Pengolahan Citra Digital

4. PENINGKATAN KUALITAS CITRA II *(FILTERING)*

IFK15036, 3 credits

Team Teaching :

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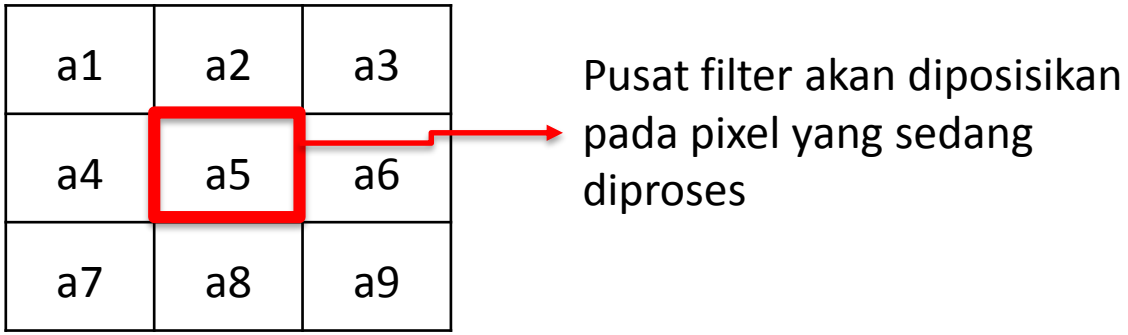
OUTLINE BAB-4

- Dasar Spatial Filter
- Konvolusi
- *Filtering* Ketetanggaan Pada Piksel
  - *Smoothing* Spatial Filter
    - Mean Filter
    - Median Filter
    - Gaussian Filter
  - *Sharpening* Spatial Filter
    - *Laplace Filter (high pass filter)*
    - *Sobel Filter*

Dasar *Spatial Filter*

- Filter juga sering disebut window atau mask (teori konvolusi)
- Umumnya memiliki ukuran ganjil 3x3, 5x5, 7x7 dst, atau juga sering disebut matrik bujursangkar.
- Filter beroperasi dengan cara dikonvolusikan dengan citra yang difilter
  - Dalam proses konvolusi filter bergerak 1 piksel ke kanan, operasi terjadi dan bergerak sampai baris diselesaikan dan pindah 1 baris di bawahnya dst.
  - Simbol matematis yang digunakan adalah bintang (*).



Filter/Mask

$h_{i-1,j-1}$	$h_{i-1,j}$	$h_{i-1,j+1}$
$h_{i,j-1}$	$h_{i,j}$	$h_{i,j+1}$
$h_{i+1,j-1}$	$h_{i+1,j}$	$h_{i+1,j+1}$

Mask dengan ukuran 3x3

- Mask dengan ukuran 3x3 direpresentasikan dalam matrik H sebagai berikut:

$$\mathbf{H} = \begin{vmatrix} h_{i-1,j-1} & h_{i-1,j} & h_{i-1,j+1} \\ h_{i,j-1} & h_{i,j} & h_{i,j+1} \\ h_{i+1,j-1} & h_{i+1,j} & h_{i+1,j+1} \end{vmatrix}$$

- Sebuah citra ***F*** dengan ukuran **tinggi** x **lebar** (**M**x**N**)

$$\mathbf{F} = \begin{vmatrix} f_{1,1} & f_{1,2} & f_{1,3} & \cdots & f_{1,N} \\ f_{2,1} & f_{2,2} & f_{2,3} & \cdots & f_{2,N} \\ f_{3,1} & f_{3,2} & f_{3,3} & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \\ f_{M,1} & f_{M,2} & f_{M,3} & \cdots & f \end{vmatrix}$$

Konvolusi

- Konvolusi (*) antara matrik citra **F** dan **H** adalah:

$$\mathbf{G} = \mathbf{F} * \mathbf{H}$$

- Nilai elemen dari matrik citra **G** adalah $g(x, y)$

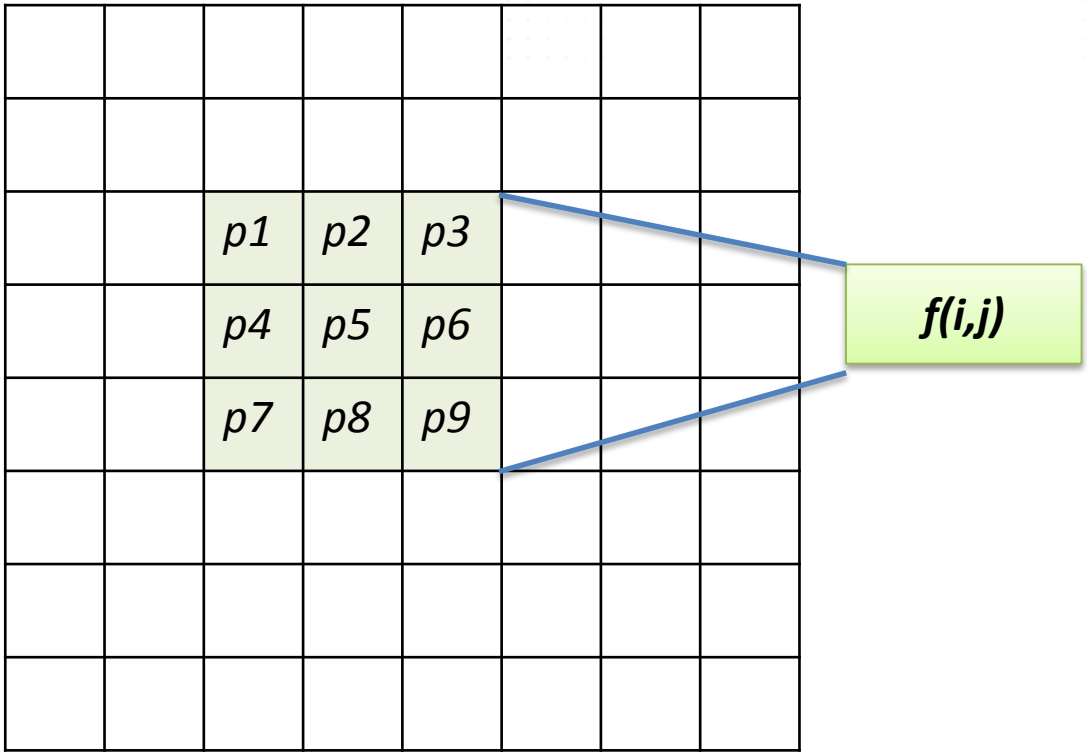
$$g(x,y) = h(-1,-1)f(x-1,y-1) + h(-1,0)f(x-$$

$$g(x,y) = \sum_{i=-1}^{1} \sum_{j=-1}^{1} h(i,j) f(x+i, y+j)$$

Ilustrasi Konvolusi

A	B	C
D	E	F
G	H	I

Kernel



Citra

$$f(i,j) = Ap1 + Bp2 + Cp3 + Dp4 + Ep5 + Fp6 + Gp7 + Hp8 + Ip9$$

Ilustrasi Konvolusi

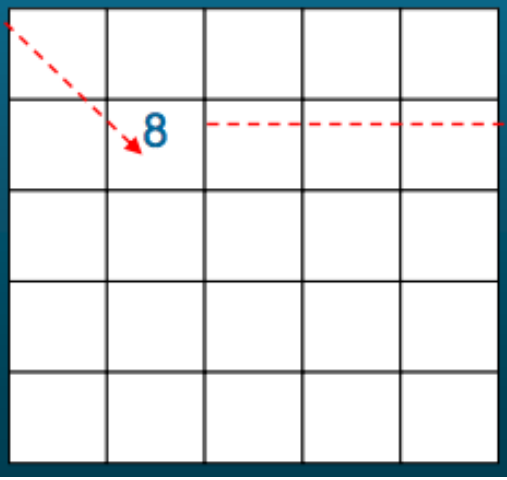
Citra dengan 5 x 5 pixel dan 8 grayscale :

0	5	5	4	4
0	**0**	5	4	4
1	6	1	3	3
1	6	7	2	3
1	6	7	6	6

→ Dikonvolusi dengan ***image mask*** :

-2	-1	0
-1	**0**	1
0	1	2

Hasilnya :



Hasil konvolusi = (0 x -2)+ (5 x -1) + (5 x 0) + (0 x -1) + (0 x 0) + (5 x 1) + (1 x 0) + (6 x 1) + (1 x 2) = 8

Smoothing Spatial Filter

- Referred to low-pass filters
- Reduce sharp transition in intensities
- Reduce noise
- Reduce ”irrelevant” details in the image
- Blur edges

Mean Filter

- Replace each pixel with the average of itself and its neighbours
- The kernel W contains only 1s
- The result is divided by the sum of the weights, i.e., with $\frac{1}{9}$
- 3×3 kernel

$$W = \frac{1}{9} * \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

- Fast operation for small neighbourhoods
- Differing pixel values will become more like their neighbours:
  - \+ Noise is reduced
  - \- Sharp edges are blurred

