CHAPTER 3

LOAD FLOW ANALYSIS

[**CONTENTS:** *Review of solution of equations, direct and iterative methods, classification of buses, importance of slack bus and Y_{BUS} based analysis, constraints involved, load flow equations, GS method: algorithms for finding the unknowns, concept of acceleration of convergence, NR method- algorithms for finding the unknowns, tap changing transformers, Fast decoupled load flow, illustrative examples*]

REVIEW OF NUMERICAL SOLUTION OF EQUATIONS

The numerical analysis involving the solution of algebraic simultaneous equations forms the basis for solution of the performance equations in computer aided electrical power system analyses, such as during linear graph analysis, load flow analysis (nonlinear equations), transient stability studies (differential equations), etc. Hence, it is necessary to review the general forms of the various solution methods with respect to all forms of equations, as under:

1. **Solution Linear equations**:

   * **Direct methods**:

        - Cramer's (Determinant) Method,

        - Gauss Elimination Method (only for smaller systems),

        - LU Factorization (more preferred method), etc.

   * **Iterative methods**:

        - Gauss Method

        - Gauss-Siedel Method (for diagonally dominant systems)

2. **Solution of Nonlinear equations:**

   Iterative methods only:

        - Gauss-Siedel Method (for smaller systems)

        - Newton-Raphson Method (if corrections for variables are small)

3. **Solution of differential equations:**

   Iterative methods only:

        - Euler and Modified Euler method,

        - RK IV-order method,

        - Milne's predictor-corrector method, etc.

It is to be observed that the nonlinear and differential equations can be solved only by the iterative methods. The iterative methods are characterized by the various performance features as under:

- Selection of initial solution/ estimates
- Determination of fresh/ new estimates during each iteration
- Selection of number of iterations as per tolerance limit
- Time per iteration and total time of solution as per the solution method selected
- Convergence and divergence criteria of the iterative solution
- Choice of the Acceleration factor of convergence, etc.

A comparison of the above solution methods is as under:

- In general, the direct methods yield exact or accurate solutions. However, they are suited for only the smaller systems, since otherwise, in large systems, the possible round-off errors make the solution process inaccurate.
- The iterative methods are more useful when the diagonal elements of the coefficient matrix are large in comparison with the off diagonal elements. The round-off errors in these methods are corrected at the successive steps of the iterative process.
- The Newton-Raphson method is very much useful for solution of non –linear equations, if all the values of the corrections for the unknowns are very small in magnitude and the initial values of unknowns are selected to be reasonably closer to the exact solution.

LOAD FLOW STUDIES

Introduction: Load flow studies are important in planning and designing future expansion of power systems. The study gives steady state solutions of the voltages at all the buses, for a particular load condition. Different steady state solutions can be obtained, for different operating conditions, to help in planning, design and operation of the power system.

Generally, load flow studies are limited to the transmission system, which involves bulk power transmission. The load at the buses is assumed to be known. Load flow

studies throw light on some of the important aspects of the system operation, such as: violation of voltage magnitudes at the buses, overloading of lines, overloading of generators, stability margin reduction, indicated by power angle differences between buses linked by a line, effect of contingencies like line voltages, emergency shutdown of generators, etc. Load flow studies are required for deciding the economic operation of the power system. They are also required in transient stability studies. Hence, load-flow studies play a vital role in power system studies.

Thus the load flow problem consists of finding the power flows (real and reactive) and voltages of a network for given bus conditions. At each bus, there are four quantities of interest to be known for further analysis: the real and reactive power, the voltage magnitude and its phase angle. Because of the nonlinearity of the algebraic equations, describing the given power system, their solutions are obviously, based on the iterative methods only. The constraints placed on the load flow solutions could be:

- The Kirchhoff's relations holding good,
- Capability limits of reactive power sources,
- Tap-setting range of tap-changing transformers,
- Specified power interchange between interconnected systems,
- Selection of initial values, acceleration factor, convergence limit, etc.

Classification of buses for LFA: Different types of buses are present based on the specified and unspecified variables at a given bus as presented in the table below:

Table 1.   Classification of buses for LFA

Sl. No.	Bus Types	Specified Variables	Unspecified variables	Remarks
1	Slack/ Swing Bus	$|V|$, δ	P_G, Q_G	$|V|$, δ: are assumed if not specified as 1.0 and 0^0
2	Generator/ Machine/ PV Bus	P_G, $|V|$	Q_G, δ	A generator is present at the machine bus
3	Load/ PQ Bus	P_G, Q_G	$|V|$, δ	About 80% buses are of PQ type
4	Voltage Controlled Bus	P_G, Q_G, $|V|$	δ, a	'a' is the % tap change in tap-changing transformer

Importance of swing bus: The slack or swing bus is usually a PV-bus with the largest capacity generator of the given system connected to it. The generator at the swing bus supplies the power difference between the "specified power into the system at the other buses" and the "total system output plus losses". Thus swing bus is needed to supply the additional real and reactive power to meet the losses. Both the magnitude and phase angle of voltage are specified at the swing bus, or otherwise, they are assumed to be equal to 1.0 p.u. and 0^0 , as per flat-start procedure of iterative solutions. The real and reactive powers at the swing bus are found by the computer routine as part of the load flow solution process. It is to be noted that the source at the swing bus is a perfect one, called the swing machine, or slack machine. It is voltage regulated, i.e., the magnitude of voltage fixed. The phase angle is the system reference phase and hence is fixed. The generator at the swing bus has a torque angle and excitation which vary or swing as the demand changes. This variation is such as to produce fixed voltage.

Importance of Y_{BUS} based LFA: The majority of load flow programs employ methods using the bus admittance matrix, as this method is found to be more economical. The bus admittance matrix plays a very important role in load floe analysis. It is a complex, square and symmetric matrix and hence only $n(n+1)/2$ elements of Y_{BUS} need to be stored for a n-bus system. Further, in the Y_{BUS} matrix, Y_{ij} = 0, if an incident element is not present in the system connecting the buses 'i' and 'j'. since in a large power system, each bus is connected only to a fewer buses through an incident element, (about 6-8), the coefficient matrix, Y_{BUS} of such systems would be highly sparse, i.e., it will have many zero valued elements in it. This is defined by the sparsity of the matrix, as under:

Percentage sparsity of a given matrix of nth order:	$=$	$\dfrac{\text{Total no. of zero valued elements of } Y_{BUS}}{\text{Total no. of entries of } Y_{BUS}}$

$$\mathbf{S} \quad = \quad (\mathbf{Z} / \mathbf{n^2}) \text{ x } \mathbf{100} \text{ \%} \qquad\qquad (1)$$

The percentage sparsity of Y_{BUS}, in practice, could be as high as 80-90%, especially for very large, practical power systems. This sparsity feature of Y_{BUS} is extensively

used in reducing the load flow calculations and in minimizing the memory required to store the coefficient matrices. This is due to the fact that only the non-zero elements Y_{BUS} can be stored during the computer based implementation of the schemes, by adopting the suitable optimal storage schemes. While Y_{BUS} is thus highly sparse, it's inverse, Z_{BUS}, the bus impedance matrix is not so. It is a FULL matrix, unless the optimal bus ordering schemes are followed before proceeding for load flow analysis.

