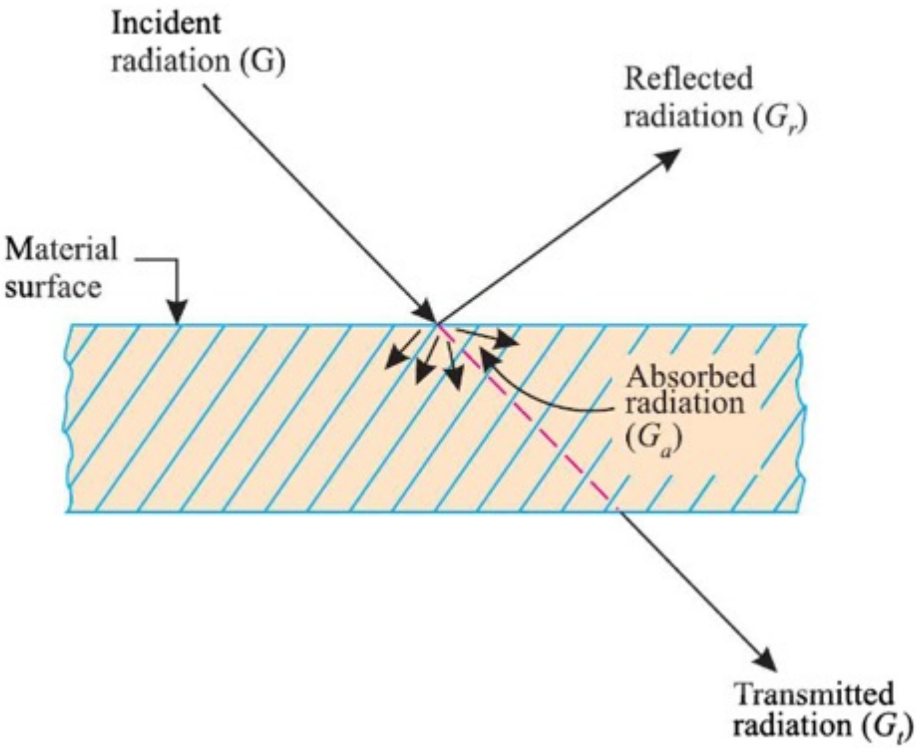


Fig. 11.2. Absorption, reflection and transmission of radiation.

By the conservation of energy principle,

$$G_a + G_r + G_t = G$$

Dividing both sides by G, we get

$$\frac{G_a}{G} + \frac{G_r}{G} + \frac{G_t}{G} = \frac{G}{G}$$

$$\alpha + \rho + \tau = 1 \qquad ...(11.5)$$

α = *absorptivity* (or fraction of incident radiation absorbed),
ρ = *reflectivity* (or fraction of incident radiation reflected), and
τ = *transmittivity* (or fraction of incident radiation transmitted).

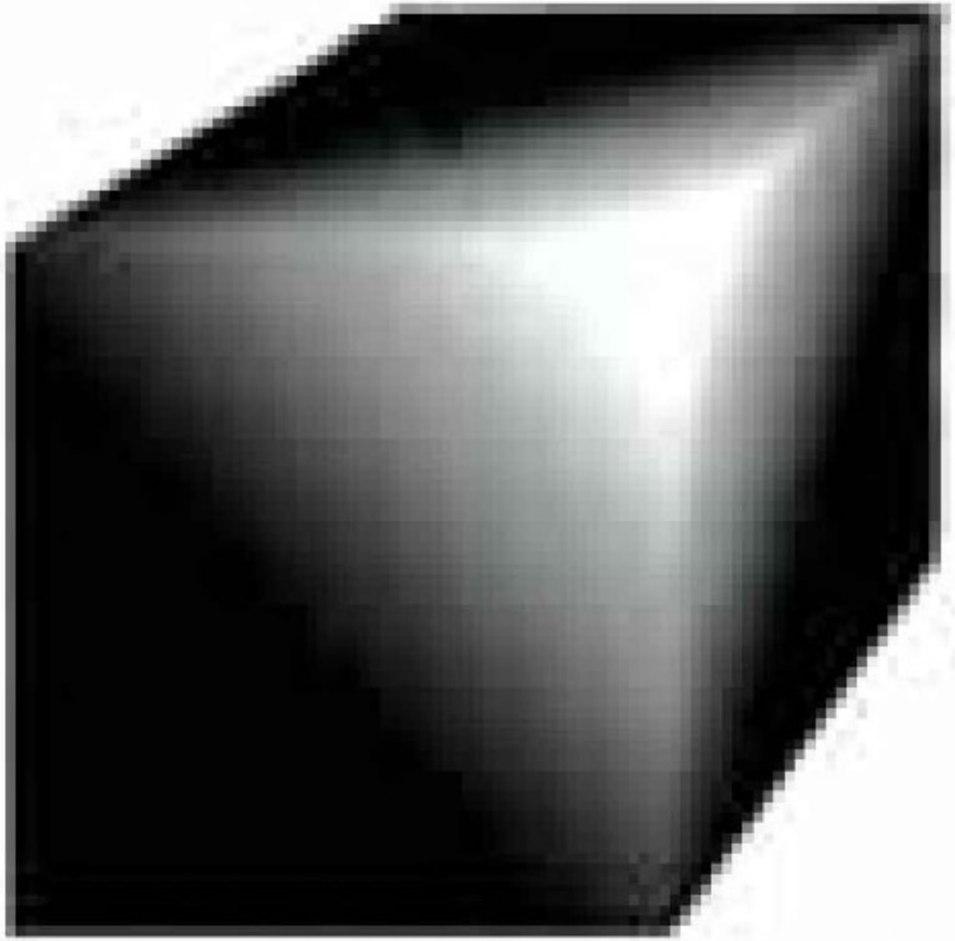
Black body: For perfectly absorbing body, $\alpha = 1$, $\rho = 0$, $\tau = 0$. Such a body is called a '*black body*' (i.e., a *black body is one which neither reflects nor transmits any part of the incident radiation but absorbs all of it*). In practice, a perfect black body ($\alpha = 1$) does not exist. However its concept is very important.

Opaque body: When no incident radiation is transmitted through the body, it is called an '*opaque body*'.

For the opaque body $\tau = 0$, and eqn. (11.5) reduces to

$$\alpha + \rho = 1 \qquad ...(11.6)$$

Solids generally do not transmit unless the material is of very thin section. Metals absorb radiation within a fraction of a micrometre, and insulators within a fraction of a millimetre. Glasses and liquids are, therefore, generally considered as opaque.



A black body is theoretical perfect absorber, which absorbs radiation of all wavelength falling on it.

White body: If all the incident radiation falling on the body are reflected, it is called a '*white body*'.

For a white body, $\rho = 1$, $\alpha = 0$ and $\tau = 0$.

Gases such as hydrogen, oxygen and nitrogen (and their mixtures such as air) have a transmissivity of practically *unity*.

Reflections are of two types: Refer Fig. 11.3.

