Given:

$$F = 2\ kN, \qquad V = 1.2 m/s, \qquad d_{pulley} = 350\ mm$$

$$S_{ultimate} = 720\ Mpa, \qquad S_{yield} = 620\ MPa, \qquad Hardness = 350\ HB$$

$$\eta_{gears} = 0.96, \qquad \eta_{bearings} = 0.99, \qquad k_m = 2, \qquad k_t = 1.5$$

Solution:

1- Calculating the required power and velocity:

$$P_{out} = F_{conveyor} * V_{conveyor} = 2.4\ kW$$

$$\omega_{out} = \frac{2V_{conveyor}}{d_{pulley}} = \frac{48}{7} rad/s$$

$$N_{out} = \frac{30\omega_{out}}{\pi} = 65.48\ rpm$$

2- Motor selection:

Using a 2-stage gearbox the required input power can be calculated as follows:

$$P_{in} = \frac{P_{out}}{{\eta_{gears}}^2 * {\eta_{bearings}}^6} = 2.766\ kW$$

From Appendix 1. Series 4A Three-Phase Induction Motors, we choose a suitable motor for the required input power.

Selection: (4A112MB8) P = 3 kW & N = 700 rpm.

$$Selected\ motor \rightarrow 4A112MB8$$

3- Reduction ratios:

$$i_{total} = \frac{N_{in}}{N_{out}} = 10.69 \cong 3 * 3.55$$

$$\therefore i_{g12} = 3, \qquad i_{g34} = 3.55$$

4- Calculating torque:

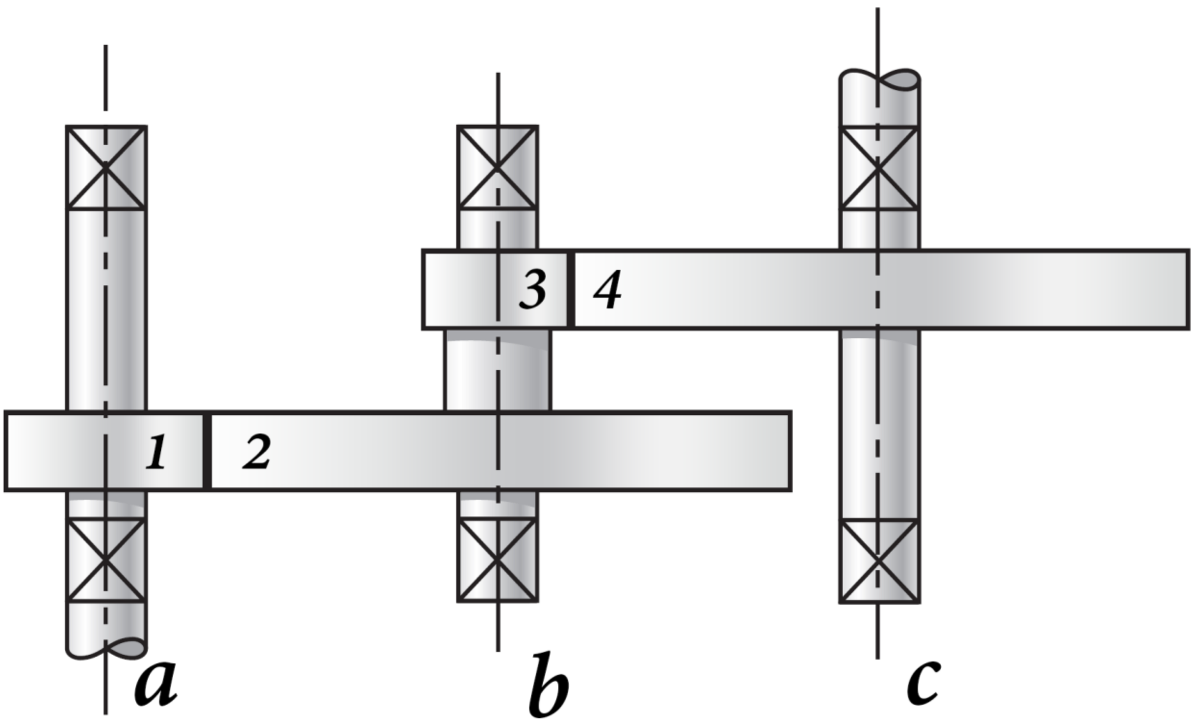
$$\omega_{motor} = \frac{2\pi N}{60} = \frac{2\pi * 700}{60} = 73.3\, rad/s$$