THE LOAD FLOW PROBLEM

Here, the analysis is restricted to a balanced three-phase power system, so that the analysis can be carried out on a single phase basis. The per unit quantities are used for all quantities. The first step in the analysis is the formulation of suitable equations for the power flows in the system. The power system is a large interconnected system, where various buses are connected by transmission lines. At any bus, complex power is injected into the bus by the generators and complex power is drawn by the loads. Of course at any bus, either one of them may not be present. The power is transported from one bus to other via the transmission lines. At any bus i, the complex power S_i (injected), shown in figure 1, is defined as
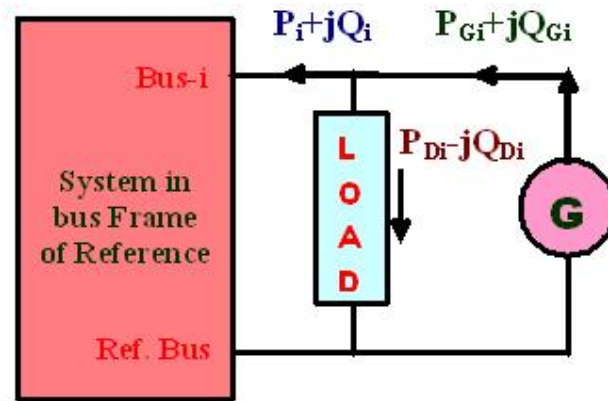
$$S_i = S_{Gi} - S_{Di} \qquad (2)$$

Fig.1   power flows at a bus-i

where S_i = net complex power injected into bus i, S_{Gi} = complex power injected by the generator at bus i, and S_{Di} = complex power drawn by the load at bus i. According to conservation of complex power, at any bus i, the complex power injected into the

bus must be equal to the sum of complex power flows out of the bus via the transmission lines. Hence,

$$S_i = \quad S_{ij} \qquad\qquad \forall\ i = 1, 2, \ldots\ldots..n \qquad\qquad (3)$$

where S_{ij} is the sum over all lines connected to the bus and n is the number of buses in the system (excluding the ground). The bus current injected at the bus-i is defined as

$$I_i = I_{Gi} - I_{Di} \qquad\qquad \forall\ i = 1, 2, \ldots\ldots..n \qquad\qquad (4)$$

where I_{Gi} is the current injected by the generator at the bus and I_{Di} is the current drawn by the load (demand) at that bus. In the bus frame of reference

$$I_{BUS} = Y_{BUS}\ V_{BUS} \qquad\qquad\qquad\qquad (5)$$

where

$$I_{BUS} = \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_n \end{bmatrix} \quad \text{is the vector of currents injected at the buses,}$$

Y_{BUS} is the bus admittance matrix, and

$$V_{BUS} = \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{bmatrix} \quad \text{is the vector of complex bus voltages.}$$

Equation (5) can be considered as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \qquad\qquad \forall\ i = 1, 2, \ldots\ldots..n \qquad\qquad (6)$$

The complex power S_i is given by

$$S_i = V_i\ I_i^{*}$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}\ V_j \right)^{*}$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}^{*} V_j^{*} \right) \qquad\qquad (7)$$

Let $\qquad V_i \triangleq |V_i| \angle u_i = |V_i|(\cos u_i + j \sin u_i)$

$$u_{ij} = u_i - u_j$$

$$Y_{ij} = G_{ij} + jB_{ij}$$

Hence from (7), we get,

$$S_i = \sum_{j=1}^{n} |V_i| |V_j| \left(\cos u_{ij} + j \sin u_{ij}\right) \left(G_{ij} - jB_{ij}\right) \qquad \textbf{(8)}$$

Separating real and imaginary parts in (8) we obtain,

$$P_i = \sum_{j=1}^{n} |V_i| |V_j| \left(G_{ij} \cos u_{ij} + B_{ij} \sin u_{ij}\right) \qquad \textbf{(9)}$$

$$Q_i = \sum_{j=1}^{n} |V_i| |V_j| \left(G_{ij} \sin u_{ij} - B_{ij} \cos u_{ij}\right) \qquad \textbf{(10)}$$

An alternate form of P_i and Q_i can be obtained by representing Y_{ik} also in polar form

as $\qquad Y_{ij} = |Y_{ij}| \angle_{ij} \qquad\qquad\qquad\qquad\qquad\qquad$ **(11)**

Again, we get from (7),

$$S_i = |V_i| \angle u_i \sum_{j=1}^{n} |Y_{ij}| \angle -_{ij} |V_j| \angle -u_j \qquad \textbf{(12)}$$

The real part of (12) gives P_i.

$$P_i = |V_i| \sum_{j=1}^{n} |Y_{ij}| |V_j| \cos\left(-_{ij} + u_i - u_j\right)$$

$$= |V_i| \sum_{j=1}^{n} |Y_{ij}| |V_j| \cos - \left(_{ij} - u_i + u_j\right) \qquad\qquad \text{or}$$

$$P_i = \sum_{j=1}^{n} |V_i| |V_j| |Y_{ij}| \cos\left(_{ij} - u_i + u_j\right) \qquad \forall\ i = 1, 2, \ldots\ldots\ldots n, \qquad \textbf{(13)}$$

Similarly, Q_i is imaginary part of (12) and is given by

$$Q_i = |V_i| \sum_{j=1}^{n} |Y_{ij}| |V_j| \sin - \left(_{ij} - u_i + u_j\right) \qquad\qquad \text{or}$$

$$Q_i = -\sum_{j=1}^{n} |V_i| |V_j| |Y_{ij}| \sin\left(_{ij} - u_i + u_j\right) \qquad \forall\ i = 1, 2, \ldots\ldots\ldots n \qquad \textbf{(14)}$$

Equations (9)-(10) and (13)-(14) are the 'power flow equations' or the 'load flow equations' in two alternative forms, corresponding to the n-bus system, where each bus-*i* is characterized by four variables, P_i, Q_i, $|V_i|$, and δ_i. Thus a total of 4n variables are involved in these equations. The load flow equations can be solved for

any 2n unknowns, if the other 2n variables are specified. This establishes the need for classification of buses of the system for load flow analysis into: PV bus, PQ bus, etc.

DATA FOR LOAD FLOW

Irrespective of the method used for the solution, the data required is common for any load flow. All data is normally in pu. The bus admittance matrix is formulated from these data. The various data required are as under:

System data: It includes: number of buses-n, number of PV buses, number of loads, number of transmission lines, number of transformers, number of shunt elements, the slack bus number, voltage magnitude of slack bus (angle is generally taken as 0°), tolerance limit, base MVA, and maximum permissible number of iterations.

Generator bus data: For every PV bus i, the data required includes the bus number, active power generation P_{Gi}, the specified voltage magnitude $\left|V_{i,sp}\right|$, minimum reactive power limit $Q_{i,min}$, and maximum reactive power limit $Q_{i,max}$.

Load data: For all loads the data required includes the the bus number, active power demand P_{Di}, and the reactive power demand Q_{Di}.

Transmission line data: For every transmission line connected between buses i and k the data includes the starting bus number i, ending bus number k,.resistance of the line, reactance of the line and the half line charging admittance.

Transformer data:

For every transformer connected between buses i and k the data to be given includes: the starting bus number i, ending bus number k, resistance of the transformer, reactance of the transformer, and the off nominal turns-ratio a.

Shunt element data: The data needed for the shunt element includes the bus number where element is connected, and the shunt admittance ($G_{sh} + j\ B_{sh}$).

GAUSS – SEIDEL (GS) METHOD

The GS method is an iterative algorithm for solving non linear algebraic equations. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At every subsequent iteration, the solution is updated

till convergence is reached. The GS method applied to power flow problem is as discussed below.

Case (a): Systems with PQ buses only:

Initially assume all buses to be PQ type buses, except the slack bus. This means that (n–1) complex bus voltages have to be determined. For ease of programming, the slack bus is generally numbered as bus-1. PV buses are numbered in sequence and PQ buses are ordered next in sequence. This makes programming easier, compared to random ordering of buses. Consider the expression for the complex power at bus-i, given from (7), as:

$$S_i = V_i \left(\sum_{j=1}^{n} Y_{ij} V_j \right)^*$$

This can be written as

$$S_i^* = V_i^* \left(\sum_{j=1}^{n} Y_{ij} V_j \right) \tag{15}$$

Since $S_i^* = P_i - jQ_i$, we get,

$$\frac{P_i - jQ_i}{V_i^*} = \sum_{j=1}^{n} Y_{ij} V_j$$

So that,

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \tag{16}$$

Rearranging the terms, we get,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \right] \qquad \forall\ i = 2, 3, \ldots\ldots..n \tag{17}$$

Equation (17) is an implicit equation since the unknown variable, appears on both sides of the equation. Hence, it needs to be solved by an iterative technique. Starting from an initial estimate of all bus voltages, in the RHS of (17) the most recent values of the bus voltages is substituted. One iteration of the method involves computation of all the bus voltages. In Gauss–Seidel method, the value of the updated voltages are used in the computation of subsequent voltages in the same iteration, thus speeding up

convergence. Iterations are carried out till the magnitudes of all bus voltages do not change by more than the tolerance value. Thus the algorithm for GS method is as under:

Algorithm for GS method

1. Prepare data for the given system as required.
2. Formulate the bus admittance matrix Y_{BUS}. This is generally done by the rule of inspection.
3. Assume initial voltages for all buses, 2,3,…n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses (except slack bus) are taken to be $1.0 \angle 0^0$. This is normally refered as the *flat start* solution.
4. Update the voltages. In any $(k+1)^{st}$ iteration, from (17) the voltages are given by

$$V_i^{(k+1)} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{(V_i^{(k)})^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} - \sum_{j=i+1}^{n} Y_{ij} V_j^{(k)} \right] \quad \forall \text{ i=2,3,…n} \qquad \textbf{(18)}$$

Here note that when computation is carried out for bus-i, updated values are already available for buses 2,3….(i-1) in the current $(k+1)^{st}$ iteration. Hence these values are used. For buses $(i+1).....n$, values from previous, k^{th} iteration are used.