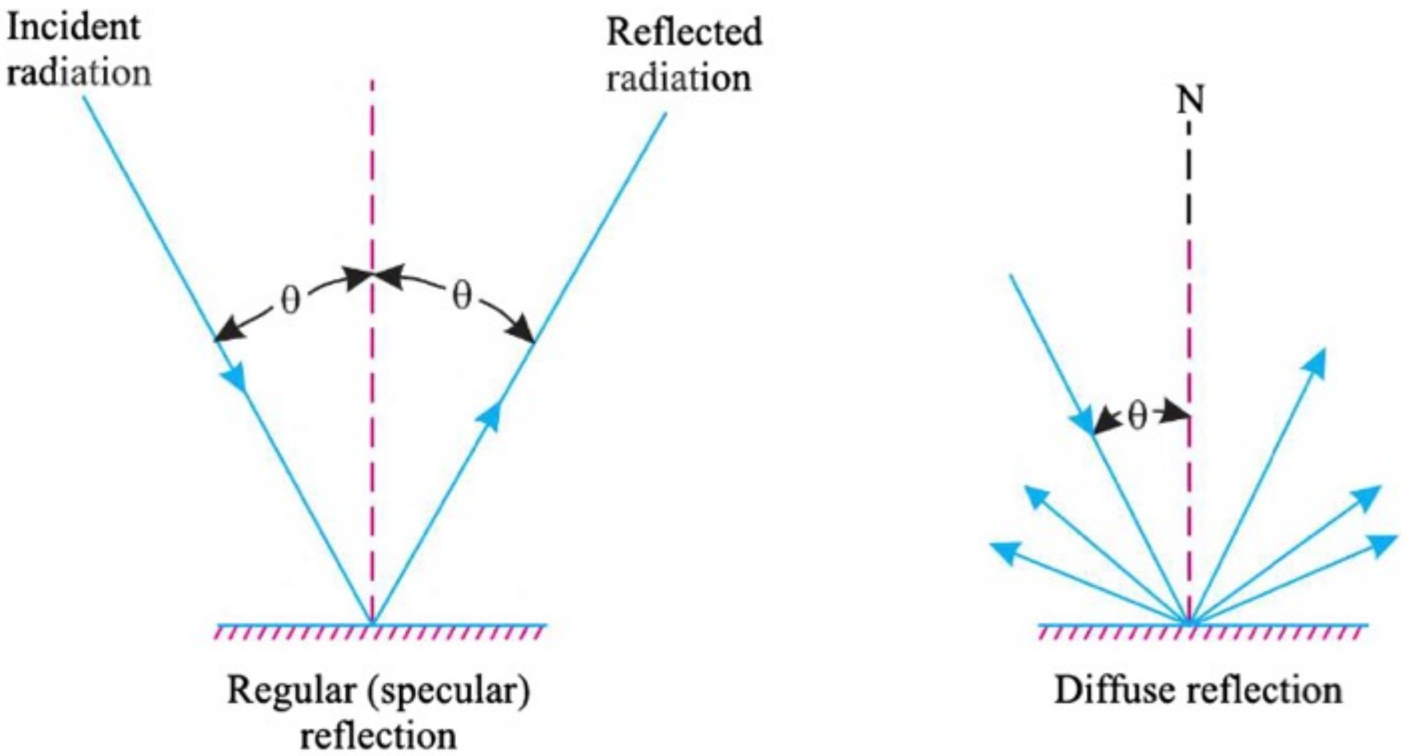


Fig. 11.3. Regular and diffuse reflections.

1. Regular (specular) reflection
2. Diffuse reflection.

Regular reflection implies that angle between the reflected beam and the normal to the *surface equals* the angle made by the incident radiation with the same normal. Reflection from highly polished and smooth surfaces approaches specular characteristics.

In a *diffused reflection*, the incident beam is reflected in *all directions*. Most of the engineering materials have rough surfaces, and these rough surfaces give diffused reflections.

Gray body: If the radiative properties, α, ρ, τ of a body are assumed to be uniform over the entire wavelength spectrum, then such a body is called *gray body*. A *gray body* is also defined as one *whose absorptivity of a surface does not vary with temperature and wavelength of the incident radiation* [$\alpha = (\alpha)_\lambda$ = constant.].

A *coloured body* is one whose absorptivity of a surface varies with the wavelength of radiation [$\alpha \neq (\alpha)_\lambda$].

THE STEFAN-BOLTZMANN LAW

The law states that *the emissive power of a black body is directly proprtional to the fourth power of its absolute temperature.*

i.*e*., $$E_b = \sigma T^4 \qquad \text{...(11.7)}$$

where, E_b = Emissive power of a black body, and

σ = Stefan-Boltzmann constant

$= 5.67 \times 10^{-8}\ \text{W/m}^2\ \text{K}^4.$

Equation (11.7) can be rewritten as:

$$E_b = 5.67 \left(\frac{T}{100}\right)^4 \qquad \text{...(11.8)}$$

KIRCHHOFF'S LAW

The law states that *at any temperature the ratio of total emissive power E to the total absorptivity α is a constant for all substances which are in thermal equilibrium with their environment.*

Gustav Kirchhoff (1824-1887)

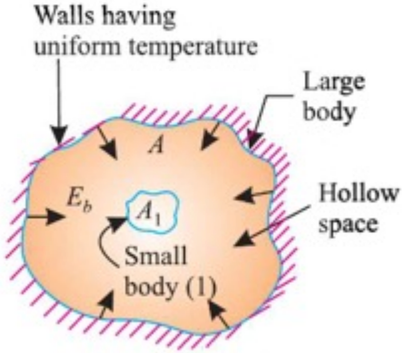


Fig. 11.5. Derivation of Kirchhoff's law.

Let us consider a large radiating body of surface area A which encloses a small body (1) of surface area A_1 (as shown in Fig. 11.5). Let the energy fall on the unit surface of the body at the rate E_b. Of this energy, generally, a fraction α, will be absorbed by the small body. Thus, this energy absorbed by the small body (1) is $\alpha_1 A_1 E_b$, in which α_1 is the absorptivity of the body. When thermal equilibrium is attained, the *energy absorbed* by the body (1) must be equal to the *energy emitted*, say, E_1 per unit surface. Thus, at equilibrium, we may write

$$A_1 E_1 = \alpha_1 A_1 E_b \quad ...(11.9)$$

Now we remove body (1) and replace it by body (2) having absorptivity α_2. The radiative energy impinging on the surface of this body is again E_b. In this case, we may write

$$A_2 E_2 = \alpha_2 A_2 E_b \quad ...(11.10)$$

By considering generality of bodies, we obtain

$$E_b = \frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E}{\alpha} \quad ...(11.11)$$

