Institut Teknologi Sepuluh Nopember
Surabaya

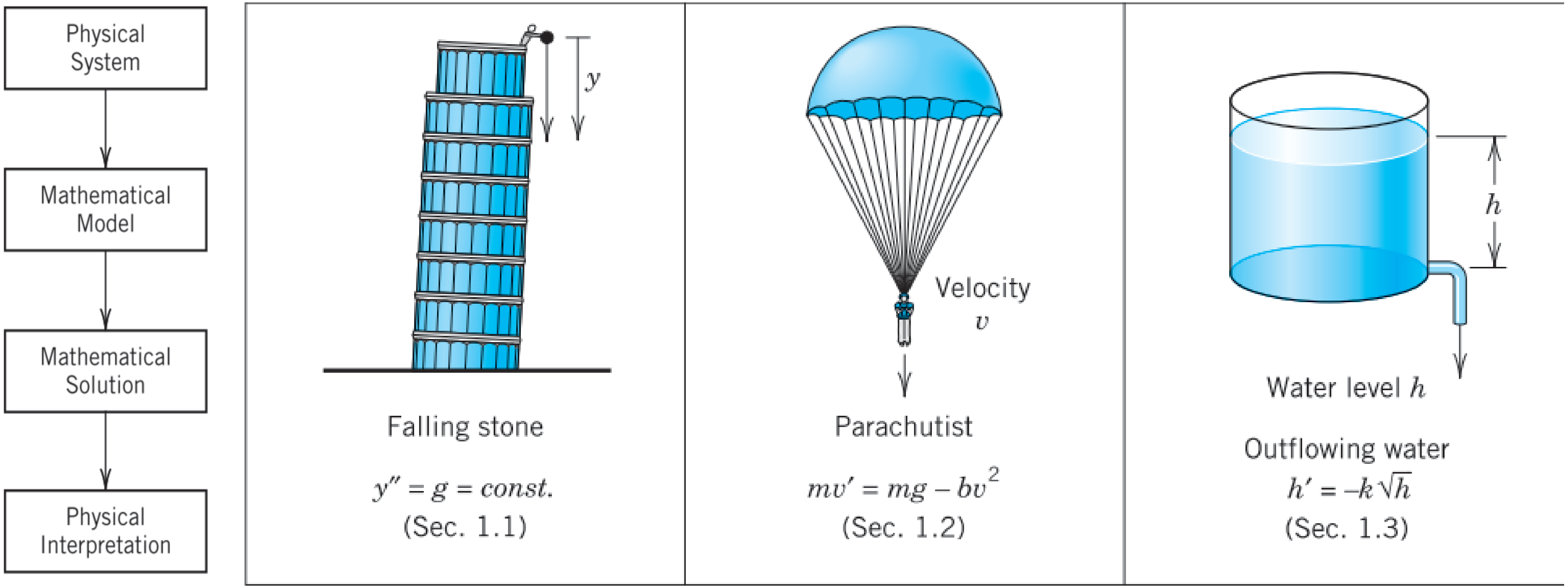
JURUSAN TEKNIK FISIKA - FTI

M1. PERSAMAAN DIFFERENSIAL

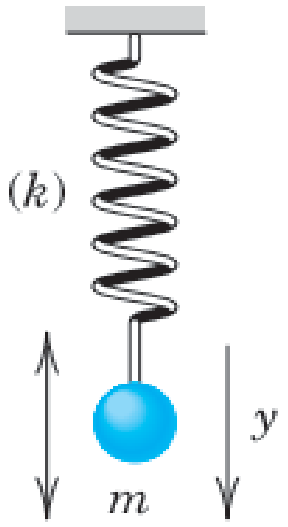
Aulia Siti Aisjah

MATEMATIKA REKAYASA 1

Bidang Teknik Fisika



Sistem Mekanik, Elektrik

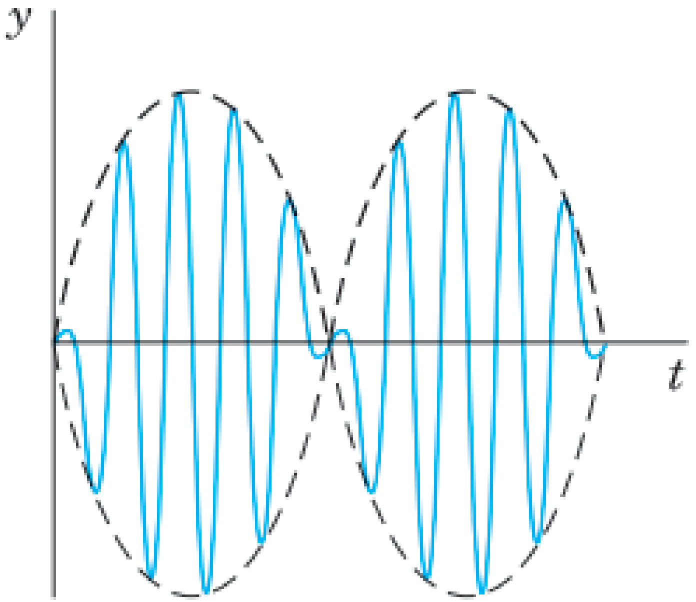


Displacement y

Vibrating mass on a spring

$my'' + ky = 0$

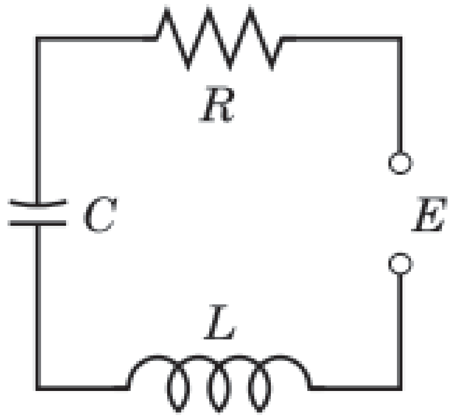
(Secs. 2.4, 2.8)



Beats of a vibrating system

$y'' + \omega_0^2 y = \cos \omega t, \quad \omega_0 \approx \omega$

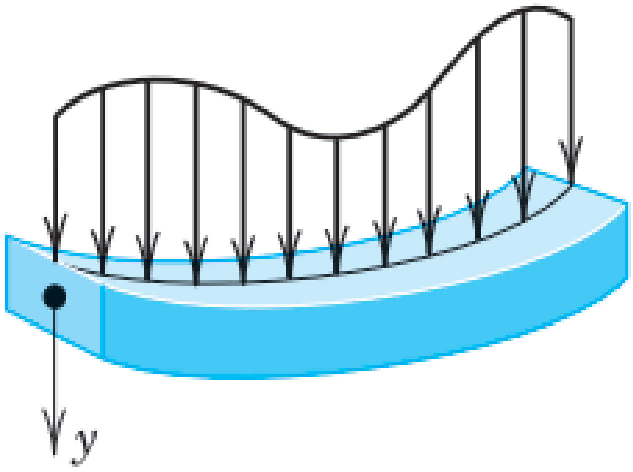
(Sec. 2.8)



Current I in an RLC circuit

$LI'' + RI' + \frac{1}{C} I = E'$