5. Continue iterations till

$$\left| \Delta V_i^{(k+1)} \right| = \left| V_i^{(k+1)} - V_i^{(k)} \right| < \vee \qquad\qquad \forall \text{ i = 2,3,…n} \qquad \textbf{(19)}$$

Where, $\vee$ is the tolerance value. Generally it is customary to use a value of 0.0001 pu.

6. Compute slack bus power after voltages have converged using (15) [assuming bus 1 is slack bus].

$$S_1^* = P_1 - jQ_1 = V_1^* \left(\sum_{j=1}^{n} Y_{1j} V_j \right) \qquad\qquad \textbf{(20)}$$

7. Compute all line flows.
8. The complex power loss in the line is given by $S_{ik} + S_{ki}$. The total loss in the system is calculated by summing the loss over all the lines.

Case (b): Systems with PV buses also present:

At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of Q_i to be used in (18). From (15) we have

$$Q_i = -\,\text{Im}\,\left\{ V_i^* \sum_{j=1}^{n} Y_{ij}\ V_j \right\}$$

Where *Im* stands for the imaginary part. At any $(k+1)^{st}$ iteration, at the PV bus-*i*,

$$Q_i^{(k+1)} = -\,\text{Im}\,\left\{ (V_i^{(k)})^* \sum_{j=1}^{i-1} Y_{ij}\ V_j^{(k+1)} + (V_i^{(k)})^* \sum_{j=i}^{n} Y_{ij}\ V_j^{(k)} \right\} \qquad \textbf{(21)}$$

The steps for i^{th} PV bus are as follows:

    1. Compute $Q_i^{(k+1)}$ using (21)

    2. Calculate V_i using (18) with $Q_i = Q_i^{(k+1)}$

    3. Since $|V_i|$ is specified at the PV bus, the magnitude of V_i obtained in step 2 has to be modified and set to the specified value $|V_{i,sp}|$. Therefore,

$$V_i^{(k+1)} = |V_{i,sp}|\ \angle u_i^{(k+1)} \qquad \textbf{(22)}$$

The voltage computation for PQ buses does not change.

Case (c): Systems with PV buses with reactive power generation limits specified:

In the previous algorithm if the Q limit at the voltage controlled bus is violated during any iteration, i.e $Q_i^{(k+1)}$ computed using (21) is either less than $Q_{i,\,min}$ or greater than $Q_{i,max}$, it means that the voltage cannot be maintained at the specified value due to lack of reactive power support. This bus is then treated as a PQ bus in the $(k+1)^{st}$ iteration and the voltage is calculated with the value of Q_i set as follows:

      If $Q_i < Q_{i,min}$ If $Q_i > Q_{i,max}$

      Then $Q_i = Q_{i,min.}$ Then $Q_i = Q_{i,max.}$

$$\textbf{(23)}$$

If in the subsequent iteration, if Q_i falls within the limits, then the bus can be switched back to PV status.

<u>Acceleration of convergence</u>

It is found that in GS method of load flow, the number of iterations increase with increase in the size of the system. The number of iterations required can be reduced if

the correction in voltage at each bus is accelerated, by multiplying with a constant ,
called the acceleration factor. In the $(k+1)^{st}$ iteration we can let

$$V_i^{(k+1)}(accelerate\ d) = V_i^{(k)} + r\left(V_i^{(k+1)} - V_i^{(k)}\right) \tag{24}$$

where is a real number. When =1, the value of $V_i^{(k+1)}$ is the computed value. If
1 < < 2, then the value computed is extrapolated. Generally is taken between 1.2
to 1.6, for GS load flow procedure. At PQ buses (pure load buses) if the voltage
magnitude violates the limit, it simply means that the specified reactive power
demand cannot be supplied, with the voltage maintained within acceptable limits.

Examples on GS load flow analysis:

Example-1: *Obtain the voltage at bus 2 for the simple system shown in Fig 2, using
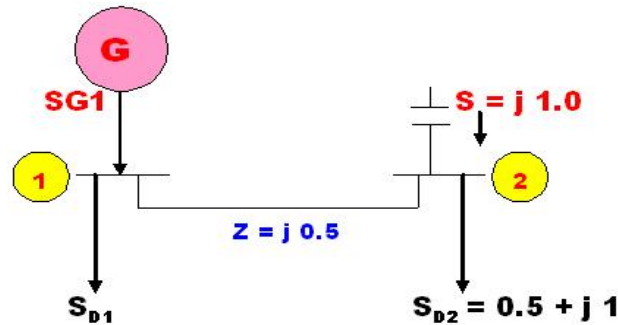the Gauss–Seidel method, if $V_1 = 1 \angle 0^0$ pu.*

Fig : System of Example 1

Solution:

Here the capacitor at bus 2, injects a reactive power of 1.0 pu. The complex power
injection at bus 2 is

$$S_2 = j1.0 - (0.5 + j\ 1.0) = -0.5\ \text{pu.}$$
$$V_1 = 1 \angle 0^0$$
$$Y_{BUS} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$
$$V_2^{(k+1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{\left(V_2^{(k)}\right)^*} - Y_{21}\ V_1\right]$$

Since V_1 is specified it is a constant through all the iterations. Let the initial voltage at
bus 2, $V_2^0 = 1 + j\ 0.0 = 1 \angle 0^0$ pu.

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$$V_2^1 = \frac{1}{-j2}\left[\frac{-0.5}{1\angle 0^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 1.0 - j0.25 = 1.030776 \angle -14.036^0$$

$$V_2^2 = \frac{1}{-j2}\left[\frac{-0.5}{1.030776\angle 14.036^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.94118 - j\,0.23529 = 0.970145 \angle -14.036^0$$

$$V_2^3 = \frac{1}{-j2}\left[\frac{-0.5}{0.970145\angle 14.036^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.9375 - j\,0.249999 = 0.970261 \angle -14.931^0$$

$$V_2^4 = \frac{1}{-j2}\left[\frac{-0.5}{0.970261\angle 14.931^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.933612 - j\,0.248963 = 0.966237 \angle -14.931^0$$

$$V_2^5 = \frac{1}{-j2}\left[\frac{-0.5}{0.966237\angle 14.931^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.933335 - j\,0.25 = 0.966237 \angle -14.995^0$$

Since the difference in the voltage magnitudes is less than 10^{-6} pu, the iterations can be stopped. To compute line flow

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{1\angle 0^0 - 0.966237 \angle -14.995^0}{j0.5}$$

$$= 0.517472 \angle -14.931^0$$

$$S_{12} = V_1 I_{12}^* = 1\angle 0^0 \times 0.517472 \angle 14.931^0$$

$$= 0.5 + j\,0.133329 \text{ pu}$$

$$I_{21} = \frac{V_2 - V_1}{Z_{12}} = \frac{0.966237 \angle -14.995^0 - 1\angle 0^0}{j0.5}$$

$$= 0.517472 \angle -194.93^0$$

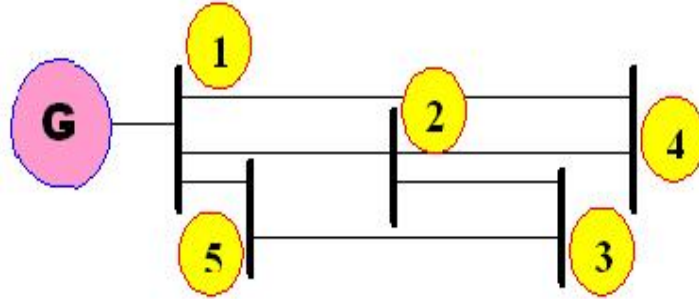$$S_{21} = V_2 I_{21}^* = -0.5 + j\,0.0 \text{ pu}$$

The total loss in the line is given by

$$S_{12} + S_{21} = j\,0.133329 \text{ pu}$$

Obviously, it is observed that there is no real power loss, since the line has no resistance.