$$T_{motor} = \frac{P_{motor}}{\omega_{motor}} = \frac{3 * 10^3}{73.3} = 40.93\ N.m$$

$$T_a = T_{motor} * {\eta_{bearings}}^2 = 40.93 * 0.99^2 = 40.11\ N.m$$

$$T_b = T_a * i_{g12} * \eta_{g12} * {\eta_{bearings}}^2 = 40.11\ * 3 * 0.96 * 0.99^2$$
$$= 113.22\ N.m$$

$$T_c = T_b * i_{g34} * \eta_{g34} * {\eta_{bearings}}^2 = 113.22 * 3.55 * 0.96 * 0.99^2$$
$$= 378.2\ N.m$$



5- 1st Stage analysis:

$T = 40.11\ N.m$

$N = 700\ rpm$

$Z_p = 20$

$Z_g = 60$

$\varphi = 20°$

Insert data to the software included in this project and obtain the results.

$m = 2.5\ mm, \quad F = 32\ mm$

$D_p = 50\ mm, \quad D_g = 150\ mm$

$W_t = 1.6\ kN, \quad W_r = 0.58\ kN$

$Fatigue\ F.O.S = 1.87$

$Surface\ durability\ F.O.S = 1.49$

Checking software calculations:

A. Fatigue check:

$m = 2.5\ mm$

$d_p = mZ_p = 2.5 * 20 = 50\ mm$

$F = 4\pi m = 31.4\ mm \cong 32\ mm$

$K_o = 1.25, \qquad K_m = 1.6$

$$J = 0.35804 + \left(\frac{60-50}{85-50}\right)(0.36532 - 0.35804) = 0.36012$$

$$V = \frac{\pi d_p N}{60} = \frac{\pi * 50 * 700}{60 * 10^3} = 1.83\ m/s$$

$$K_v = \frac{50}{50+\sqrt{200V}} = \frac{50}{50+\sqrt{200 * 1.83}} = 0.723$$

$$W_t = \frac{2T}{d_p} = \frac{2 * 40.11 * 10^3}{50} = 1604.4\ N = 1.6\ kN$$

$W_r = W_t \tan\varphi = 1.6 \tan 20° = 0.58\ kN$

$$\overline{S_e} = Smallest\ of\ \begin{cases} 0.5S_u = 360\ MPa \\ 0.8S_y = 496\ MPa \end{cases} = 360\ MPa$$

$S_e = K_a K_b K_c K_d K_e K_f \overline{S_e}$

$K_a = 0.715, \qquad K_b = 0.974, \qquad K_c = 0.868$

$K_d = 1, \qquad K_e = 1, \qquad K_f = 1.33$

$S_e = 0.715 * 0.974 * 0.868 * 1.33 * 360 = 289.43\ MPa$

$$n_f = \frac{S_e K_v F m J}{K_o K_m W_t} = \frac{289.43 * 0.723 * 32 * 2.5 * 0.36012}{1.25 * 1.6 * 1604.4} = 1.88$$

B. Surface durability check:

$$S_c = 2.76HB - 70 = 2.76 * 350 - 70 = 896\ MPa$$

$$S_H = \frac{C_L C_H}{C_T C_R} S_c$$

$$C_L = 1, \quad C_H = 1, \quad C_T = 1, \quad C_R = 0.8$$

$$S_H = \frac{896}{0.8} = 1120\ MPa$$

$$I = \frac{\cos\varphi \sin\varphi}{2}\left(\frac{m_G}{m_G + 1}\right) = \frac{\cos 20° \sin 20°}{2}\left(\frac{3}{3+1}\right) = 0.12$$

$$C_p = 191\ MPa^{0.5}, \quad C_v = 0.723, \quad C_o = 1.25, \quad C_m = 1.6$$

$$n_s = \frac{\left(\frac{S_H}{C_p}\right)^2 C_v F d_p I}{C_o C_m W_t} = \frac{\left(\frac{1120}{191}\right)^2 * 0.723 * 32 * 50 * 0.12}{1.25 * 1.6 * 1604.4} = 1.49$$

Hence, the software calculations are correct.