Also, as per definition of emissivity ε, we have

$$\varepsilon = \frac{E}{E_b}$$

or,

$$E_b = \frac{E}{\varepsilon} \quad ...(11.12)$$

By comparing eqns. (11.11) and (11.12), we obtain

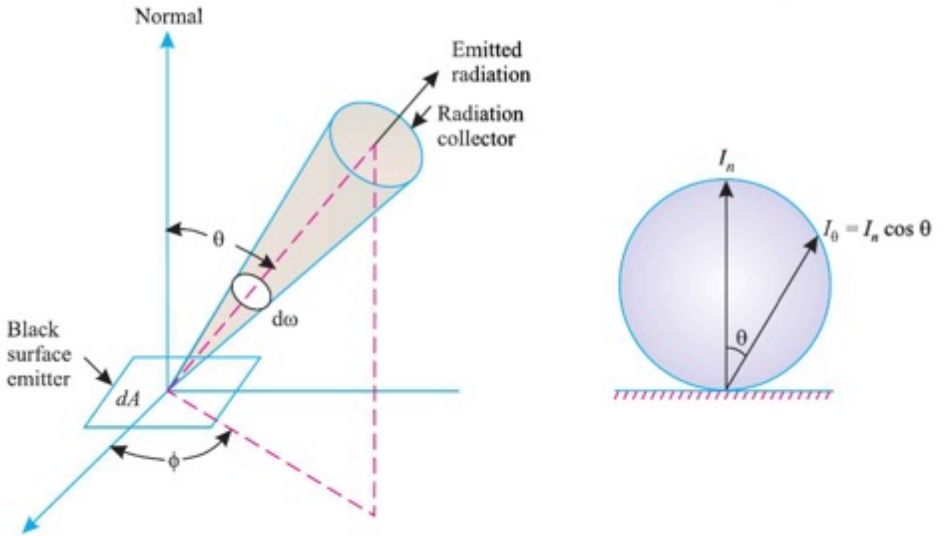
$$\varepsilon = \alpha \quad ...(11.13)$$

(α is always smaller than 1. Therefore, the emissive power E is always smaller than the emissive power of a black body at equal temperature.)

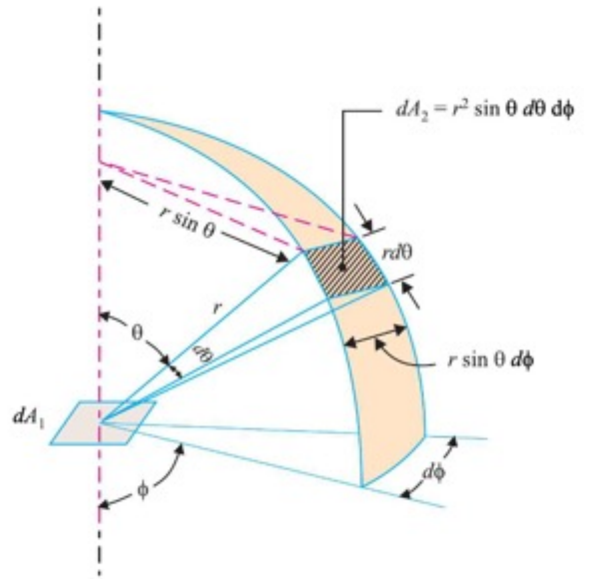
Thus, Kirchhoff's law also states that *the emissivity of a body is equal to its absorptivity when the body remains in thermal equilibrium with its surroundings.*

INTENSITAS RADIASI

- Intensitas radiasi (I) didefinisikan sebagai jumlah energi yang meninggalkan suatu permukaan dalam arah tertentu per satuan sudut benda padt per satuan luas normal permukaan pancaran terhadap rata-rata arah dalam ruang.
- Solid angle didefinisikan sebagai sebagian ruangan di dalam bola tertutup oleh permukaan berbentuk kerucut dengan puncak kerucut pada pusat bola.



(*a*) Spacial distribution of radiations emitted from a surface



(*b*) Illustration for evaluating area dA_2

Fig. 11.7. Radiation from an elementary surface.

- Gambar 11.7a menunjukan suatu permukaan hitam yang kecil dari luas dA yang memmancarkan radiasi pada arah yang berbeda.
- Gambar 11.7b, radiasi dari luas elemen dA1 pada pusat bola

The projected area of dA_1 on a plane perpendicular to the line joining dA_1 and $dA_2 = dA_1 \cos\theta$.

The solid angle subtended by $dA_2 = \dfrac{dA_2}{r^2}$

$\therefore$ The intensity of radiation,

$$I = \frac{dQ_{1-2}}{dA_1 \cos\theta \times \dfrac{dA_2}{r^2}} \qquad ...(11.20)$$

where, dQ_{1-2} is the rate of radiation heat transfer from dA_1 to dA_2.

It is evident from the Fig. 11.7 (*b*) that

$$dA_2 = rd\theta\,(r \sin\theta\, d\phi)$$

or,

$$dA_2 = r^2 \sin\theta . d\theta . d\phi \qquad ...(11.21)$$

From eqns. (11.20) and (11.21), we obtain

$$dQ_{1-2} = I\, dA_1 . \sin\theta . \cos\theta .\, d\theta . d\phi$$

The total radiation through the hemisphere is given by

$$Q = IdA_1 \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{\theta=0}^{\phi=2\pi} \sin\theta . \cos\theta\, d\theta\, d\phi$$

$$= 2\pi IdA_1 \int_{\theta=0}^{\theta=\frac{\pi}{2}} \sin\theta \cos\theta\, d\theta$$

$$= \pi IdA_1 \int_{\theta=0}^{\theta=\frac{\pi}{2}} \sin\theta \cos\theta\, d\theta$$

$$= \pi IdA_1 \int_{\theta=0}^{\theta=\frac{\pi}{2}} \sin 2\theta\, d\theta$$

or,

$$Q = \pi I\, dA_1 \qquad ...(11.22)$$

Also

$$Q = E.dA_1$$

$\therefore$

$$EdA_1 = \pi I\, dA_1$$

or,

$$E = \pi I \qquad ...(11.23)$$

i.e., The *total emissive power of a diffuse surface is equal to* π *times its intensity of radiation.*

LAMBERT'S COSINE LAW

- Menyatakan bahwa daya emisivitas lokal E_θ dari suatu permukaan radiasi dalam setiap arah berbanding langsung terhadap sudu cosinus pancaran.
- Jika E_n adalah daya emisi total permukaan radiasi dalam arah normal maka:

$$E_\theta = E_n \cos \theta$$

Example 11.1. *The effective temperature of a body having an area of 0.12 m^2 is 527°C. Calculate the following:*

(i) *The total rate of energy emission,*
(ii) *The intensity of normal radiation, and*
(iii) *The wavelength of maximum monochromatic emissive power.*

Solution. *Given*: $A = 0.12\ m^2$; $T = 527 + 273 = 800\ K$

(*i*) **The total rate of energy emission, E_b:**

$$E_b = \sigma A T^4 \text{ W (watts)} \qquad \text{...[Eqn. (11.2a)]}$$

$$= 5.67 \times 10^{-8} \times 0.12 \times (800)^4 = 5.67 \times 0.12 \times \left(\frac{800}{100}\right)^4 = \mathbf{2786.9\ W\ (Ans.)}$$