Example-2:

For the power system shown in fig. below, with the data as given in tables below, obtain the bus voltages at the end of first iteration, by applying GS method.

Power System of Example 2

Line data of example 2

SB	EB	R (pu)	X (pu)	$\frac{B_C}{2}$
1	2	0.10	0.40	-
1	4	0.15	0.60	-
1	5	0.05	0.20	-
2	3	0.05	0.20	-
2	4	0.10	0.40	-
3	5	0.05	0.20	-

Bus data of example 2

Bus No.	P_G (pu)	Q_G (pu)	P_D (pu)	Q_D (pu)	$\left\vert V_{SP}\right\vert$ (pu)	
1	-	-	-	-	1.02	0°
2	-	-	0.60	0.30	-	-
3	1.0	-	-	-	1.04	-
4	-	-	0.40	0.10	-	-
5	-	-	0.60	0.20	-	-

Solution: In this example, we have,

- Bus 1 is slack bus, Bus 2, 4, 5 are PQ buses, and Bus 3 is PV bus

- The lines do not have half line charging admittances

$P_2 + jQ_2 = P_{G2} + jQ_{G2} - (P_{D2} + jQ_{D2}) = -0.6 - j0.3$

$$P_3 + jQ_3 = P_{G3} + jQ_{G3} - (P_{D3} + jQ_{D3}) = 1.0 + jQ_{G3}$$

Similarly $P_4 + jQ_4 = -0.4 - j0.1$, $\qquad$ $P_5 + jQ_5 = -0.6 - j0.2$

The Y_{bus} formed by the rule of inspection is given by:

$Y_{bus} =$

2.15685 -j8.62744	-0.58823 +j2.35294	0.0+j0.0	-0.39215 +j1.56862	-1.17647 +j4.70588
-0.58823 +j2.35294	2.35293 -j9.41176	-1.17647 +j4.70588	-0.58823 +j2.35294	0.0+j0.0
0.0+j0.0	-1.17647 +j4.70588	2.35294 -j9.41176	0.0+j0.0	-1.17647 +j4.70588
-0.39215 +j1.56862	-0.58823 +j2.35294	0.0+j0.0	0.98038 -j3.92156	0.0+j0.0
-1.17647 +j4.70588	0.0+j0.0	-1.17647 +j4.70588	0.0+j0.0	2.35294 -j9.41176

The voltages at all PQ buses are assumed to be equal to $1+j0.0$ pu. The slack bus voltage is taken to be $V_1^0 = 1.02+j0.0$ in all iterations.

$$V_2^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21} V_1^o - Y_{23} V_3^0 - Y_{24} V_4^0 - Y_{25} V_5^0 \right]$$

$$= \frac{1}{Y_{22}} \left[\frac{-0.6 + j0.3}{1.0 - j0.0} - \left\{(-0.58823 + j2.35294) \times 1.02\angle 0^o\right\} \right.$$

$$\left. - \left\{(-1.17647 + j4.70588) \times 1.04\angle 0^o\right\} - \left\{(-0.58823 + j2.35294) \times 1.0\angle 0^0\right\}\right]$$

$$= 0.98140 \angle -3.0665^o = 0.97999 - j0.0525$$

Bus 3 is a PV bus. Hence, we must first calculate Q_3. This can be done as under:

$$Q_3 = |V_3||V_1| \left(G_{31} \sin u_{31} - B_{31} \cos u_{31}\right) + |V_3||V_2| \left(G_{32} \sin u_{32} - B_{32} \cos u_{32}\right)$$

$$+ |V_3|^2 \left(G_{33} \sin u_{33} - B_{33} \cos u_{33}\right) + |V_3||V_4| \left(G_{34} \sin u_{34} - B_{34} \cos u_{34}\right)$$

$$+ |V_3||V_5| \left(G_{35} \sin u_{35} - B_{35} \cos u_{35}\right)$$

We note that $\quad_1 = 0^o; \quad _2 = -3.0665^o; \quad _3 = 0^o; \quad _4 = 0^o \quad$ and $\quad _5 = 0^o$

$\therefore \quad _{31} = \quad _{33} = \quad _{34} = \quad _{35} = 0^o (\quad _{ik} = \quad _i - \quad _k); \qquad _{32} = 3.0665^o$

$Q_3 = 1.04 [1.02 (0.0+j0.0) + 0.9814 \{-1.17647 \times \sin(3.0665^o) - 4.70588$

$\times\cos(3.0665^o)\}+1.04\{-9.41176 \times\cos(0^o)\}+1.0 \{0.0 + j0.0\}+1.0\{-4.70588\times\cos(0^0)\}]$

$$= 1.04 [-4.6735 + 9.78823 - 4.70588] = 0.425204 \text{ pu.}$$

$$V_3^1 = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31} V_1^o - Y_{32} V_2^1 - Y_{34} V_4^0 - Y_{35} V_5^0 \right]$$

$$= \frac{1}{Y_{33}} \left[\frac{1.0 - j0.425204}{1.04 - j0.0} - \{(-1.7647 + j4.70588) \times (0.98140 \angle -3.0665^{\circ})\} \right.$$

$$\left. - \{(-1.17647 + j4.70588) \times (1 \angle 0^{\circ})\} \right]$$

$$= 1.05569 \angle 3.077^{\circ} = 1.0541 + j0.05666 \text{ pu}.$$

Since it is a PV bus, the voltage magnitude is adjusted to specified value and V_3^1 is

computed as: $V_3^1 = 1.04 \angle 3.077^0 \text{ pu}$

$$V_4^1 = \frac{1}{Y_{44}} \left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41} V_1^o - Y_{42} V_2^1 - Y_{43} V_3^1 - Y_{45} V_5^0 \right]$$

$$= \frac{1}{Y_{44}} \left[\frac{-0.4 + j0.1}{1.0 - j0.0} - \{(-0.39215 + j1.56862) \times 1.02 \angle 0^{\circ}\} \right.$$

$$\left. - \{(-0.58823 + j2.35294) \times (0.98140 \angle -3.0665^{\circ})\} \right]$$

$$= \frac{0.45293 - j3.8366}{0.98038 - j3.92156} = 0.955715 \angle -7.303^{\circ} \text{ pu} = 0.94796 - j0.12149$$

$$V_5^1 = \frac{1}{Y_{55}} \left[\frac{P_5 - jQ_5}{V_5^{o*}} - Y_{51} V_1^o - Y_{52} V_2^1 - Y_{53} V_3^1 - Y_{54} V_4^1 \right]$$

$$= \frac{1}{Y_{55}} \left[\frac{-0.6 + j0.2}{1.0 - j0.0} - \{(-1.17647 + j4.70588) \times 1.02 \angle 0^{\circ}\} \right.$$

$$\left. - \{(-1.17647 + j4.70588) \times 1.04 \angle 3.077^{\circ}\} \right]$$

$$= 0.994618 \angle -1.56^{\circ} = 0.994249 - j0.027$$

Thus at end of 1st iteration, we have,

$$V_1 = 1.02 \angle 0^0 \text{ pu} \qquad\qquad V_2 = 0.98140 \angle -3.066^0 \text{ pu}$$
$$V_3 = 1.04 \angle 3.077^0 \text{ pu} \qquad\qquad V_4 = 0.955715 \angle -7.303^0 \text{ pu}$$
$$\text{and} \qquad V_5 = 0.994618 \angle -1.56^0 \text{ pu}$$

Example-3:

Obtain the load flow solution at the end of first iteration of the system with data as given below. The solution is to be obtained for the following cases

  (i)   All buses except bus 1 are PQ Buses

  (ii)   Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

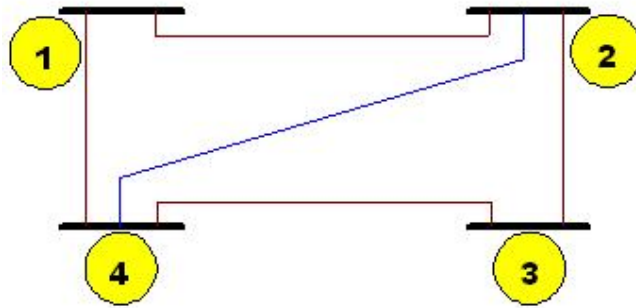 *(iii) Bus 2 is PV bus, with voltage magnitude specified as 1.04 and 0.25 Q₂ 1.0 pu.*