6- 2nd Stage analysis:

$T = 113.22\ N.m$

$N = 233.33\ rpm$

$Z_p = 20$

$Z_g = 71$

$\varphi = 20°$

Insert data to the software included in this project and obtain the results.

$$m = 3.5\ mm, \qquad F = 44\ mm$$

$$D_p = 70\ mm, \qquad D_g = 248.5\ mm$$

$$W_t = 3.23\ kN, \qquad W_r = 1.18\ kN$$

$$Fatigue\ F.O.S = 1.91$$

$$Surface\ durability\ F.O.S = 1.63$$

Checking software calculations:

A. Fatigue check:

$m = 3.5\ mm$

$d_p = mZ_p = 3.5 * 20 = 70\ mm$

$F = 4\pi m = 44\ mm$

$K_o = 1.25, \qquad K_m = 1.6$

$$J = 0.35804 + \left(\frac{71 - 50}{85 - 50}\right)(0.36532 - 0.35804) = 0.362408$$

$$V = \frac{\pi d_p N}{60} = \frac{\pi * 70 * 233.33}{60 * 10^3} = 0.855\ m/s$$

$$K_v = \frac{50}{50 + \sqrt{200V}} = \frac{50}{50 + \sqrt{200 * 0.855}} = 0.793$$

$$W_t = \frac{2T}{d_p} = \frac{2 * 113.22 * 10^3}{70} = 3234.86\ N = 3.23\ kN$$

$W_r = W_t \tan\varphi = 3.23 \tan 20° = 1.18\ kN$

$\overline{S_e} = 360\ MPa$

$S_e = K_a K_b K_c K_d K_e K_f \overline{S_e}$

$K_a = 0.715, \qquad K_b = 0.942, \qquad K_c = 0.868$

$K_d = 1, \qquad K_e = 1, \qquad K_f = 1.33$

$S_e = 0.715 * 0.942 * 0.868 * 1.33 * 360 = 279.92\ MPa$

$$n_f = \frac{S_e K_v F m J}{K_o K_m W_t} = \frac{279.92 * 0.793 * 44 * 3.5 * 0.362408}{1.25 * 1.7 * 3234.86} = 1.92$$

B. Surface durability check:

$$S_c = 2.76HB - 70 = 2.76 * 350 - 70 = 896\ MPa$$

$$S_H = \frac{C_L C_H}{C_T C_R} S_c$$

$$C_L = 1, \qquad C_H = 1, \qquad C_T = 1, \qquad C_R = 0.8$$

$$S_H = \frac{896}{0.8} = 1120\ MPa$$

$$I = \frac{\cos\varphi \sin\varphi}{2}\left(\frac{m_G}{m_G + 1}\right) = \frac{\cos 20° \sin 20°}{2}\left(\frac{3.55}{3.55 + 1}\right) = 0.125$$

$$C_p = 191\ MPa^{0.5}, \qquad C_v = 0.723, \qquad C_o = 1.25, \qquad C_m = 1.6$$

$$n_s = \frac{\left(\frac{S_H}{C_p}\right)^2 C_v F d_p I}{C_o C_m W_t} = \frac{\left(\frac{1120}{191}\right)^2 * 0.793 * 44 * 70 * 0.12}{1.25 * 1.6 * 3234.86} = 1.62$$

Hence, the software calculations are correct.

7- Shaft design & bearing selection:

$$\tau_{all} = Smallest\ of \begin{cases} 0.75 * 0.18 * S_u = 97.2\ MPa \\ 0.75 * 0.3 * S_y = 139.5\ MPa \end{cases} = 97.2\ Mpa$$