Hasil Output Mean Filter

Original image

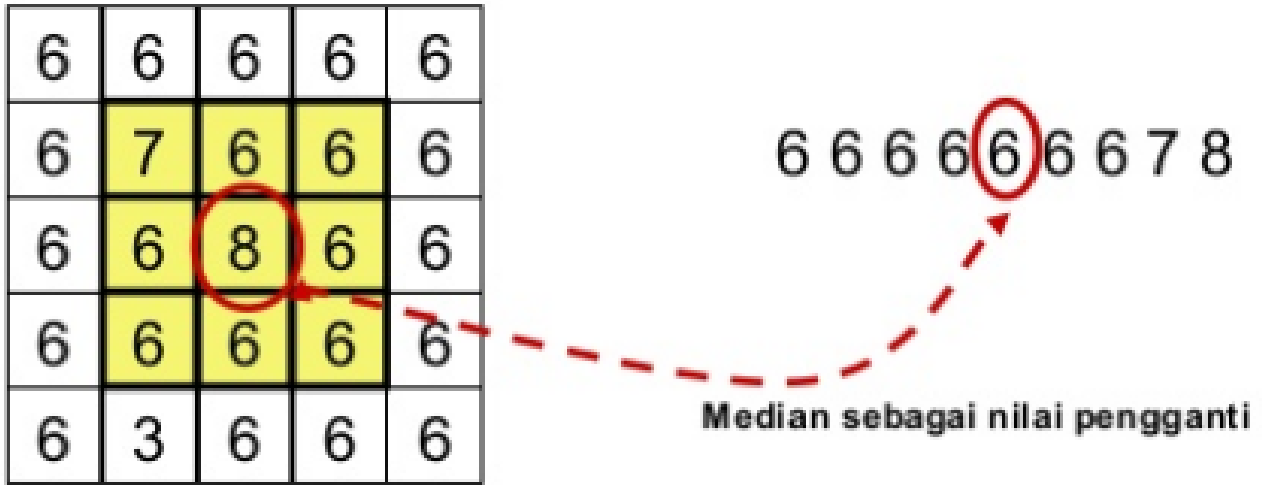
Original image

Mean Filtering image

Mean Filtering image

Median Filter

- Digunakan untuk menghilangkan noise
- Menggunakan nilai tengah dari pixel-pixel yang tertutup filter



Hasil Output *Median Filter*

Original image with noise

Median Filtering image

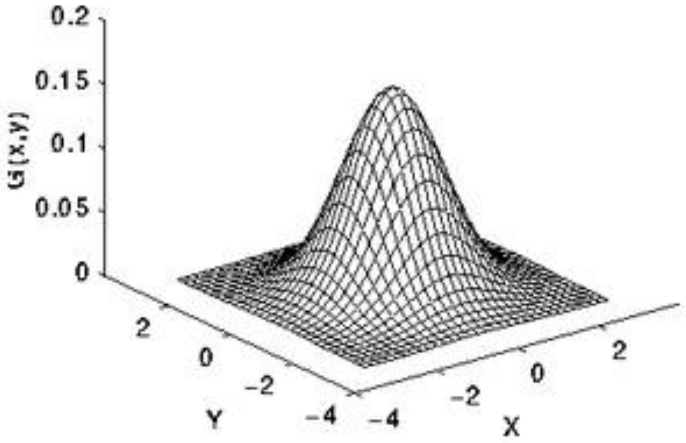
Gaussian Filter

Sifat-sifat filter atau mask

- Elemennya adalah berdistribusi Gaussian dan seluruh elemennya berjumlah 1
- Filter Gaussian setidaknya berukuran 5x5
- Semua nilai elemennya adalah positip, berikut adalah mask Gaussian filter 5x5 dengan $\sigma = 1$

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

σ adalah standar deviasi
pusat (x,y) adalah pusat piksel, pusat(0,0) mendapat bobot terbesar 1



$$\frac{1}{273}$$

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

Kernel Gaussian Filter

filter 5x5 dengan $\sigma = 1$

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

1	3	6	3	1
3	15	25	15	3
6	25	41	25	6
3	15	25	15	3
1	3	6	3	1

- $G(0,0) = e^{-0} = 1$
- $G(1,0) = G(0,1) = G(-1,0) = G(0,-1) = e^{-1/2} = 0.6065$
- $G(1,1) = G(1,-1) = G(-1,1) = G(-1,-1) = e^{-1} = 0.3679$
- $G(2,1) = G(1,2) = G(-2,1) = G(-2,-1) = e^{-(5/2)} = 0.0821$
- $G(2,0) = G(0,2) = G(0,-2) = G(-2,0) = e^{-2} = 0.1353$
- $G(2,2) = G(-2,-2) = G(-2,2) = G(2,-2) = e^{-4} = 0.0183$

Contoh : Ketika mengalikan nilai G(x,y) di depan dengan 41 dan dibulatkan ke atas

G(0,0) = 1*41 = 41
G(1,0) = G(0,1) = G(-1,0) = G(0,-1)= 0.6065*41 = 25
G(1,1) = G(1,-1) = G(-1,1) = G(-1,-1)= 0.3679*41 = 15
G(2,1) = G(1,2) = G(-2,1) = G(-2,-1)= 0.0821*41 = 3
G(2,0) = G(0,2) = G(0,-2) = G(-2,0)= 0.1353*41 = 6
G(2,2) = G(-2,-2) = G(-2,2) = G(2,-2)= 0.0183*41 = 1

Hasil Output *Gaussian Filter*

Sharpening Spatial Filter

- Filter penghalusan (smoothing) akan menghilangkan kedetilan dari citra hasil pemfilteran.
- *Sharpening spatial filter* akan meningkatkan kedetilan kenampakan citra
  - Menghilangkan keburaman (blurring) pada citra
  - Menonjolkan tepi
- Filter sharpening didasarkan pada diferensiasi spasial (***spatial differentiation***)