Fig. System for Example 3

Table: Line data of example 3

SB	EB	R (pu)	X (pu)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

Table: Bus data of example 3

Bus No.	P_i (pu)	Q_i (pu)	V_i
1	–	–	$1.04 \angle 0^0$
2	0.5	– 0.2	–
3	– 1.0	0.5	–
4	– 0.3	– 0.1	–

Solution: Note that the data is directly in terms of injected powers at the buses. The bus admittance matrix is formed by inspection as under:

$$
Y_{BUS} = \begin{array}{|c|c|c|c|}
\hline
3.0 - j9.0 & -2.0 + j6.0 & -1.0 + j3.0 & 0 \\
\hline
-2.0 + j6.0 & 3.666 - j11.0 & -0.666 + j2.0 & -1.0 + j3.0 \\
\hline
-1.0 + j3.0 & -0.666 + j2.0 & 3.666 - j11.0 & -2.0 + j6.0 \\
\hline
0 & -1.0 + j3.0 & -2.0 + j6.0 & 3.0 - j9.0 \\
\hline
\end{array}
$$

Case(i): All buses except bus 1 are PQ Buses

Assume all initial voltages to be $1.0 \angle 0^0$ pu.

$$V_2^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21} V_1^o - Y_{23} V_3^0 - Y_{24} V_4^0 \right]$$

$$= \frac{1}{Y_{22}} \left[\frac{0.5 + j0.2}{1.0 - j0.0} - \left\{ (-2.0 + j6.0) \times (1.04\angle 0^o) \right\} \right.$$

$$\left. - \left\{ (-0.666 + j2.0) \times (1.0\angle 0^o) \right\} - \left\{ (-1.0 + j3.0) \times (1.0\angle 0^0) \right\} \right]$$

$$= 1.02014 \angle 2.605^o$$

$$V_3^1 = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31} V_1^o - Y_{32} V_2^1 - Y_{34} V_4^0 \right]$$

$$= \frac{1}{Y_{33}} \left[\frac{-1.0 - j0.5}{1.0 - j0.0} - \left\{ (-1.0 + j3.0) \times (1.04\angle 0.0^o) \right\} \right.$$

$$\left. - \left\{ (-0.666 + j2.0) \times (1.02014\angle 2.605^o) \right\} - \left\{ (-2.0 + j6.0) \times (1.0\angle 0^0) \right\} \right]$$

$$= 1.03108 \angle -4.831^o$$

$$V_4^1 = \frac{1}{Y_{44}} \left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41} V_1^o - Y_{42} V_2^1 - Y_{43} V_3^1 \right]$$

$$= \frac{1}{Y_{44}} \left[\frac{0.3 + j0.1}{1.0 - j0.0} - \left\{ (-1.0 + j3.0) \times (1.02014\angle 2.605^o) \right\} \right.$$

$$\left. - \left\{ (-2.0 + j6.0) \times (1.03108\angle -4.831^o) \right\} \right]$$

$$= 1.02467 \angle -0.51^o$$

Hence

$$V_1^1 = 1.04 \angle 0^0 \text{ pu} \qquad\qquad V_2^1 = 1.02014 \angle 2.605^0 \text{ pu}$$

$$V_3^1 = 1.03108 \angle -4.831^0 \text{ pu} \qquad\qquad V_4^1 = 1.02467 \angle -0.51^0 \text{ pu}$$

Case(ii): Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

We first compute Q_2.

$$Q_2 = |V_2| \left[|V_1| (G_{21} \sin u_{21} - B_{21} \cos u_{21}) + |V_2| (G_{22} \sin u_{22} - B_{22} \cos u_{22}) \right.$$

$$\left. + |V_3| (G_{23} \sin u_{23} - B_{23} \cos u_{23}) + |V_4| (G_{24} \sin u_{24} - B_{24} \cos u_{24}) \right]$$

$$= 1.04 \, [1.04 \, \{-6.0\} + 1.04 \, \{11.0\} + 1.0\{-2.0\} + 1.0 \, \{-3.0\} = 0.208 \text{ pu}.$$

$$V_2^1 = \frac{1}{Y_{22}} \left[\frac{0.5 - j0.208}{1.04\angle 0^0} - \left\{ (-2.0 + j6.0) \times (1.04\angle 0^o) \right\} \right.$$

$$\left. - \left\{ (-0.666 + j2.0) \times (1.0\angle 0^o) \right\} - \left\{ (-1.0 + j3.0) \times (1.0\angle 0^0) \right\} \right]$$

$$= 1.051288 + j0.033883$$

The voltage magnitude is adjusted to 1.04. Hence $V_2^1 = 1.04 \angle 1.846^0$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0 \angle 0^0} - \left\{(-1.0 + j3.0)\times(1.04\angle 0.0^o)\right\}\right.$$

$$\left. - \left\{(-0.666 + j2.0)\times(1.04\angle 1.846^o)\right\} - \left\{(-2.0 + j6.0)\times(1.0\angle 0^0)\right\}\right]$$

$$= 1.035587 \angle - 4.951^o \text{ pu}.$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \left\{(-1.0 + j3.0)\times(1.04\angle 1.846^o)\right\}\right.$$

$$\left. - \left\{(-2.0 + j6.0)\times(1.035587\angle - 4.951^o)\right\}\right]$$

$$= 0.9985 \angle - 0.178^o$$

Hence at end of 1st iteration we have:

$$V_1^1 = 1.04 \angle 0^0 \text{ pu} \qquad\qquad V_2^1 = 1.04 \angle 1.846^0 \text{ pu}$$

$$V_3^1 = 1.035587 \angle -4.951^0 \text{ pu} \qquad V_4^1 = 0.9985 \angle -0.178^0 \text{ pu}$$

Case (iii): Bus 2 is PV bus, with voltage magnitude specified as 1.04 & 0.25 Q_2 1 pu. If 0.25 Q_2 1.0 pu then the computed value of Q_2 = 0.208 is less than the lower limit. Hence, Q_2 is set equal to 0.25 pu. Iterations are carried out with this value of Q_2. The voltage magnitude at bus 2 can no longer be maintained at 1.04. Hence, there is no necessity to adjust for the voltage magnitude. Proceeding as before we obtain at the end of first iteration,

$$V_1^1 = 1.04 \angle 0^0 \text{ pu} \qquad\qquad V_2^1 = 1.05645 \angle 1.849^0 \text{ pu}$$

$$V_3^1 = 1.038546 \angle -4.933^0 \text{ pu} \qquad V_4^1 = 1.081446 \angle 4.896^0 \text{ pu}$$

Limitations of GS load flow analysis:

GS method is very useful for very small systems. It is easily adoptable, it can be generalized and it is very efficient for systems having less number of buses. However, GS LFA fails to converge in systems with one or more of the features as under:

- Systems having large number of radial lines
- Systems with short and long lines terminating on the same bus
- Systems having negative values of transfer admittances
- Systems with heavily loaded lines, etc.

GS method successfully converges in the absence of the above problems. However, convergence also depends on various other set of factors such as: selection of slack bus, initial solution, acceleration factor, tolerance limit, level of accuracy of results needed, type and quality of computer/ software used, etc.