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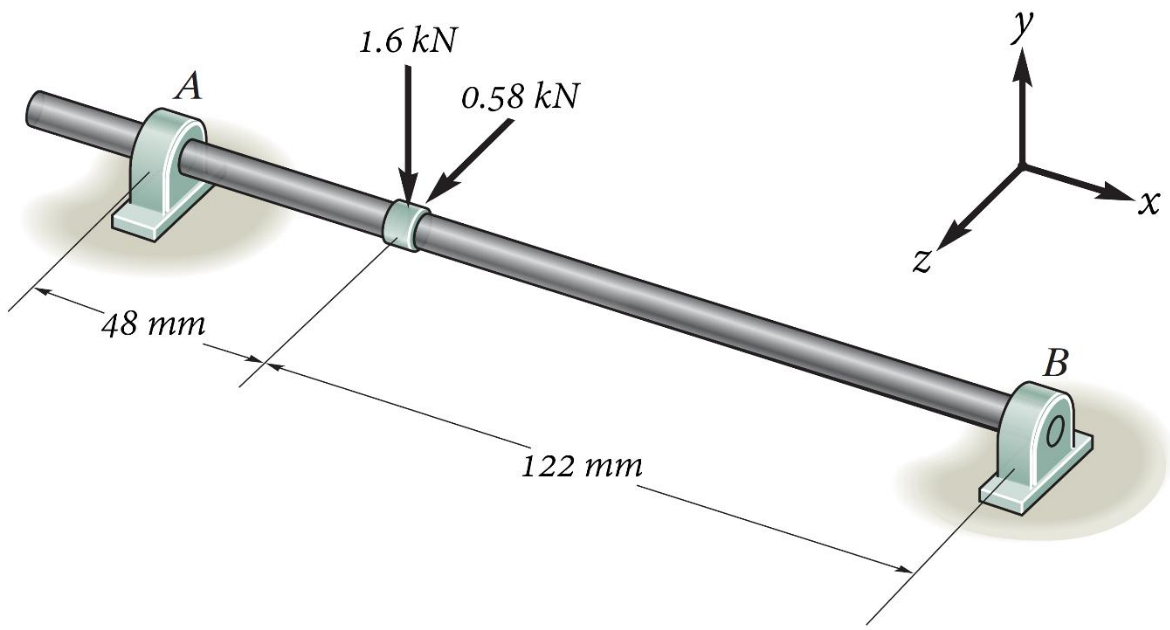
1) Input shaft:



$$T = 40.11\ N.m$$

$$\mathrm{F}_g = \sqrt{(1.6)^2 + (0.58)^2} = 1.7\ kN$$

$$\sum M_B = 0$$

$$1.7 * 122 - \mathrm{R}_A * 170 = 0 \rightarrow \mathrm{R}_A = 1.22\ kN$$

$$M_{max} = \mathrm{R}_A * 48 = 58.56\ N.m$$

$$d_{min} = \sqrt[3]{\frac{16\sqrt{(K_m M)^2 + (K_t T)^2}}{\pi \tau_{all}}}$$

$$d_{min} = \sqrt[3]{\frac{16 * 10^3 * \sqrt{(2 * 58.56)^2 + (1.5 * 40.11)^2}}{97.2\pi}} = 19\ mm$$

$$d_{min} = 20\ mm$$

- **Bearing selection:**

Apparently the maximum load is on bearing A which we will choose and use the same bearing at B for symmetry.

$$(F_r = 1.22\ kN, \qquad F_a = 0) \rightarrow \left(\frac{F_a}{F_r} = 0 < e\right) \rightarrow X = 1, \qquad Y = 0$$

$$P = XVF_r + YF_a = F_r = 1220\ N$$

$$Taking\ L_h = 20{,}000\ hr$$

$$C = P * \left(\frac{60N\ L_h}{10^6}\right)^{\frac{1}{k}} \rightarrow L_h = 20{,}000\ hr, \qquad N = 700\ rpm, \qquad k = 3$$

$$C_{calculated} = 11511\ N \rightarrow From\ table\ 23.8\ we\ choose\ bearing\ 6304$$