(*ii*) **The intensity of normal radiation, I_{bn} :**

$$I_{bn} = \frac{E_b}{\pi}, \qquad \text{where } E_b \text{ is in W/m}^2 K^4 \qquad \text{...(Eqn. 11.23)}$$

$$= \frac{\sigma T^4}{\pi} = \frac{5.67 \times \left(\frac{800}{100}\right)^4}{\pi} = \mathbf{7392.5\ W/m^2.sr\ \ (Ans.)}$$

(*iii*) **The wavelength of maximum monochromatic emissive power, l_{max} :**

From Wien's displacement law,

$$\lambda_{max} T = 2898\ \mu m K \qquad \text{...[Eqn. (11.18)]}$$

or,

$$\lambda_{\max} = \frac{2898}{T} = \frac{2898}{800} = \mathbf{3.622\,\mu m\ \ (Ans.)}$$

Example 11.2. *Assuming the sun to be a black body emitting radiation with maximum intensity at* $\lambda = 0.49\ \mu m$, *calculate the following :*

(i) *The surface temperature of the sun, and*

(ii) *The heat flux at surface of the sun.*

Solution. *Given*: $l_{max} = 0.49\ \mu\text{m}$

(*i*) The surface temperature of the sun, *T*:

According to Wien's displacement law,

$$\lambda_{max} T = 2898\ \mu mK$$

$$\therefore \quad T = \frac{2898}{\lambda_{\text{max}}} = \frac{2898}{0.48} = \mathbf{5914\ K\ \ (Ans.)}$$

(*ii*) The heat flux at the surface of the sun, $(E)_{sun}$:

$$(E)_{sun} = \sigma T^4 = 5.67 \times 10^{-8}\ T^4 = 5.67 \left(\frac{T}{100}\right)^4$$

$$= 5.67 \times \left(\frac{5914}{100}\right)^4 = \mathbf{6.936 \times 10^7\ W/m^2\ (Ans.)}$$

Example 11.3. *Calculate the following for an industrial furnace in the form of a black body and emitting radiation at 2500°C :*

(*i*) *Monochromatic emissive power at 1.2 μm length,*

(*ii*) *Wavelength at which the emission is maximum,*

(*iii*) *Maximum emissive power,*

(*iv*) *Total emissive power, and*

(*v*) *Total emissive power of the furnace if it is assumed as a real surface with emissivity equal to 0.9.*

Solution. *Given* : $T = 2500 + 273 = 2773 K$; $\lambda = 1.2\ \mu m$, $\varepsilon = 0.9$

(*i*) **Monochromatic emissive power at 1.2 μm length, $(E_\lambda)_b$:**

According to Planck's law,

$$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right) - 1} \qquad \text{...[Eqn. (11.15)}$$

where, $C_1 = 3.742 \times 10^8$ W.μm^4/m^2 = 0.3742×10^{-15} W.m^4/m^2, and

$C_2 = 1.4388 \times 10^{-2}$ mK

Substituting the values, we get

$$(E_\lambda)_b = \frac{0.3742 \times 10^{-15} \times (1.2 \times 10^{-6})^{-5}}{\exp\left(\frac{1.4388 \times 10^{-2}}{1.2 \times 10^{-6} \times 2773}\right) - 1} = \frac{1.5 \times 10^{14}}{74.48} = \mathbf{2.014 \times 10^{12}\ W/m^2\ (Ans.)}$$

(*ii*) Wavelength at which the emission is maximum, λ_{max}:

Accodrding to Wien's displacement law,

$$\lambda_{max} = \frac{2898}{T} = \frac{2898}{2773} = \mathbf{1.045\ \mu m\ \ (Ans.)}$$

(*iii*) Maximum emissive power, $(E_{\lambda b})$max:

$$(E_{\lambda b})_{\text{max}} = 1.285 \times 10^{-5}\ \text{T}^5\ \text{W/m}^2 \text{ per metre length} \qquad \text{[Eqn. (11.19)]}$$

$$= 1.285 \times 10^{-5} \times (2773)^5 = \mathbf{2.1 \times 10^{12}\ W/m^2\ per\ metre\ length\ (Ans.)}$$

[**Note:** At high temperature the difference between $(E_\lambda)_b$ and $(E_{\lambda b})_{max}$ is very small].

(*iv*) Total emissive power, E_b :

$$E_b = \sigma T^4 = 5.67 \times 10^{-8} (2773)^4 = 5.67 \left(\frac{2773}{100}\right)^4 = \mathbf{3.352 \times 10^6\ W/m^2.\ (Ans.)}$$

(*v*) Total emissive power, E with emissivity (ε) = 0.9 :

$$E = \varepsilon\, \sigma T^4 = 0.9 \times 5.67 \left(\frac{2773}{100}\right)^4 = \mathbf{3.017 \times 10^6\ W/m^2.\ (Ans.)}$$

Example 11.4. *Assuming the sun (diameter = 1.4 × 10^9 m) as a black body having a surface temperature of 5750 K and at a mean distance of 15 × 10^{10} m from the earth (diameter = 12.8 × 10^6 m), estimate the following:*

(i) *The total energy emitted by the sun,*

(ii) *The emission received per* m^2 *just outside the atmosphere of the earth,*

(iii) *The total energy received by the earth if no radiation is blocked by the atmosphere of the earth, and*

(iv) *The energy received by a 1.6 m × 1.6 m solar collector whose normal is inclined at 50° to the sun. The energy loss through the atmosphere is 42 percent and diffuse radiation is 22 percent of direct radiation.*

Solution: Radius of the sun, $r_s = \dfrac{1.4 \times 10^9}{2} = 0.7 \times 10^9$ m

Mean distance of the sun from the earth,

$$R = 15 \times 10^{10} \text{ m}$$

Radius of the earth $\qquad r_e = \dfrac{12.8 \times 10^6}{2} = 6.4 \times 10^6$ m

Surface temperature of the sun, $\qquad T = 5750\ K$

(i) **The total energy emitted by the sun, E_b:**

$$E_b = \sigma A T^4 = 5.67 \times 10^{-8} \times 4\pi\, r_s^2 \times (5750)^4$$

$$= 5.67 \times 4\pi \times (0.7 \times 10^9)^2 \times \left(\frac{5750}{100}\right)^4$$

$$= \mathbf{3.816 \times 10^{26}\ W\ (Ans.)}$$

(*ii*) The emission received per m^2:

The sun may be regarded as a point source at a distance of 15×10^{10} m from the earth. The *mean area* just outside the earth's atmosphere over which the radiation will fall is

$$= 4\pi R^2 = 4\pi \times (15 \times 10^{10})^2 \text{ m}^2$$

∴ The emission received outside the earth's atmosphere

$$= \frac{3.816 \times 10^{26}}{4\pi \times (15 \times 10^{10})^2} = \mathbf{1349.6\ W/m^2\ (Ans.)}$$

(*iii*) The total energy received by the earth:

Assuming the earth a spherical body, the energy received by it will be proportional to the perpendicular projected area, *i.e.*, that of a circle ($= \pi r_e^2$).

$$\therefore \quad \text{Energy received by the earth} = 1349.6 \times \pi \times (6.4 \times 10^6)^2$$

$$= \mathbf{1.736 \times 10^{17}\ W\ (Ans.)}$$

(*iv*) The energy received by the solar collector:

The direct energy reaching the earth $= (1 - 0.42) \times 1349.6 = 782.77$ W/m^2

The diffuse radiation $= 0.22 \times 782.77 = 172.21$ W/m^2

∴ Total radiation reaching the collector $= 782.77 + 172.21 \simeq 955$ W/m^2

The projected area $= A \cos\theta = 1.6 \times 1.6 \times \cos 40° = 1.961$ m^2

∴ Energy received by the solar collector

$$= 955 \times 1.961 = \mathbf{1872.7\ W\ (Ans.)}$$

HIGHLIGHTS

1. '*Radiation*' *heat transfer* is defined as "the transfer of energy across a system boundary by means of an electromagnetic mechanism which is caused solely by a temperature difference.
2. The *emissive power* (E) is defined as the total amount of radiation emitted by a body per unit area per unit time; it is expressed in W/m^2.
3. *Emissivity* (ε) is defined as the ability of the surface of a body to radiate heat. It is also defined as the ratio of the emissive power of any body to the emissive power of a black body of equal temperature $\left(i.e., \ \varepsilon = \dfrac{E}{E_b}\right)$.
4. A *black body* is one which neither reflects nor transmits any part of the incident radiation but absorbs all of it.
5. A *gray body* is one whose absorptivity of a surface does not vary with temperature and wavelength of the incident radiation $[\alpha = (\alpha)_\lambda = \text{constant}]$.
6. The *Stefan-Boltzmann* law states that the emissive power of a black body is directly proportional to the fourth power of its absolute temperature.

   i.e., $E_b = \sigma T^4$

   where, E_b = Emissive power of a black body, and

   σ = Stefan-Boltzmann constant = $5.67 \times 10^{-8}\ W/m^2 K^4$
7. *Kirchhoff's law* states that at any temperature the ratio of total emissive power E to the total absorptivity α is constant for all substances which are in thermal equilibrium with their environment.
8. *Planck's law* is given by:

   $$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{\exp\left[\dfrac{C_2}{\lambda T}\right] - 1}$$

   where, $C_1 = 2\pi c^2 h = 3.742 \times 10^8\ W.\mu m^4/m^2$;

$$C_2 = \frac{ch}{k} = 1.4388 \times 10^4 \ \mu\text{mK}$$

[C = velocity of light in vacuum $\simeq 3 \times 10^8$ m/s; h = Planck's constant = 6.625×10^{-34} Js; k = Boltzmann constant = 1.3805×10^{-23} J/K]

9. *Wien's displacement law* states that the product of λ_{max} and T is constant *i.e.*,

$$\lambda_{max} T = \text{constant } (\simeq 2900 \ \mu \ m \ \text{K})$$

A combination of Planck's law and Wien's displacement law yields the condition for the maximum monochromatic emissive power for a black body.

$$(E_{\lambda b})_{max} = 1.285 \times 10^{-5} \ \text{T}^5 \ \text{W/m}^2 \text{ per metre length.}$$

10. The *intensity of radiation* (I) is defined as the rate of energy leaving a space in a given direction per unit solid angle per unit area of the emitting surface normal to the mean direction in space.

11. *Lambert's cosine law* states that the total emissive power E_θ from a radiating plane surface in any direction is directly proportional to the cosine of the angle of emission.

or $$E_\theta = E_n \cos \theta \qquad \text{... (true only for diffuse radiation surface)}$$

where E_n is the total emissive power of the radiating surface in the direction of its normal.

HIGHLIGHTS

List of formulae :

1. $F_{1-2} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{\pi r^2}$
2. $F_{1-1} + F_{1-2} + F_{1-3} + \ldots F_{1-m} = 1$
3. $A_1 F_{1-2} = A_2 F_{2-1}$ ... Reciprocity theorem
4. $F_{1-1} = 0$ for convex and flat surface
5. For *two black bodies* :

$$(Q_{12})_{net} = A_1 F_{1-2} \sigma (T_1^4 - T_2^4)$$

6. For *two gray bodies* :

$$(Q_{12})_{net} = A_1 (F_g)_{1-2} \sigma (T_1^4 - T_2^4)$$

where $$(F_g)_{1-2} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{1-2}} + \frac{1-\varepsilon_2}{\varepsilon_2} \cdot \frac{A_1}{A_2}}$$

7. Two black surfaces connected by a single refractory surface :

$$(Q_{12})_{net} = A_1 \overline{F}_{1-2} (E_{b_1} - E_{b_2}) = A_1 F_{1-2} \sigma (T_1^4 - T_2^4)$$

where, $$\overline{F}_{1-2} = F_{1-2} + \left[\frac{1}{\frac{1}{(1 - F_{1-2})} + \frac{A_1}{A_2}\left(\frac{1}{1 - F_{2-1}}\right)} \right]$$

$$= \frac{A_2 - A_1 F_{1-2}^2}{A_1 + A_2 - 2A_1 F_{1-2}}$$

8. Two gray surfaces connected by a refractory surface :

$$(Q_{12})_{net} = A_1 (F_g)_{1-2} (E_{b1} - E_{b2}) = A_1 (F_g)_{1-2} \sigma (T_1^4 - T_2^4)$$

where,

$$(F_g)_{1-2} = \left[\frac{1}{\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right) + \frac{1}{\bar{F}_{1-2}}} \right]$$

and,

$$\bar{F}_{1-2} = \frac{A_2 - A_1 F_{1-2}^2}{A_1 + A_2 - 2A_1 F_{1-2}}$$

9. Radiation from gases and vapours

$$Q_{net} = \sigma A (\varepsilon_g T_g^4 - \alpha_g T_w^4).$$

Example 12.1. *Assuming the sun to radiate as a black body, calculate its temperature from the data given below:*

The average radiant energy flux incident upon the earth's atmosphere (solar constant) = 1380 W/m²

Radius of the sun = 7.0×10^8 m

Distance between the sun and the earth = 15×10^{10} m

Solution. *Given:* Solar constant = 1380 W/m²; r_s (radius of the sun) = 7×10^8 m

r (distance between the sun and the earth) = 15×10^{10} m

Surface temperature of the sun, T :

Refer Fig. 12.6. The heat flow from small area dA_1 (on the surface of sun) to the small area dA_2 (on the surface of the earth), is given by,