Turunan Pertama dan Kedua

Turunan pertama dari fungsi

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

Turunan kedua dari fungsi

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$

Turunan Kedua Digunakan untuk Perbaikan Citra *(Image Enhancement)*

- Turunan kedua digunakan untuk perbaikan citra dikarenakan memiliki respon yang baik untuk meningkatkan kedetilan (sharpening) dan implementasinya yang sederhana

Laplacian didefinisikan sbg:

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

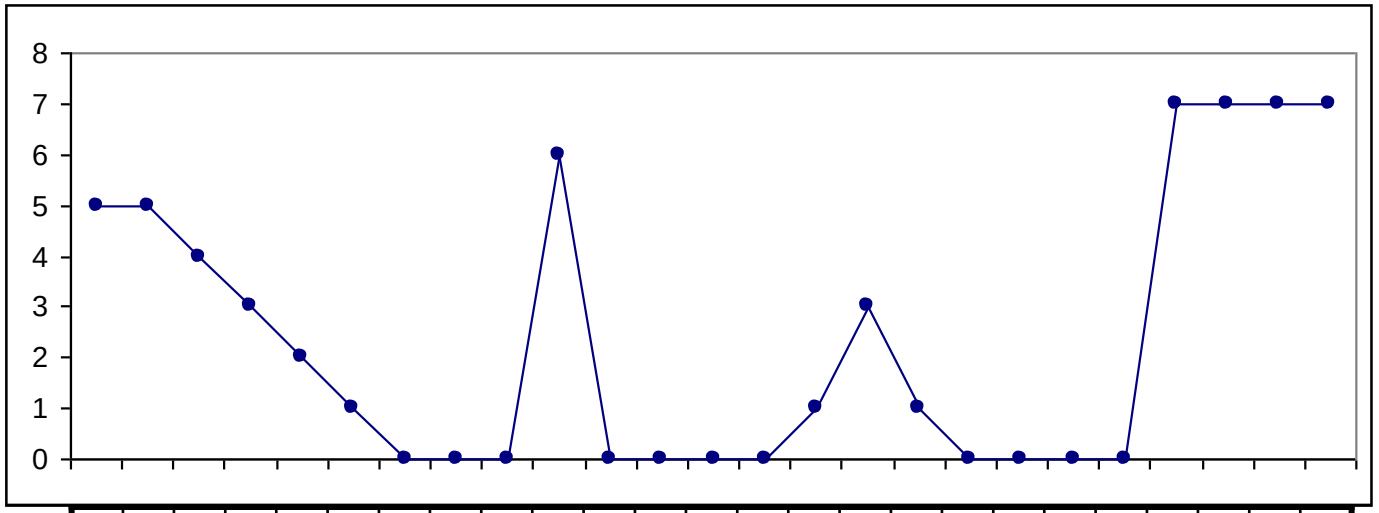
Turunan parsial dalam arah ***x*** didefinisikan sbb. :

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

Dan dalam arah **y** adalah:

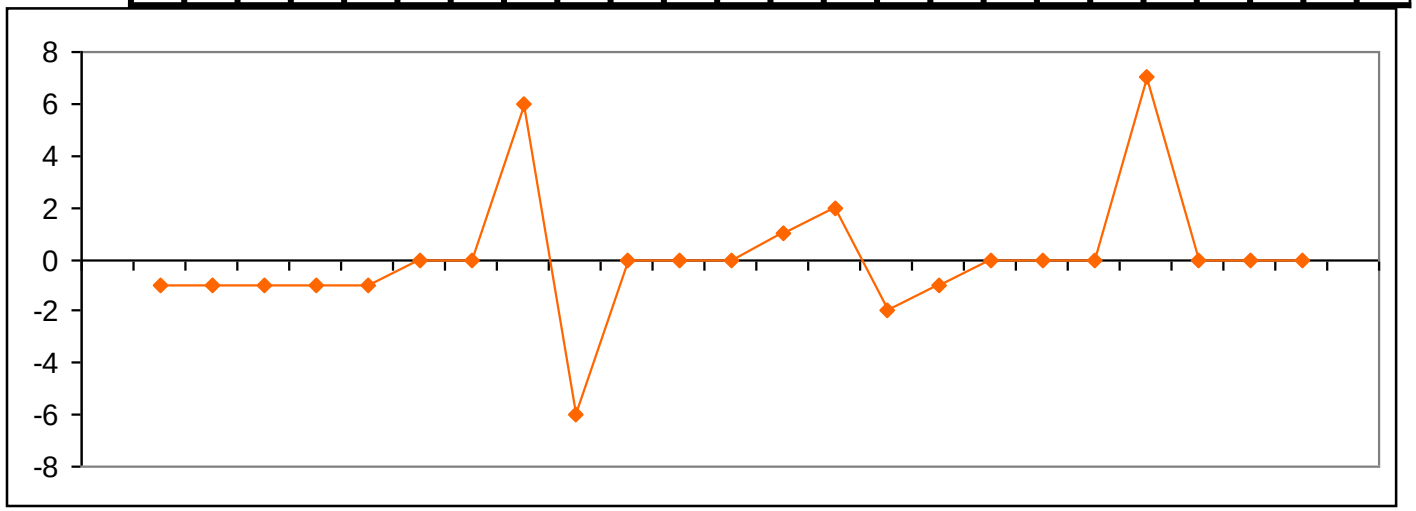
$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