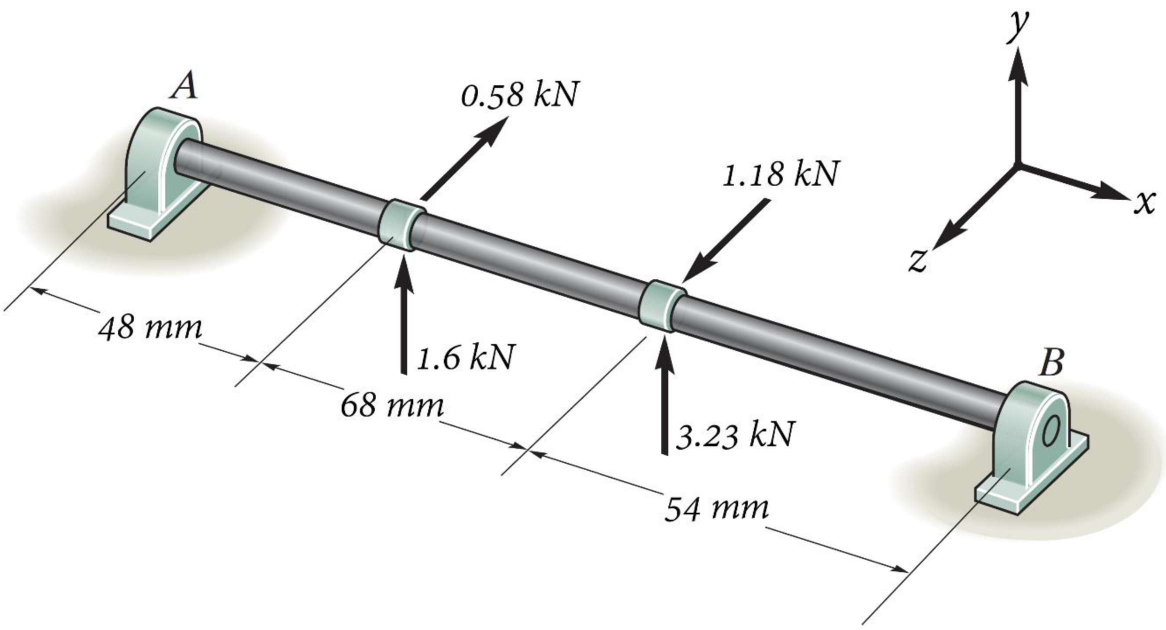
$$P = 1220\ N, \qquad C_{tabulated} = 12200\ N$$

$$L_h = \left(\frac{C}{P}\right)^k * \frac{10^6}{60N} = 23{,}810\ hr$$

Hence for the input shaft, we will use 6304 bearings.

$$Selected\ bearing \rightarrow 6304$$

2) Intermediate shaft:



$T = 113.22\ N.m$

$\sum (M_B)_z = 0$

$-(R_A)_y * 170 - 1.6 * 122 - 3.23 * 54 = 0 \rightarrow (R_A)_y = -2.17\ kN$

$\sum (M_B)_y = 0$

$(R_A)_z * 170 + 1.18 * 54 - 0.58 * 122 = 0 \rightarrow (R_A)_z = 0.04\ kN$

$\sum F_y = 0 \rightarrow (R_B)_y = -2.66\ kN$

$\sum F_z = 0 \rightarrow (R_B)_z = -0.64\ kN$

$R_A = \sqrt{(R_A)_y{}^2 + (R_A)_z{}^2} = 2.17\ kN$

$R_B = \sqrt{(R_B)_y{}^2 + (R_B)_z{}^2} = 2.74\ kN$

$M_{max} = R_B * 54 = 148\ N.m$

$$d_{min} = \sqrt[3]{\frac{16\sqrt{(K_m M)^2 + (K_t T)^2}}{\pi \tau_{all}}}$$

$$d_{min} = \sqrt[3]{\frac{16 * 10^3 * \sqrt{(2 * 148)^2 + (1.5 * 113.22)^2}}{97.2\pi}} = 26.15\ mm$$

$$d_{min} = 30\ mm$$

- **Bearing selection:**

Apparently the maximum load is on bearing B which we will choose and use the same bearing at A for symmetry.

$(F_r = 2.74\ kN, \quad F_a = 0) \rightarrow \left(\frac{F_a}{F_r} = 0 < e\right) \rightarrow X = 1, \quad Y = 0$

$P = XVF_r + YF_a = F_r = 2740\ N$

$Taking\ L_h = 20{,}000\ hr$

$C = P * \left(\frac{60N\ L_h}{10^6}\right)^{\frac{1}{k}} \rightarrow L_h = 20{,}000\ hr, \quad N = 233.33\ rpm, \quad k = 3$

$C_{calculated} = 17925\ N \rightarrow From\ table\ 23.8\ we\ choose\ bearing\ 6306$