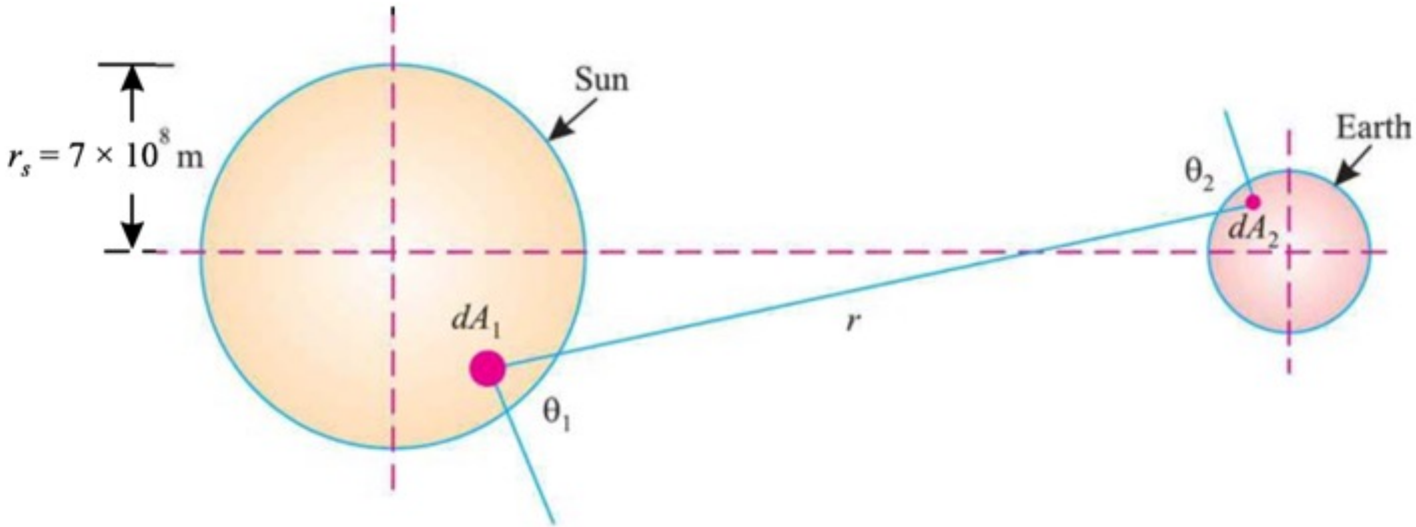


Fig. 12.6

$$dQ_{1-2} = \frac{I_{b1} \cos \theta_1 \cos \theta_2 \, dA_1 \, dA_2}{r^2} \qquad \text{...(Eqn. (12.2)]}$$

or,
$$\frac{dQ_{1-2}}{dA_2 \cos \theta_2} = \frac{E_b}{\pi r^2} dA_1 \cos \theta_1 \qquad \left(\because I_b = \frac{E_b}{\pi}\right)$$

Integrating both sides, we get

$$\int \frac{dQ_{1-2}}{dA_2 \cos \theta_2} = \frac{E_b}{\pi r^2} \int dA_1 \cos \theta_1$$

$$\text{Also, solar constant} = \int \frac{dQ_{1-2}}{dA_2 \cos \theta_2}$$

$\therefore$
$$\text{Solar constant} = \frac{E_b}{\pi r^2} \int dA_1 \cos \theta_1$$

$\int dA_1 \cos \theta_1 = A_1 = \pi r_s^2$, here θ_1 is taken as zero because all rays from the sun falling on the earth due to extremely long distance are considered parallel to each other; therefore, $\cos 0° = 1$.

$$\therefore \quad \text{Solar constant} = \frac{E_b}{\pi r^2} \times \pi r_s^2 = 1380$$

But,
$$E_b = \sigma T^4$$

$$\therefore \quad \frac{\sigma T^4}{\pi r^2} \times \pi r_s^2 = 1380$$

or,
$$\sigma T^4 = 1380 \times \left(\frac{r}{r_s}\right)^2$$

or,
$$5.67 \times \left(\frac{T}{100}\right)^4 = 1380 \times \left(\frac{15 \times 10^{10}}{7 \times 10^8}\right)^2$$

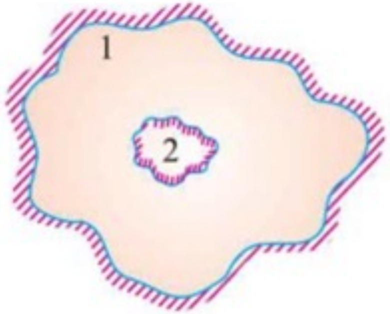
or,
$$\left(\frac{T}{100}\right)^4 = 1117.58 \times 10^4 \text{ or } \frac{T}{100} = (1117.58 \times 10^4)^{1/4} = 57.82$$

or,
$$T = \mathbf{5782\ K\ (Ans.)}$$

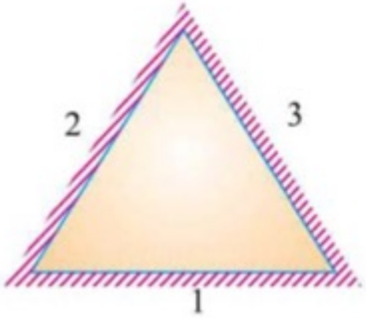
Example 12.2. *Calculate the shape factors for the configurations shown in the Fig. 12.7.*

Solution. The shape factors can be worked out by using *summation rule*, the *reciprocity theorem* and from the *inspection of geometry*.

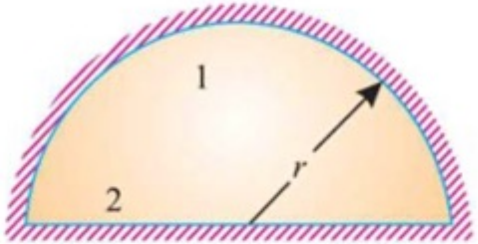
(*i*) *A black body inside a black enclosure:*



A black body inside a black enclosure
(*i*)



A tube with cross-section of an equilateral triangle
(*ii*)



Hemispherical surface and a plane surface
(*iii*)

Fig. 12.7

$$F_{2-1} = 1$$ ...Because all radiation emanating from the black surface is intercepted by the enclosing surface 1.

$$F_{1-1} + F_{1-2} = 1$$ ... By *summation rule* for radiation from surface 1

$$A_1 F_{1-2} = A_2 F_{2-1}$$ ... By *reciprocity theorem*

or,
$$F_{1-2} = \frac{A_2}{A_1} F_{2-1}$$

∴
$$F_{1-1} = 1 - F_{1-2} = 1 - \frac{A_2}{A_1} F_{2-1} = 1 - \frac{A_2}{A_1} \qquad (\because F_{2-1} = 1)$$

Hence,
$$F_{1-1} = \mathbf{1 - \frac{A_2}{A_1}} \quad \textbf{(Ans.)}$$

(*ii*) *A tube with cross-section of an equilateral triangle:*

$$F_{1-1} + F_{1-2} + F_{1-3} = 1$$ ... By summation rule

$$F_{1-1} = 0$$ ... Because the flat surface 1 *cannot see itself.*

$$\therefore \quad F_{1-2} + F_{1-3} = 1$$

$$F_{1-2} = F_{1-3} = \mathbf{0.5\ (Ans.)}$$ ... By symmetry

Similarly, considering radiation from surface 2 :

$$F_{2-1} + F_{2-2} + F_{2-3} = 1$$

or, $$F_{2-1} + F_{2-3} = 1 \qquad (\because F_{2-2} = 0)$$

or, $$F_{2-3} = 1 - F_{2-1}$$

$$A_1 F_{1-2} = A_2 F_{2-1}$$ ... By reciprocity theorem

or, $$F_{2-1} = \frac{A_1}{A_2} F_{1-2} = F_{1-2} \qquad (\because A_1 = A_2)$$

$$\therefore \quad F_{2-3} = 1 - F_{1-2} = 1 - 0.5 = \mathbf{0.5\ (Ans.)}$$

(*iii*) *Hemispherical surface and a plane surface:*

$$F_{1-1} + F_{1-2} = 1$$ ... By summation rule

$$A_1 F_{1-2} = A_2 F_{2-1}$$ ... By reciprocity theorem

or, $$F_{1-2} = \frac{A_2}{A_1} F_{2-1}$$

But, $$F_{2-1} = 1$$ ... Because all radiation emanating from the black surface 2 are interecpted by the enclosing surface 1.

$$\therefore \quad F_{1-2} = \frac{A_2}{A_1} = \frac{\pi r^2}{2 \pi r^2} = \mathbf{0.5\ (Ans.)}$$

Thus in case of a hemispherical surface half the radiation falls on surface 2 and the other half is intercepted by the hemisphere itself.