Turunan Pertama

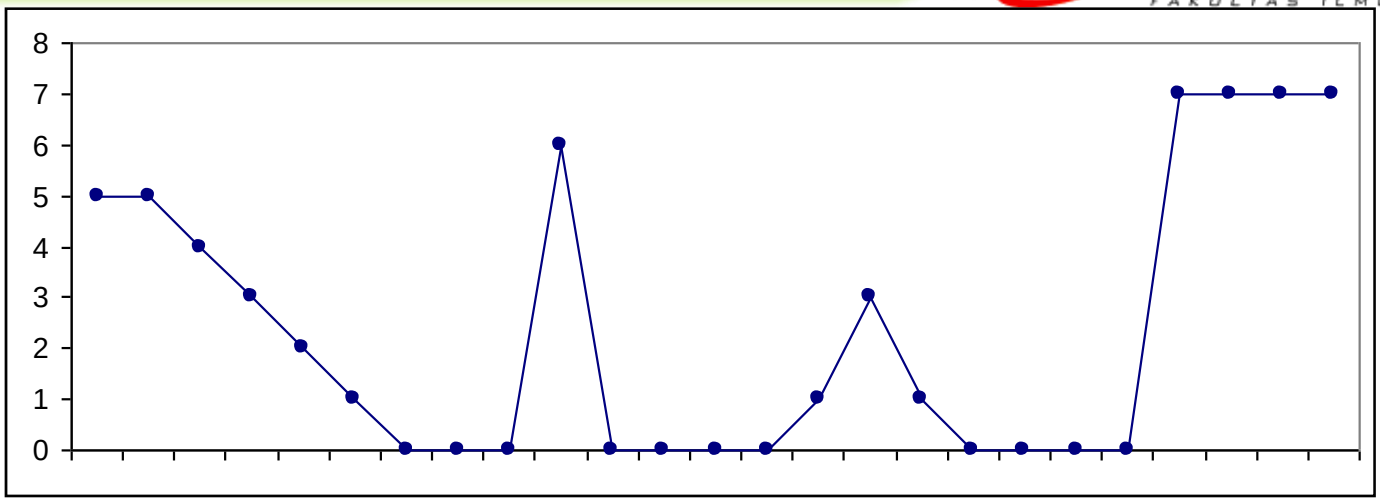


	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7

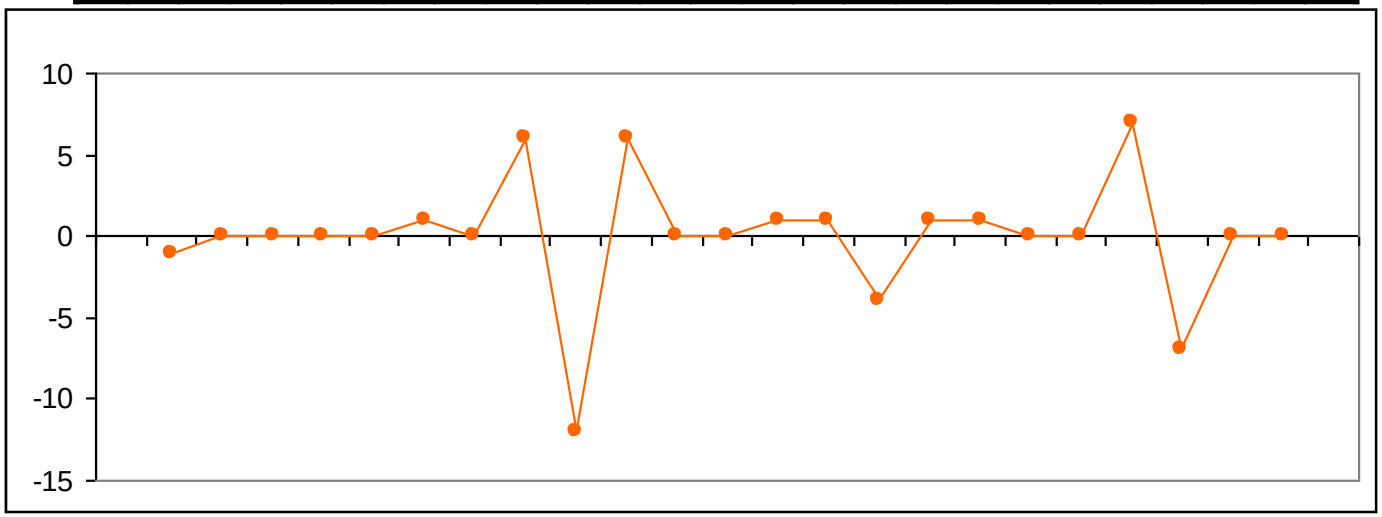
-1	-1	-1	-1	-1	0	0	6	-6	0	0	0	1	2	-2	-1	0	0	0	7	0	0	0	