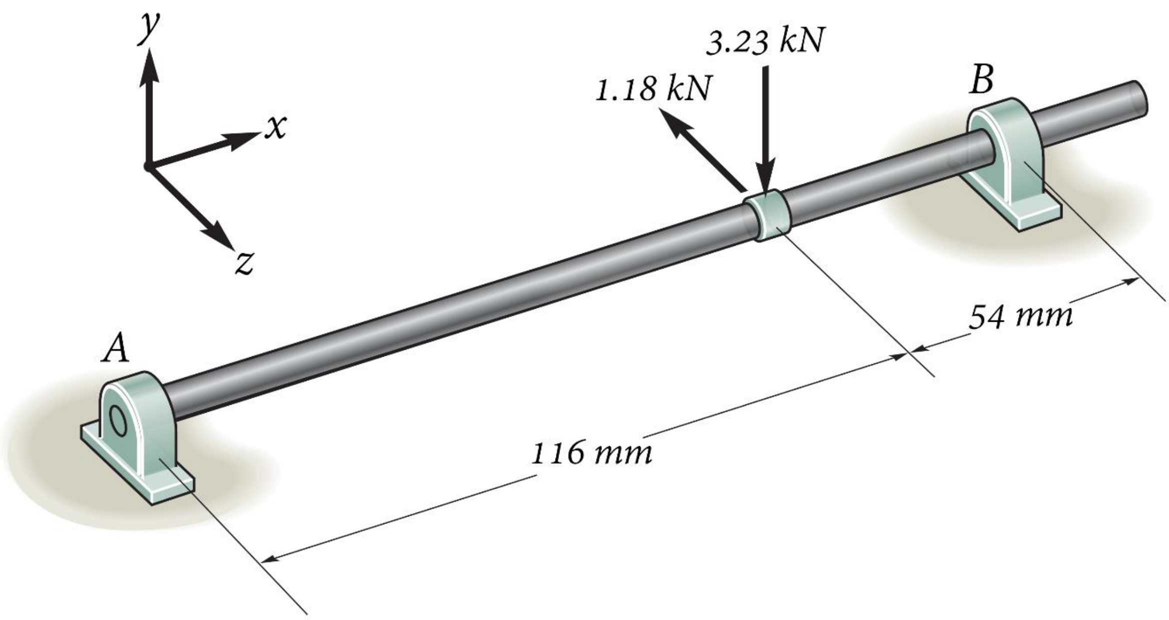
$P = 2740\ N, \quad C_{tabulated} = 21600\ N$

$L_h = \left(\frac{C}{P}\right)^k * \frac{10^6}{60N} = 35{,}000\ hr$

Hence for the intermediate shaft, we will use <u>6306</u> bearings.

$$Selected\ bearing \rightarrow 6306$$

3) Output shaft:



$T = 378.2\ N.m$

$F_g = \sqrt{(3.23)^2 + (1.18)^2} = 3.44\ kN$

$\sum M_A = 0$

$R_B * 170 - 3.44 * 116 = 0 \rightarrow R_B = 2.35\ kN$

$M_{max} = R_B * 54 = 127\ N.m$

$$d_{min} = \sqrt[3]{\frac{16\sqrt{(K_m M)^2 + (K_t T)^2}}{\pi \tau_{all}}}$$

$$d_{min} = \sqrt[3]{\frac{16 * 10^3 * \sqrt{(2 * 127)^2 + (1.5 * 378.2)^2}}{97.2\pi}} = 31.9\ mm$$

$$d_{min} = 35\ mm$$

- **Bearing selection:**

Apparently the maximum load is on bearing B which we will choose and use the same bearing at A for symmetry.

$$(\mathrm{F}_r = 2.35\, kN, \qquad \mathrm{F}_a = 0) \rightarrow \left(\frac{\mathrm{F}_a}{\mathrm{F}_r} = 0 < e\right) \rightarrow X = 1, \qquad Y = 0$$

$$P = XV\mathrm{F}_r + Y\mathrm{F}_a = \mathrm{F}_r = 2350\, N$$

$$Taking\ L_h = 20{,}000\ hr$$

$$C = P * \left(\frac{60N\ L_h}{10^6}\right)^{\frac{1}{k}} \rightarrow L_h = 20{,}000\ hr, \qquad N = 65.73\ rpm, \qquad k = 3$$

$$C_{calculated} = 10078\ N \rightarrow From\ table\ 23.8\ we\ choose\ bearing\ 6007$$

$$P = 2350\ N, \qquad C_{tabulated} = 12200\ N$$

$$L_h = \left(\frac{C}{P}\right)^k * \frac{10^6}{60N} = 35{,}500\ hr$$

Hence for the output shaft, we will use <u>6007</u> bearings.

$$Selected\ bearing \rightarrow 6007$$