NEWTON –RAPHSON METHOD

Newton-Raphson (NR) method is used to solve a system of non-linear algebraic equations of the form f(x) =0. Consider a set of n non-linear algebraic equations given by

$$f_i(x_1, x_2 \ldots \ldots x_n) = 0 \qquad i = 1,2 \ldots \ldots n \qquad (25)$$

Let $x_1^0, x_2^0 \ldots \ldots x_n^0$, be the initial guess of unknown variables and $\Delta x_1^0, \Delta x_2^0 \ldots \ldots \Delta x_n^0$ be the respective corrections. Therefore,

$$f_i(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0 \ldots \ldots x_n^0 + \Delta x_n^0) = 0 \qquad i = 1,2 \ldots \ldots n \qquad (26)$$

The above equation can be expanded using Taylor's series to give

$$f_i(x_1^0, x_2^0 \ldots \ldots x_n^0) + \left[\left(\frac{\partial f_i}{\partial x_1} \right)^0 \Delta x_1^0 + \left(\frac{\partial f_i}{\partial x_2} \right)^0 \Delta x_2^0 + \ldots + \left(\frac{\partial f_i}{\partial x_n} \right)^0 \Delta x_n^0 \right]$$

$$+ \text{ Higher order terms} = 0 \qquad \forall \ i = 1,2 \ldots \ldots n \qquad (27)$$

Where, $\left(\frac{\partial f_i}{\partial x_1} \right)^0, \left(\frac{\partial f_i}{\partial x_2} \right)^0, \ldots \ldots \left(\frac{\partial f_i}{\partial x_n} \right)^0$ are the partial derivatives of f_i with respect to $x_1, x_2 \ldots \ldots x_n$ respectively, evaluated at ($x_1^0, x_2^0 \ldots \ldots x_0^n$). If the higher order terms are neglected, then (27) can be written in matrix form as

$$\begin{bmatrix} f_1^0 \\ f_2^0 \\ . \\ . \\ . \\ f_n^0 \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1} \right)^0 & \left(\frac{\partial f_1}{\partial x_2} \right)^0 & . & . & . & \left(\frac{\partial f_1}{\partial x_n} \right)^0 \\ \left(\frac{\partial f_2}{\partial x_1} \right)^0 & \left(\frac{\partial f_2}{\partial x_2} \right)^0 & . & . & . & \left(\frac{\partial f_2}{\partial x_n} \right)^0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \left(\frac{\partial f_n}{\partial x_1} \right) & \left(\frac{\partial f_n}{\partial x_2} \right) & . & . & . & \left(\frac{\partial f_n}{\partial x_n} \right) \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ . \\ . \\ . \\ \Delta x_n^0 \end{bmatrix} = 0 \qquad (28)$$

In vector form (28) can be written as

$$F^0 + J^0 \Delta X^0 = 0$$

Or
$$F^0 = -J^0 \Delta X^0$$

Or
$$\Delta X^0 = -[J^0]^{-1} F^0 \qquad (29)$$

And
$$X^1 = X^0 + \Delta X^0 \qquad (30)$$

Here, the matrix [J] is called the **Jacobian** matrix. The vector of unknown variables is updated using (30). The process is continued till the difference between two successive iterations is less than the tolerance value.

NR method for load flow solution in polar coordinates

In application of the NR method, we have to first bring the equations to be solved, to the form $f_i(x_1, x_2, ... x_n) = 0$, where $x_1, x_2, ... x_n$ are the unknown variables to be determined. Let us assume that the power system has n_1 PV buses and n_2 PQ buses. In polar coordinates the unknown variables to be determined are:

(i) θ_i, the angle of the complex bus voltage at bus i, at all the PV and PQ buses. This gives us $n_1 + n_2$ unknown variables to be determined.

(ii) $|V_i|$, the voltage magnitude of bus i, at all the PQ buses. This gives us n_2 unknown variables to be determined.

Therefore, the total number of unknown variables to be computed is: $n_1 + 2n_2$, for which we need $n_1 + 2n_2$ consistent equations to be solved. The equations are given by,

$$\Delta P_i = P_{i,sp} - P_{i,cal} = 0 \tag{31}$$

$$\Delta Q_i = Q_{i,sp} - Q_{i,cal} = 0 \tag{32}$$

Where $\quad\quad P_{i,sp} = $ Specified active power at bus i

$\quad\quad\quad\quad\quad\quad Q_{i,sp} = $ Specified reactive power at bus i

$\quad\quad\quad\quad\quad\quad P_{i,cal} = $ Calculated value of active power using voltage estimates.

$\quad\quad\quad\quad\quad\quad Q_{i,cal} = $ Calculated value of reactive power using voltage estimates

$\quad\quad\quad\quad\quad\quad \Delta P = $ Active power residue

$\quad\quad\quad\quad\quad\quad \Delta Q = $ Reactive power residue

The real power is specified at all the PV and PQ buses. Hence (31) is to be solved at all PV and PQ buses leading to $n_1 + n_2$ equations. Similarly the reactive power is specified at all the PQ buses. Hence, (32) is to be solved at all PQ buses leading to n_2 equations.

We thus have $n_1 + 2n_2$ equations to be solved for $n_1 + 2n_2$ unknowns. (31) and (32) are of the form $F(x) = 0$. Thus NR method can be applied to solve them. Equations (31) and (32) can be written in the form of (30) as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta |V| \end{bmatrix}$$

(33)

Where J_1, J_2, J_3, J_4 are the negated partial derivatives of ΔP and ΔQ with respect to corresponding u and $|V|$. The negated partial derivative of ΔP, is same as the partial derivative of P_{cal}, since P_{sp} is a constant. The various computations involved are discussed in detail next.

Computation of P_{cal} and Q_{cal}:

The real and reactive powers can be computed from the load flow equations as:

$$P_{i,Cal} = P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos u_{ik} + B_{ik}\sin u_{ik})$$

$$= G_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik}\cos u_{ik} + B_{ik}\sin u_{ik})$$

(34)

$$Q_{i,Cal} = Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin u_{ik} - B_{ik}\cos u_{ik})$$

$$= -B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik}\sin u_{ik} - B_{ik}\cos u_{ik})$$

(35)

The powers are computed at any $(r+1)^{st}$ iteration by using the voltages available from previous iteration. The elements of the Jacobian are found using the above equations as:

Elements of J_1

$$\frac{\partial P_i}{\partial u_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|\{G_{ik}(-\sin u_{ik}) + B_{ik}\cos u_{ik}\}$$

$$= -Q_i - B_{ii}|V_i|^2$$

$$\frac{\partial P_i}{\partial u_k} = |V_i||V_k|(G_{ik}(-\sin u_{ik})(-1) + B_{ik}(\cos u_{ik})(-1))$$

Elements of J₃

$$\frac{\partial Q_i}{\partial u_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik} \cos u_{ik} + B_{ik} \sin u_{ik}) = P_i - G_{ii}|V_i|^2$$

$$\frac{\partial Q_i}{\partial u_k} = -|V_i||V_k|(G_{ik} \cos u_{ik} + B_{ik} \sin u_{ik})$$

Elements of J₂

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = 2|V_i|^2 G_{ii} + |V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_k|(G_{ik} \cos u_{ik} + B_{ik} \sin u_{ik}) = P_i + |V_i|^2 G_{ii}$$

$$\frac{\partial P_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik} \cos u_{ik} + B_{ik} \sin u_{ik})$$

Elements of J₄

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = -2|V_i|^2 B_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik} \sin u_{ik} - B_{ik} \cos u_{ik}) = Q_i - |V_i|^2 B_{ii}$$

$$\frac{\partial Q_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik} \sin u_{ik} - B_{ik} \cos u_{ik})$$

Thus, the linearized form of the equation could be considered again as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta u \\ \frac{\Delta |V|}{|V|} \end{bmatrix}$$

The elements are summarized below:

(i) $H_{ii} = \dfrac{\partial P_i}{\partial u_i} = -Q_i - B_{ii}|V_i|^2$

(ii) $H_{ik} = \dfrac{\partial P_i}{\partial u_k} = a_k f_i - b_k e_i = |V_i||V_k|(G_{ik} \sin u_{ik} - B_{ik} \cos u_{ik})$

(iii) $N_{ii} = \dfrac{\partial P_i}{\partial |V_i|}|V_i| = P_i + G_{ii}|V_i|^2$

(iv) $N_{ik} = \dfrac{\partial P_i}{\partial |V_k|}|V_k| = a_k e_i + b_k f_i = |V_i||V_k|(G_{ik} \cos u_{ik} + B_{ik} \sin u_{ik})$

(v) $M_{ii} = \dfrac{\partial Q_i}{\partial u_i} = P_i - G_{ii}|V_i|^2$

(vi) $M_{ik} = \dfrac{\partial Q_i}{\partial u_k} = -(a_k e_i + b_k f_i) = -N_{ik}$

(vii) $L_{ii} = \dfrac{\partial Q_i}{\partial |V_i|} |V_i| = Q_i - B_{ii} |V_i|^2$

(viii) $L_{ik} = \dfrac{\partial Q_i}{\partial |V_k|} |V_k| = a_k f_i - b_k e_i = H_{ik}$

In the above equations,

$Y_{ik} = G_{ik} + jB_{ik}$

$e_k + jf_k = |V_k|(\cos u_k + j \sin u_k)$

And $\quad a_k + jb_k = (G_{ik} + jB_{ik})(e_k + jf_k)$ $\qquad$ **(36)**

If $Y_{ik} = 0.0 + j0.0$ (if there is no line between buses i and k) then the corresponding off-diagonal elements in the Jacobian matrix will also be zero. Hence, the Jacobian is also a sparse matrix.