Turunan Kedua



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7
	-1	0	0	0	0	1	0	6	-12	6	0	0	1	1	-4	1	1	0	0	7	-7	0	0	



Laplacian Filter

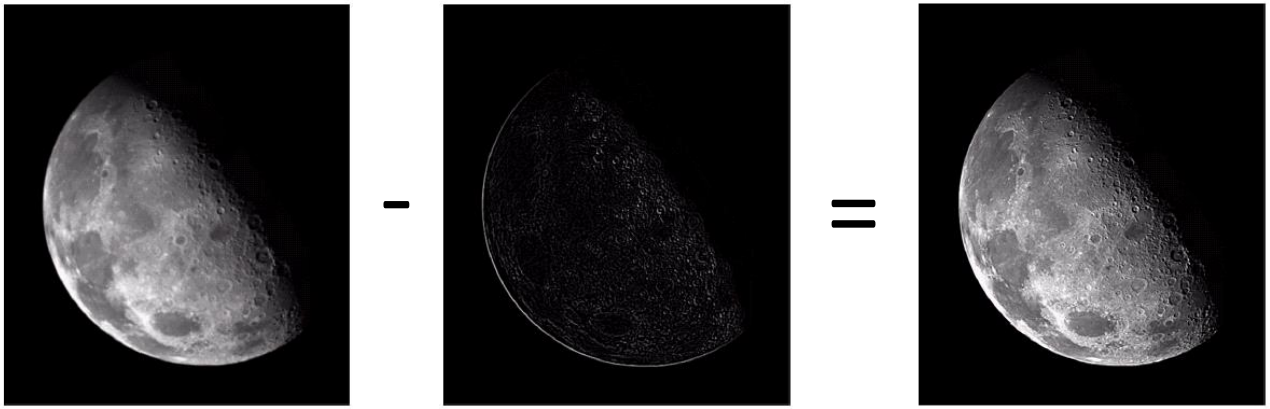
Laplacian dapat dibentuk sbb:

$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] - 4f(x,y)$$

Berdasarkan formula tersebut dapat dibentuk spasial filter sbb.:

0	1	0
1	-4	1
0	1	0

Laplacian Image Enhancement



Citra asli

Citra terfilter Laplacian

Citra akhir yg Lbh tajam kenampakannya

$$g(x,y) = f(x,y) - \nabla^2 f$$

Pada citra hasil akhir, batas tepi dan kedetilan unsur tampak lebih jelas

Laplacian Image Enhancement

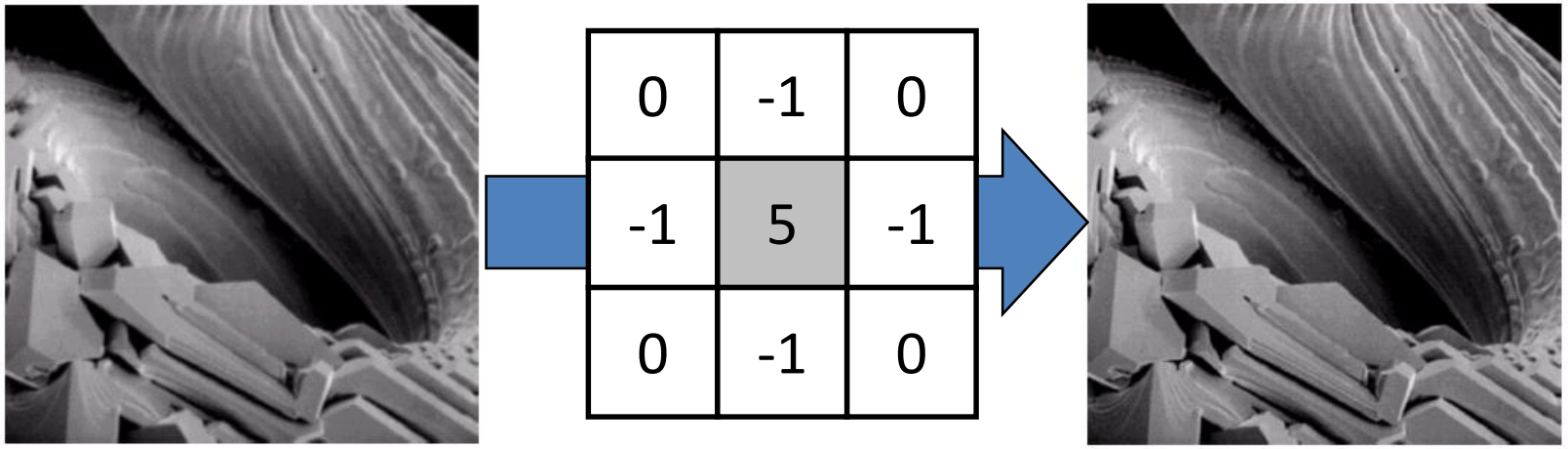
Penyederhanaan Perbaikan Citra
(Filter Laplacian)