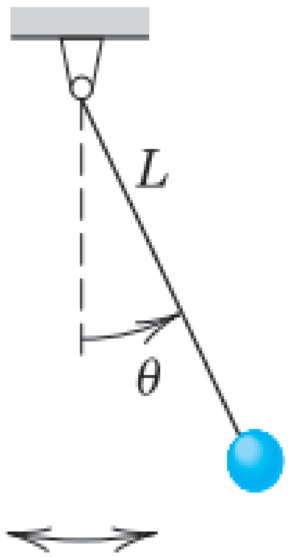
(Sec. 2.9)



Deformation of a beam

$$EIy^{iv} = f(x)$$

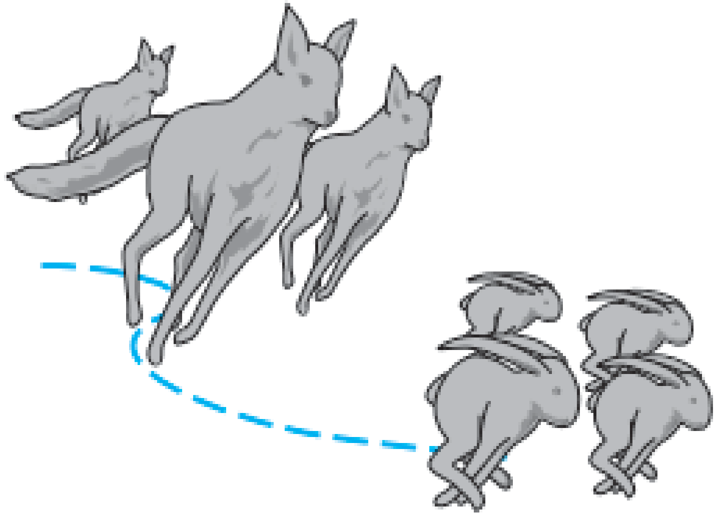
(Sec. 3.3)



Pendulum

$$L\theta'' + g \sin \theta = 0$$

(Sec. 4.5)



Lotka–Volterra predator–prey model

$$y_1' = ay_1 - by_1y_2$$
$$y_2' = ky_1y_2 - ly_2$$

(Sec. 4.5)

Persamaan Differensial – Biasa (*Ordinary Diif. Eq*) - ODE

- Pers. Differensial yang terdiri dari sebuah fungsi yang tidak diketahui dan turunan nya dikatakan sebagai pers. Differensial (PD)
- PD merupakan sebuah bentuk yang mempunyai peran yang sangat penting (di dalam bidang Teknik fisika PD merupakan model system / model proses) sebagai sebuah formula matematik.