Example 12.3. *Explain the meaning of the term geometric factor in relation to heat exchange by radiation. Derive an expression for the geometric factor F_{11} for the inside surface of a black hemispherical cavity of radius R with respect to itself.* **(U.P.S.C., 1994)**

Solution. • **Geometric factor** *is defined as the fraction of radiative energy that is diffused from one surface element and strikes the other surface directly with no intervening reflection.*

- The *geometric factor* depends only on the *specific geometry of the emitter and the collection surfaces.*
- The geometric factor is represented by the symbol F_{i-j} which means the shape factor from a surface A_i to another surface A_j. Thus, the geometric factor F_{1-2} of surface A_1 to surface A_2 is

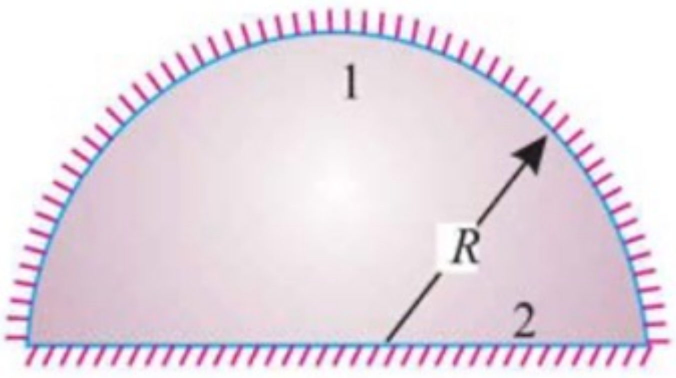


Fig. 12.8

$$F_{1-2} = \frac{\text{Direct radiation from surface 1 incident upon surface 2}}{\text{Total radiation from emitting surface}}$$

Geometric factor F_{1-1} for the inside surface of a black hemispherical cavity of radius R with respect to itself.

$$F_{1-1} = 1 - \frac{A_2}{A_1} = 1 - \frac{\pi R^2}{2\pi R^2} = 1 - \frac{1}{2} = \mathbf{0.5\ (Ans.)}$$

Example 12.4. *Derive expressions for shape factors of the cavities (each enclosed on its surface with a flat surface) shown in the Fig. 12.9. Also, calculate the net radiative heat transfer from the cavities, if h = 20 cm, d = 15 cm, temperature inside surface of each cavity = 400° C and the emissivity of each cavity surface is 0.8.*

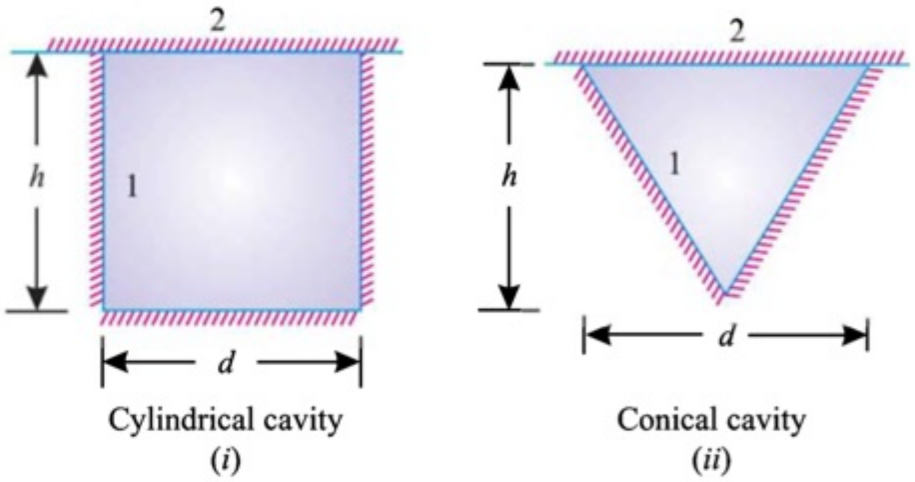


Fig. 12.9

Solution :

(i) Cylindrical cavity:

$$F_{1-1} + F_{1-2} = 1 \quad \text{... By summation rule}$$

or, $$F_{1-1} = 1 - F_{1-2}$$

Also, $$F_{2-1} + F_{2-2} = 1 \quad \text{... By summation rule}$$

$$F_{2-2} = 0$$ ... Being a flat surface (flat surface cannot see itself).

$$F_{2-1} = 1$$ ... Because all radiation emitted by the black surface 2 is intercepted by the enclosing surface 1.

$$A_1 F_{1-2} = A_2 F_{2-1} \quad \text{... By reciprocity theorem}$$

or, $$F_{1-2} = \frac{A_2}{A_1} F_{2-1} = \frac{A_2}{A_1}$$

or,
$$F_{1-1} = 1 - \frac{\frac{\pi}{4} d^2}{\frac{\pi}{4} d^2 + \pi dh} = 1 - \frac{d}{d + 4h} = \frac{d + 4h - d}{4h + d} = \frac{\mathbf{4h}}{\mathbf{4h + d}} \quad \textbf{(Ans.)}$$

(*ii*) *Conical cavity:*

$$F_{1-1} = 1 - \frac{A_2}{A_1}$$

... This relation (calculated above) is applicable in this case (and all such cases) also.

$$= 1 - \frac{\frac{\pi}{4} d^2}{\left[\frac{\pi d \times \text{slant height}}{2}\right]} = 1 - \frac{\frac{\pi}{4} d^2}{\frac{\pi d}{2} \times \left[\sqrt{h^2 + \left(\frac{d}{2}\right)^2}\right]}$$

or,
$$F_{1-1} = 1 - \frac{\mathbf{d}}{\sqrt{\mathbf{4h^2 + d^2}}} \quad \textbf{(Ans.)}$$

Net radiative heat transfer:

The net radiative heat transfer from a cavity can be calculated by using the following formulae:

$$Q_1 = A_1\,\varepsilon_1\,\sigma\,T_1^4\left[\frac{1 - F_{1-1}}{1 - (1 - \varepsilon_1)\,F_{1-1}}\right] \quad ...(12.17)$$