Seluruh proses pada slide sebelumnya dapat dikombinasikan menjadi operasi filter tunggal:

$$
\begin{aligned}
g(x,y) &= f(x,y) - \nabla^2 f \\
&= f(x,y) - [\, f(x+1,y) + f(x-1,y) \\
&\quad + f(x,y+1) + f(x,y-1) - 4f(x,y)\,] \\
&= 5f(x,y) - f(x+1,y) - f(x-1,y) \\
&\quad - f(x,y+1) - f(x,y-1)
\end{aligned}
$$

Penyederhanaan Perbaikan Citra
(Filter Laplacian)

Filter spasial baru yg menggabungkan seluruh operasi menjadi operasi filter tunggal:



Varian Pada Filter Laplacian Sederhana

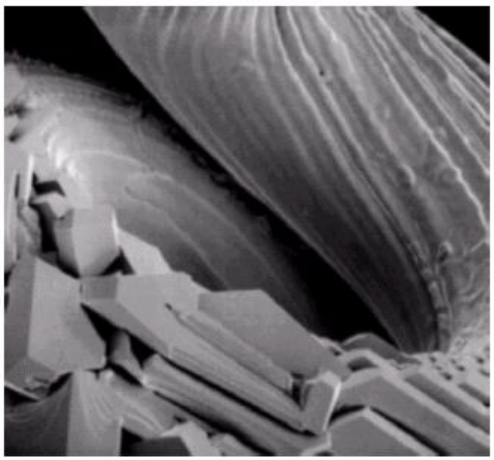
Beberapa jenis varian dr filter laplacian:

0	1	0
1	-4	1
0	1	0

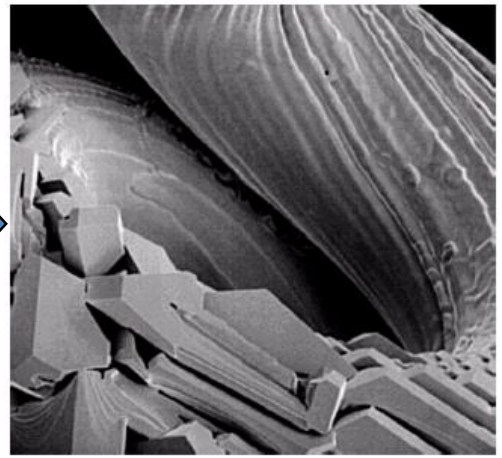
Simple Laplacian

1	1	1
1	-8	1
1	1	1

Variant of Laplacian



-1	-1	-1
-1	9	-1
-1	-1	-1



Laplacian Filter

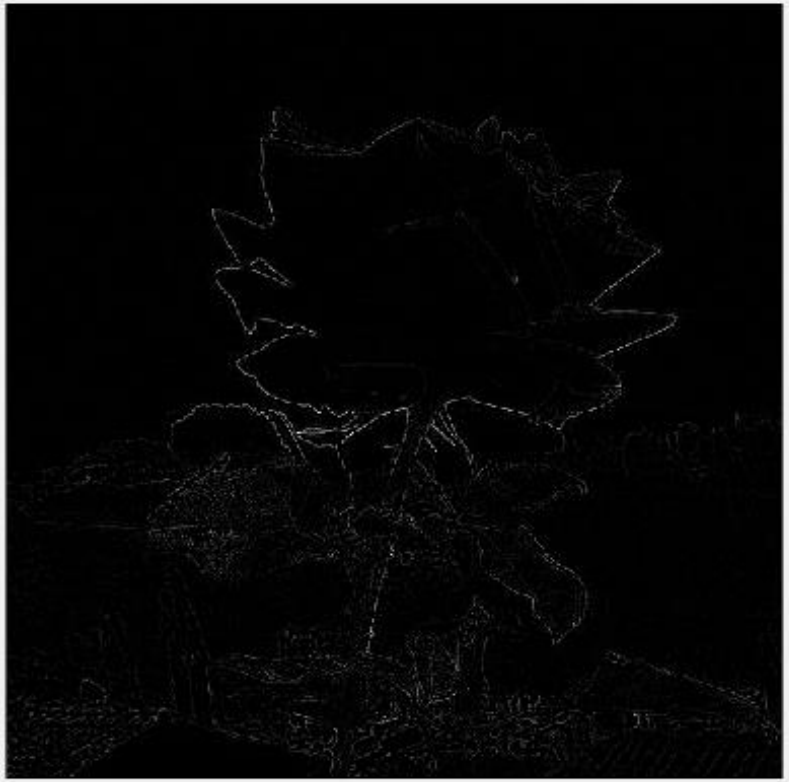
+ Rotation invariant; needs only one convolution mask
- Responds doubly to some edges
- Noise sensitive