Size of the sub-matrices of the Jacobian: The dimensions of the various sub-matrices are as per the table below:

Matrix	size
H	$(n_1+n_2) \times (n_1+n_2)$
N	$(n_1+n_2) \times (n_2)$
M	$(n_2) \times (n_1+n_2)$
L	$(n_2) \times (n_2)$
J	$(n_1+2n_2) \times (n_1+2n_2)$
P	$(n_1+n_2) \times 1$
Q	$n_2 \times 1$
	$(n_1+n_2) \times 1$
$\Delta|V|/|V|$	$n_2 \times 1$

ALGORITHM FOR NR METHOD

IN POLAR COORDINATES

1. Formulate the Y_{BUS}

2. Assume initial voltages as follows:

$$V_i = |V_{i,sp}| \angle 0^0 \quad \text{(at all PV buses)}$$

$$V_i = 1 \angle 0^0 \quad \text{(at all PQ buses)}$$

3. At $(r+1)^{st}$ iteration, calculate $P_i^{(r+1)}$ at all the PV and PQ buses and $Q_i^{(r+1)}$ at all the

PQ buses, using voltages from previous iteration, $V_i^{(r)}$. The formulae to be used are

$$P_{i,Cal} = P_i = G_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik} \cos u_{ik} + B_{ik} \sin u_{ik})$$

$$Q_{i,Cal} = Q_i = -B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik} \sin u_{ik} - B_{ik} \cos u_{ik})$$

4. Calculate the power mismatches (power residues)

$$\Delta P_i^{(r)} = P_{i,sp} - P_{i,cal}^{(r+1)} \quad \text{(at PV and PQ buses)}$$

$$\Delta Q_i^{(r)} = Q_{i,sp} - Q_{i,cal}^{(r+1)} \quad \text{(at PQ buses)}$$

5. Calculate the Jacobian $[J^{(r)}]$ using $V_i^{(r)}$ and its elements spread over H, N, M, L

sub- matrices using the relations derived as in (36).

6. Compute

$$\begin{bmatrix} \Delta u^{(r)} \\ \dfrac{\Delta |V^{(r)}|}{|V|} \end{bmatrix} = [J^{(r)}]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$$

7. Update the variables as follows:

$$u_i^{(r+1)} = u_i^{(r)} + \Delta u_i^{(r)} \quad \text{(at all buses)}$$

$$|V_i|^{(r+1)} = |V_i|^{(r)} + \Delta |V_i|^{(r)}$$

8. Go to step 3 and iterate till the power mismatches are within acceptable tolerance.

DECOUPLED LOAD FLOW

In the NR method, the inverse of the Jacobian has to be computed at every iteration. When solving large interconnected power systems, alternative solution methods are possible, taking into account certain observations made of practical systems. These are,

- Change in voltage magnitude $|V_i|$ at a bus primarily affects the flow of reactive power Q in the lines and leaves the real power P unchanged. This observation implies that $\dfrac{\partial Q_i}{\partial |V_j|}$ is much larger than $\dfrac{\partial P_i}{\partial |V_j|}$. Hence, in the Jacobian, the elements of the sub-matrix $[N]$, which contains terms that are partial derivatives of real power with respect to voltage magnitudes can be made zero.

- Change in voltage phase angle at a bus, primarily affects the real power flow P over the lines and the flow of Q is relatively unchanged. This observation implies that $\dfrac{\partial P_i}{\partial u_j}$ is much larger than $\dfrac{\partial Q_i}{\partial u_j}$. Hence, in the Jacobian the elements of the sub-matrix $[M]$, which contains terms that are partial derivatives of reactive power with respect to voltage phase angles can be made zero.

These observations reduce the NRLF linearised form of equation to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta u \\ \dfrac{\Delta |V|}{|V|} \end{bmatrix} \tag{37}$$

From (37) it is obvious that the voltage angle corrections Δu are obtained using real power residues ΔP and the voltage magnitude corrections $|\Delta V|$ are obtained from reactive power residues ΔQ. This equation can be solved through two alternate strategies as under:

Strategy-1

(i) Calculate $\Delta P^{(r)}, \Delta Q^{(r)}$ and $J^{(r)}$

(ii) Compute $\begin{bmatrix} \Delta u^{(r)} \\ \dfrac{\Delta|V^{(r)}|}{|V^{(r)}|} \end{bmatrix} = \left[J^{(r)} \right]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$

(iii) Update u and $|V|$.

(iv) Go to step (i) and iterate till convergence is reached.

Strategy-2

(i) Compute $\Delta P^{(r)}$ and Sub-matrix $H^{(r)}$. From (37) find $\Delta u^{(r)} = \left[H^{(r)} \right]^{-1} \Delta P^{(r)}$

(ii) Up date u using $u^{(r+1)} = u^{(r)} + \Delta u^{(r)}$.

(iii) Use $u^{(r+1)}$ to calculate $\Delta Q^{(r)}$ and $L^{(r)}$

(iv) Compute $\dfrac{\Delta|V^{(r)}|}{|V^{(r)}|} = \left[L^{(r)} \right]^{-1} \Delta Q^{(r)}$

(v)Update, $|V^{(r+1)}| = |V^{(r)}| + |\Delta V^{(r)}|$

(vi) Go to step (i) and iterate till convergence is reached.

In the first strategy, the variables are solved simultaneously. In the second strategy the iteration is conducted by first solving for Δu and using updated values of u to calculate $\Delta|V|$. Hence, the second strategy results in faster convergence, compared to the first strategy.

FAST DECOUPLED LOAD FLOW

If the coefficient matrices are constant, the need to update the Jacobian at every iteration is eliminated. This has resulted in development of fast decoupled load Flow (FDLF). Here, certain assumptions are made based on the observations of practical power systems as under:

- $B_{ij} >> G_{ij}$ (Since the X/R ratio of transmission lines is high in well designed systems)

- The voltage angle difference $\left(u_i - u_j\right)$ between two buses in the system is very small. This means $\cos\left(u_i - u_j\right) \cong 1$ and $\sin\left(u_i - u_j\right) = 0.0$

- $Q_i << B_{ii}\left|V_i\right|^2$

With these assumptions the elements of the Jacobian become

$$H_{ik} = L_{ik} = -\left|V_i\right|\left|V_k\right|B_{ik} \qquad \left(i \neq k\right)$$

$$H_{ii} = L_{ii} = -B_{ii}\left|V_i\right|^2$$

The matrix (37) reduces to

$$\left[\Delta P\right] = \left[\left|V_i\right|\left|V_j\right|B_{ij}'\ \right]\left[\Delta u\right]$$

$$\left[\Delta Q\right] = \left[\left|V_i\right|\left|V_j\right|B_{ij}''\ \right]\left[\frac{\Delta|V|}{|V|}\right] \tag{38}$$

Where B_{ij}' and B_{ij}'' are negative of the susceptances of respective elements of the bus admittance matrix. In (38) if we divide LHS and RHS by $\left|V_i\right|$ and assume $\left|V_j\right| \cong 1$, we get,

$$\left[\frac{\Delta P}{|V|}\right] = \left[B_{ij}'\ \right]\left[\Delta u\right]$$

$$\left[\frac{\Delta Q}{|V|}\right] = \left[B_{ij}''\ \right]\left[\frac{\Delta|V|}{|V|}\right] \tag{39}$$

Equations (39) constitute the Fast Decoupled load flow equations. Further simplification is possible by:

- Omitting effect of phase shifting transformers
- Setting off-nominal turns ratio of transformers to 1.0
- In forming B_{ij}', omitting the effect of shunt reactors and capacitors which mainly affect reactive power
- Ignoring series resistance of lines in forming the Y_{bus}.

With these assumptions we obtain a loss-less network. In the FDLF method, the matrices $[B']$ and $[B'']$ are constants and need to be inverted only once at the beginning of the iterations.