(*i*) *Cylindrical cavity:*

$$F_{1-1} = \frac{4h}{4h + d} = \frac{4 \times 0.2}{4 \times 0.2 + 0.15} = 0.842$$

$$Q_1 = \left[\frac{\pi}{4} \times (0.15)^2 + \pi \times 0.15 \times 0.2\right] \times 0.8 \times 5.67 \times \left[\frac{(400 + 273)}{100}\right]^4 \left[\frac{1 - 0.842}{1 - (1 - 0.8) \times 0.842}\right]$$

$$= 0.1119 \times 4.536 \times 2051.45 \times 0.19 = \mathbf{197.84\ W\ (Ans.)}$$

(*ii*) *Conical cavity:*

$$F_{1-1} = 1 - \frac{d}{\sqrt{4h^2 + d^2}} = 1 - \frac{0.15}{\sqrt{4 \times 0.2^2 + 0.15^2}} = 0.649$$

$$Q_1 = \frac{\pi \times 0.15}{2} \times \left[\sqrt{(0.2)^2 + \left(\frac{0.15}{2}\right)^2}\right] \times 0.8 \times 5.67 \times \left[\frac{400 + 273}{100}\right]^4 \left[\frac{1 - 0.649}{1 - (1 - 0.8 \times 0.649)}\right]$$

$$= 0.0503 \times 0.8 \times 5.67 \times 2051.45 \times 0.403 = \mathbf{188.63\ W\ (Ans.)}$$

Example 12.5. *A small sphere (outside diameter = 60 mm) with a surface temperature of 300° C is located at the geometric centre of a large sphere (inside diameter = 360 mm) with an inner surface temperature of 15° C. Calculate how much of emission from the inner surface of the large sphere is incident upon the outer surface of the small sphere; assume that both sides approach black body behaviour.*

What is the net interchange of heat between the two spheres?

60 ... = 30 mm = 0.03 m; r_2 (large sphere) = $\frac{360}{2}$ = 180 mm = 0.18 m.

Since all the radiation being emitted by the small sphere is incident upon and absorbed by the inner surface of the large sphere, therefore, configuration factor between 1 and 2 is $F_{1-2} = 1$.

Now, $$A_1 F_{1-2} = A_2 F_{2-1} \quad \text{... Reciprocity theorem}$$

or, $$4 \pi r_1^2 \times F_{1-2} = 4 \pi r_2^2 \times F_{2-1}$$

$\therefore$ $$F_{2-1} = F_{1-2} \times \frac{4 \pi r_1^2}{4 \pi r_2^2} = 1 \times \frac{r_1^2}{r_2^2} = \left(\frac{0.03}{0.18}\right)^2 = \mathbf{0.0278} \text{ } \mathbf{(Ans.)}$$

Thus 2.78% of the emission from the inner surface of the large sphere is incident upon the small sphere and absorbed by it.

Also, $F_{2-1} + F_{2-2} = 1$ ... From energy balance for the large sphere.

or, $F_{2-2} = 1 - F_{2-1} = 1 - 0.0278 = 0.9722$

Thus, 97.22% of emission from the large sphere is absorbed by the inner surface of the sphere itself.

∴ The net interchange of heat between the two spheres is,

$$Q_{net} = F_{1-2}\, A_1\, \sigma\, (T_1^4 - T_2^4)$$

$$= 1 \times (4\pi \times 0.03^2) \times 5.67 \left[\left(\frac{300 + 273}{100}\right)^4 - \left(\frac{15 + 273}{100}\right)^4\right]$$

$$= 0.0113 \times 5.67 \times 1009.2 = \mathbf{64.66\ W\ (Ans.)}$$

Example 12.6. *A 70 mm thick metal plate with a circular hole of 35 mm diameter along the thickness is maintained at a uniform temperature 250° C. Find the loss of energy to the surroundings at 27° C, assuming the two ends of the hole to be as parallel discs and the metallic surfaces and surroundings have black body characteristics.*

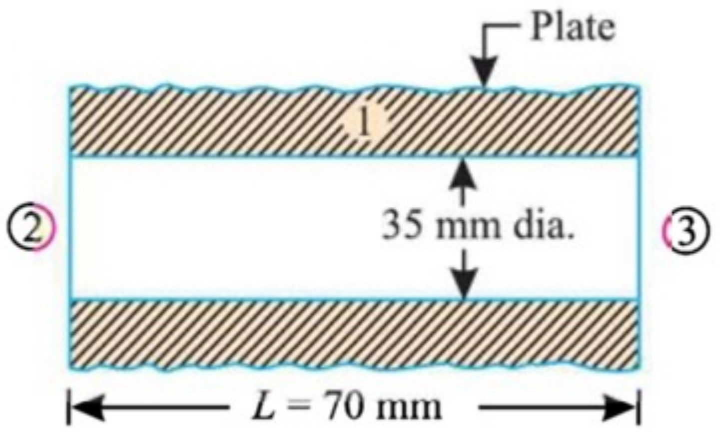


Fig. 12.10

Solution. *Given:* $r_2 = (r_3) = \frac{35}{2} = 17.5$ mm $= 0.0175$ m, $L = 70$ mm $= 0.07$ m, $T_1 = 250 + 273 = 523$ K

$$T_{surr.} = 27 + 273 = 300 \text{ K.}$$

Refer Fig. 12.10. Let suffix 1 designate the cavity and the suffices 2 and 3 denote the two ends of the 35 mm dia. hole which are behaving as discs. Thus,

$$\frac{L}{r_2} = \frac{0.07}{0.0175} = 4; \frac{r_3}{L} = \frac{0.0175}{0.07} = 0.25$$

With reference to Fig. 12.3, the configuration factor, F_{2-3} is 0.065

Now, $F_{2-1} + F_{2-2} + F_{2-3} = 1$ ... By summation rule

But, $F_{2-2} = 0$

$\therefore$ $F_{2-1} = 1 - F_{2-3} = 1 - 0.065 = 0.935$

Also, $A_1 F_{1-2} = A_2 F_{2-1}$ ... By reciprocating theorem or, $$F_{1-2} = \frac{A_2 \; F_{2-1}}{A_1} = \frac{\pi \times (0.0175)^2 \times 0.935}{\pi \times 0.035 \times 0.07} = 0.1168$$

$$F_{1-3} = F_{1-2} = 0.1168 \qquad \text{... By symmetry}$$

The total loss of energy = Loss of heat by both ends

$$= A_1 \, F_{1-2} \, \sigma \, (T_1^{\,4} - T_{surr}^{\,4}) + A_1 \, F_{1-3} \, \sigma \, (T_1^{\,4} - T_{surr}^{\,4})$$

$$= 2 \, A_1 \, F_{1-2} \, \sigma \, (T_1^{\,4} - T_{surr}^{\,4}) \qquad (\because \; F_{1-2} = F_{1-3})$$

$$= 2 \times (\pi \times 0.035 \times 0.07) \times 0.1168 \times 5.6 \left[\left(\frac{523}{100} \right)^4 - \left(\frac{300}{100} \right)^4 \right] = \mathbf{6.8 \; W \; (Ans.)}$$

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