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

−1	2	−1
2	−4	2
−1	2	−1

Hasil dari *Laplacian Filter*



Hasil dari *Laplacian Filter*

Filter Turunan Pertama (*1st Derivative Filtering*)

Implementasi turunan filter pertama sulit untuk diaplikasikan

fungsi $f(x, y)$, gradien dari f pd koordinat (x, y) diberikan sebagai vektor kolom:

$$\nabla \mathrm{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

Filter Turunan Pertama

magnitut dari vektor ini diberikan sbb:

$$\nabla f = mag(\nabla \mathrm{f})$$

$$= \left[G_x^2 + G_y^2\right]^{1/2}$$

$$= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}$$

Untuk alasan praktis dapat disederhanakan sbb:

$$\nabla f \approx |G_x| + |G_y|$$

Filter Turunan Pertama

Gradien dapat dihitung sbb:

$$\nabla f \approx \left|(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)\right|$$
$$+ \left|(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)\right|$$

Yang didasarkan pada koordinat sbb.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Operasi Sobel

Berdasarkan persamaan sebelumnya, dapat diturunkan operasi sobel:

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

Pada citra diaplikasikan masing-masing filter kemudian hasilnya ditambahkan.

Contoh Perhitungan Operasi Sobel

- Consider a matrix A with size 4x4 $\begin{bmatrix} 1 & 2 & 2 & 1 \\ 1 & 1 & 0 & 3 \\ 2 & 4 & 1 & 5 \\ 2 & 1 & 2 & 0 \end{bmatrix}$
- Consider the elements in the window 3x3 $\begin{bmatrix} 1 & 2 & 2 \\ 1 & 1 & 0 \\ 2 & 4 & 1 \end{bmatrix}$
- Find the x-direction derivative: Subtract the first row from the Third row using the mask.

$$Gx=(z_7+2z_8+z_9)-(z_1+2z_2+z_3)$$

$$Gx(1,1)=(2+(2*4)+1)-(1+(2*2)+2)=4$$

Contoh Perhitungan Operasi Sobel

- Find the y-direction derivative: Subtract the first column from the Third column using the mask.

$$Gy=(z_3+2z_6+z_9)-(z_1+2z_4+z_7)$$

$$Gy(1,1)=(2+(2*0)+1)-(1+(2*1)+2)=-2$$

- Find the Gradient:

$$Gx^2=16.\quad Gy^2=4$$

$$\Delta f=[16+4]^{1/2}=(20)^{1/2}\text{ , ie. } f(1,1)=4.4721.$$

The procedure is done for the whole image matrix.

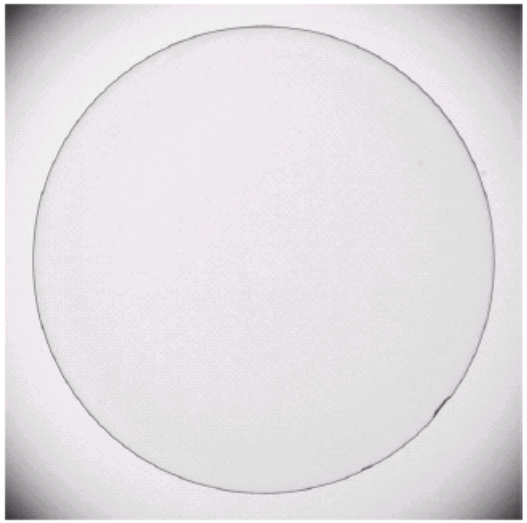
Contoh Hasil Operasi Sobel

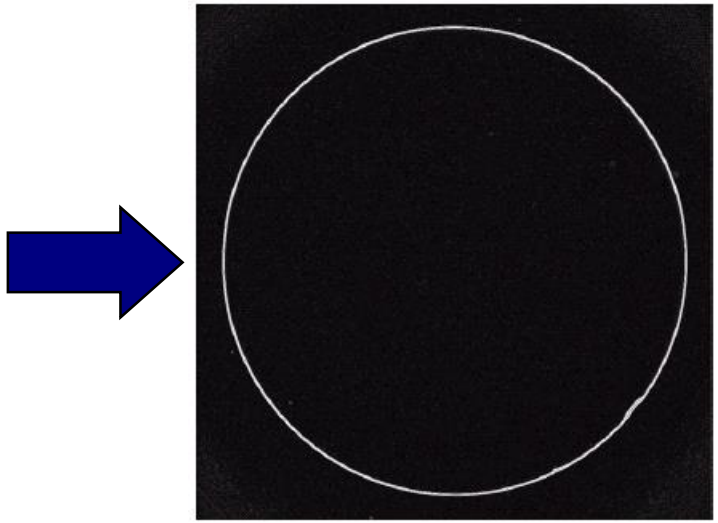
Contoh Hasil Operasi Sobel

original image

Sobel gradient

Contoh Hasil Operasi Sobel





Citra kontak lens, diaplikasikan filter utk mendeteksi cacat pd pinggirnya

Filter Sobel digunakan untuk deteksi tepi.

Thank You.....