REPRESENTATION OF TAP CHANGING TRANSFORMERS

Consider a tap changing transformer represented by its admittance connected in series with an ideal autotransformer as shown (a= turns ratio of transformer)
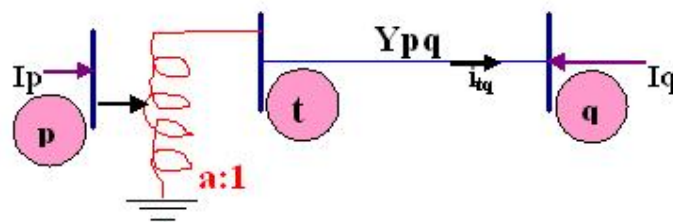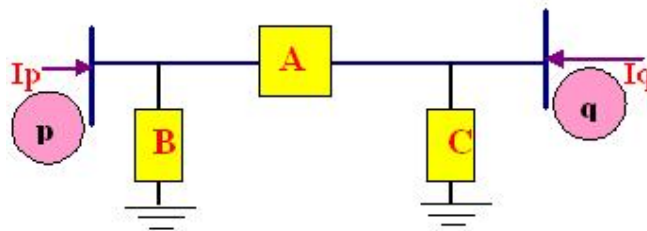

Fig. 2. Equivalent circuit of a tap setting transformer

Fig. 3.   -Equivalent circuit of Fig.2 above.

By equating the bus currents in both the mutually equivalent circuits as above, it can be shown that the   -equivalent circuit parameters are given by the expressions as under:

(i) Fixed tap setting transformers (on no load)

A = Ypq/ a

B = 1/a (1/a -1) Ypq

C = (1-1/a) Ypq

(i) Tap changing under load (TCUL) transformers (on load)

A = Ypq

B = (1/a -1) (1/a + 1 – Eq/Ep) Ypq

C = (1-1/a) (Ep/Eq) Ypq

Thus, here, in the case of TCUL transformers, the shunt admittance values are observed to be a function of the bus voltages.

COMPARISON OF LOAD FLOW METHODS

The comparison of the methods should take into account the computing time required for preparation of data in proper format and data processing, programming ease, storage requirements, computation time per iteration, number of iterations, ease and time required for modifying network data when operating conditions change, etc. Since all the methods presented are in the bus frame of reference in admittance form, the data preparation is same for all the methods and the bus admittance matrix can be formed using a simple algorithm, by the rule of inspection. Due to simplicity of the equations, Gauss-Seidel method is relatively easy to program. Programming of NR method is more involved and becomes more complicated if the buses are randomly numbered. It is easier to program, if the PV buses are ordered in sequence and PQ buses are also ordered in sequence.

The storage requirements are more for the NR method, since the Jacobian elements have to be stored. The memory is further increased for NR method using rectangular coordinates. The storage requirement can be drastically reduced by using sparse matrix techniques, since both the admittance matrix and the Jacobian are sparse matrices. The time taken for a single iteration depends on the number of arithmetic and logical operations required to be performed in a full iteration. The Gauss –Seidel method requires the fewest number of operations to complete iteration. In the NR method, the computation of the Jacobian is necessary in every iteration. Further, the inverse of the Jacobian also has to be computed. Hence, the time per iteration is larger than in the GS method and is roughly about 7 times that of the GS method, in large systems, as depicted graphically in figure below. Computation time can be reduced if

the Jacobian is updated once in two or three iterations. In FDLF method, the Jacobian is constant and needs to be computed only once. In both NR and FDLF methods, the time per iteration increases directly as the number of buses.
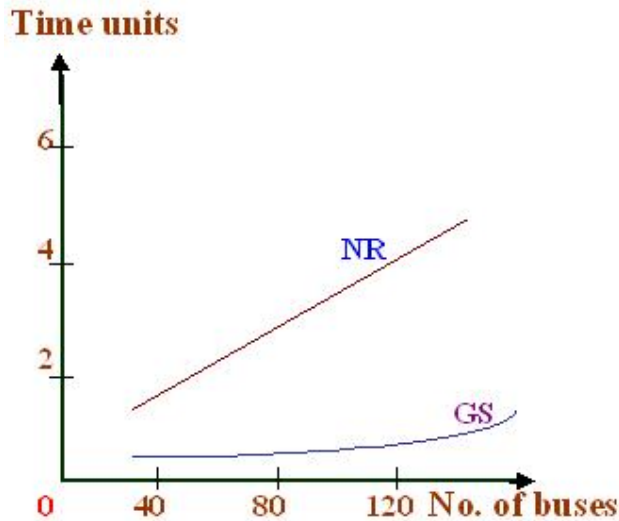

Figure 4. Time per Iteration in GS and NR methods

The number of iterations is determined by the convergence characteristic of the method. The GS method exhibits a linear convergence characteristic as compared to the NR method which has a quadratic convergence. Hence, the GS method requires more number of iterations to get a converged solution as compared to the NR method. In the GS method, the number of iterations increases directly as the size of the system increases. In contrast, the number of iterations is relatively constant in NR and FDLF methods. They require about 5-8 iterations for convergence in large systems. A significant increase in rate of convergence can be obtained in the GS method if an acceleration factor is used. All these variations are shown graphically in figure below. The number of iterations also depends on the required accuracy of the solution. Generally, a voltage tolerance of 0.0001 pu is used to obtain acceptable accuracy and the real power mismatch and reactive power mismatch can be taken as 0.001 pu. Due to these reasons, the NR method is faster and more reliable for large systems. The convergence of FDLF method is geometric and its speed is nearly 4-5 times that of NR method.
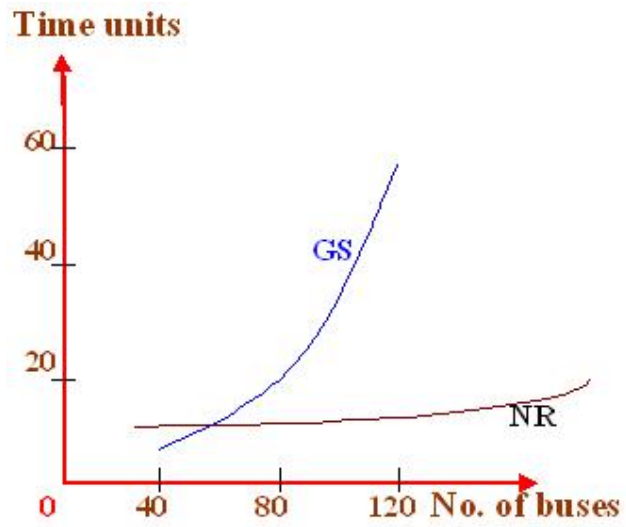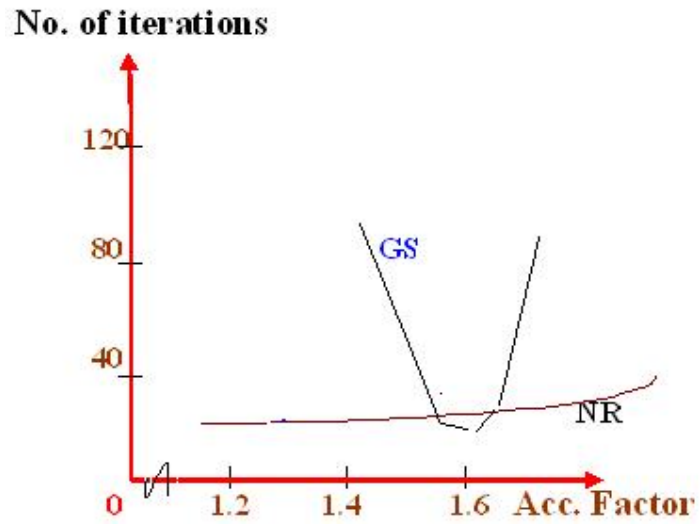
**Figure 5. Total time of Iteration in
GS and NR methods**

**Figure 6. Influence of acceleration factor
on load flow methods**

FINAL WORD

In this chapter, the load flow problem, also called as the power flow problem, has been considered in detail. The load flow solution gives the complex voltages at all the buses and the complex power flows in the lines. Though, algorithms are available using the impedance form of the equations, the sparsity of the bus admittance matrix and the ease of building the bus admittance matrix, have made algorithms using the admittance form of equations more popular.

The most popular methods are the Gauss-Seidel method, the Newton-Raphson method and the Fast Decoupled Load Flow method. These methods have been discussed in detail with illustrative examples. In smaller systems, the ease of programming and the memory requirements, make GS method attractive. However, the computation time increases with increase in the size of the system. Hence, in large systems NR and FDLF methods are more popular. There is a trade off between various requirements like speed, storage, reliability, computation time, convergence characteristics etc. No single method has all the desirable features. However, NR method is most popular because of its versatility, reliability and accuracy.