

# WHOLE NUMBERS

## 1. Definition

The **whole numbers** are the numbers without fractions and it is a collection of positive integers and zero.

It is represented by the symbol “**W**” and the set of numbers are {0, 1, 2, 3, 4, 5, 6,.....}.

## 2. Symbol

The symbol to represent whole numbers is the alphabet ‘W’ in capital letters.

$$W = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots\}$$

Thus, the **whole numbers list** includes 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, ....

Whole Numbers:  $W = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots\}$

Natural Numbers:  $N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, \dots\}$

Integers:  $Z = \{\dots -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \dots\}$

## 3. Properties

The properties of whole numbers are based on arithmetic operations such as **addition, subtraction, division and multiplication**.

### a. Closure Property

They can be closed under addition and multiplication.

If  $x$  and  $y$  are two whole numbers then  $x \cdot y$  or  $x + y$  is also a whole number.

**Example:**

2 and 7 are whole numbers.

$2 + 7 = 9$ ; a whole number

$2 \times 7 = 14$ ; a whole number

Therefore, the whole numbers are closed under addition and multiplication.

## **b. Commutative Property of Addition and Multiplication**

The sum and product of two whole numbers will be the same whatever the order they are added or multiplied in.

If  $x$  and  $y$  are two whole numbers, then  $x + y = y + x$  and  $x \cdot y = y \cdot x$

### **Example:**

Consider two whole numbers 4 and 6.

$$4 + 6 = 10$$

$$6 + 4 = 10$$

Thus,  $4 + 6 = 6 + 4$ .

Also,

$$4 \times 6 = 24$$

$$6 \times 4 = 24$$

Thus,  $4 \times 6 = 6 \times 4$

Therefore, the whole numbers are commutative under addition and multiplication.

## **c. Additive identity**

When a whole number is added to 0, its value remains unchanged.

If  $x$  is a whole number then  $x + 0 = 0 + x = x$

**Example:**

Consider two whole numbers 0 and 5.

$$0 + 5 = 5$$

$$5 + 0 = 5$$

Here,  $0 + 5 = 5 + 0$

Therefore, 0 is called the additive identity of whole numbers.

**d. Multiplicative identity**

When a whole number is multiplied by 1, its value remains unchanged.

If  $x$  is a whole number then  $x \cdot 1 = x = 1 \cdot x$

**Example:**

Consider two whole numbers 1 and 27.

$$1 \times 27 = 27$$

$$27 \times 1 = 27$$

Here,  $1 \times 27 = 27 \times 1$

Therefore, 1 is the multiplicative identity of whole numbers.

**e. Associative Property**

When whole numbers are being added or multiplied as a set, they can be grouped in any order, and the result will be the same.

If  $x$ ,  $y$  and  $z$  are whole numbers then  $x + (y + z) = (x + y) + z$  and  $x \cdot (y \cdot z) = (x \cdot y) \cdot z$

**Example:**

Consider three whole numbers 2, 3, and 4.

$$2 + (3 + 4) = 2 + 7 = 9$$

$$(2 + 3) + 4 = 5 + 4 = 9$$

Thus,  $2 + (3 + 4) = (2 + 3) + 4$

$$2 \times (3 \times 4) = 2 \times 12 = 24$$

$$(2 \times 3) \times 4 = 6 \times 4 = 24$$

Here,  $2 \times (3 \times 4) = (2 \times 3) \times 4$

Therefore, the whole numbers are associative under addition and multiplication.

## f. Distributive Property

If  $x$ ,  $y$  and  $z$  are three whole numbers, the distributive property of multiplication over addition is  $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$ , similarly, the distributive property of multiplication over subtraction is  $x \cdot (y - z) = (x \cdot y) - (x \cdot z)$

### Example:

Let us consider three whole numbers 7, 10 and 6.

$$7 \times (10 + 6) = 7 \times 16 = 112$$

$$(7 \times 10) + (7 \times 6) = 70 + 42 = 112$$

Here,  $7 \times (10 + 6) = (7 \times 10) + (7 \times 6)$

Also,

$$7 \times (10 - 6) = 7 \times 4 = \dots \text{ (let this section as a practice)}$$

$$(7 \times 10) - (7 \times 6) = \dots$$

So,  $\dots = \dots$

Hence, verified the distributive property of whole numbers.

### **g. Multiplication by zero**

When a whole number is multiplied to 0, the result is always 0.

**Example:**

$$0 \times 9 = 0$$

$$9 \times 0 = 0$$

Here,  $0 \times 9 = 9 \times 0$

Thus, for any whole number multiplied by 0, the result is always 0.

### **h. Division by zero**

The division of a whole number by 0 is not defined.

## **4. Practice Problems**

1. How many whole numbers are there between -3 and 10?
2. What is the additive inverse of the whole number 75?