$$\frac{dv}{dt} = g - \frac{c}{m} v$$

v- variable terikat (dependent)

t- variable bebas (independent)

ODE banyak digunakan di berbagai bidang

Contoh

- Semua cabang ilmu teknik
- Ekonomi
- Biologi dan kesehatan
- Kimia, fisika dll

Hukum Newton - pendinginan

Bagaimana suhu obyek (fluida di dalam cangkir) akan berubah pada saat panas dari fluida tersebut hilang ke lingkungan nya

Suhu Obyek: T_{Obj} Suhu Ruang: T_{Room}

Newton's laws states: *"The rate of change in the temperature of an object is proportional to the difference in temperature between the object and the room temperature"*

Form ODE

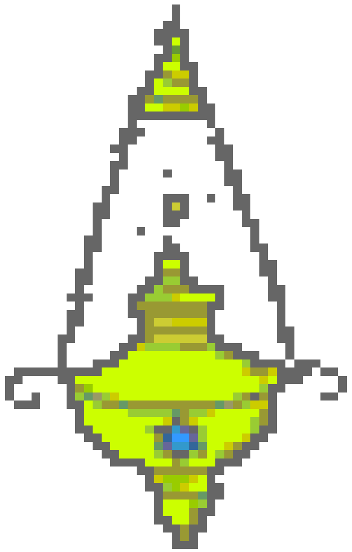
$$\frac{dT_{Obj}}{dt} = -\alpha(T_{Obj} - T_{Room})$$

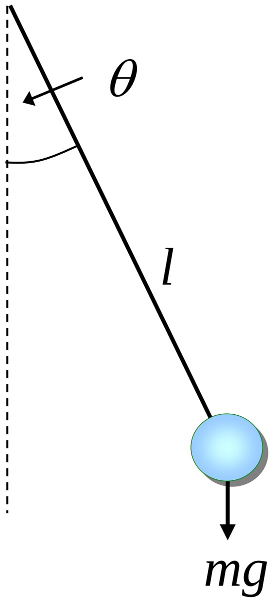
Solve ODE

$$T_{Obj} = T_{Room} + (T_{init} - T_{Room})e^{-\alpha t}$$

Where T_{init} is the initial temperature of the object.

Contoh – Sebuah pendulum





Newton's 2nd law for a rotating object:

- Moment of inertia x angular acceleration = Net external torque

$$ml^2 \cdot \frac{d^2\theta}{dt^2} = -mgl\sin\theta$$

Susun ulang dengan pembagi ml^2

$$\frac{d^2\theta}{dt^2} + \omega^2 \sin\theta = 0$$

dimana $\omega^2 = \frac{g}{l}$

Apakah sulit menyelesaikan PD di atas?

Beberapa definisi / istilah di dalam ODE

- Order
- Linier
- Homogen
- Kondisi awal / kondisi batas

Orde sebuah PD

- Orde sebuah PD ditandai oleh turunan tertinggi variable terikatnya terhadap variable bebas nya.

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} = 0 \quad \Rightarrow \quad \text{2nd order}$$

$$\frac{dx}{dt} = x\frac{d^3x}{dt^3} \quad \Rightarrow \quad \text{3rd order}$$

Linieritas

- Persamaan linier melibatkan variabel dependen (y) dan turunannya sendiri. Tidak boleh ada fungsi nonlinier yang dikategorikan "tidak biasa" dari y atau turunannya.
- Persamaan linier harus memiliki koefisien konstan, atau koefisien yang bergantung pada variabel independen (t). Jika y atau turunannya muncul di koefisien, persamaannya tidak linier.

Contoh Mana PD dikatakan Linier

$$\frac{dy}{dt} + y = 0$$

$$\frac{dx}{dt} + x^2 = 0$$

$$\frac{dy}{dt} + t^2 = 0$$

$$y\frac{dy}{dt} + t^2 = 0$$

Cari beberapa contoh Pd dalam kategori:

1. **Linier**
2. **Non Linier**

Bentuk Linier dan non Linier

Linear	Non-linear
$2y$	y^2 or $\sin(y)$
$\frac{dy}{dt}$	$y\frac{dy}{dt}$
$(2+3\sin t)y$	$(2-3y^2)y$
$t\frac{dy}{dt}$	$\left(\frac{dy}{dt}\right)^2$

Sifat Linearity

Sebuah ODE linier, akan mempunyai solusi:

$y = f(t)$ dan $y = g(t)$

maka:

$$y = a \times f(t) + b \times g(t)$$

dimana a dan b konstan,

Juga sebagai solusi

Sifat Linearity

Contoh

$$\frac{d^2y}{dt^2}+y=0$$ solusinya $y=\sin t$ dan $y=\cos t$

Cek

$$\frac{d^2(\sin t)}{dt^2}+\sin t=-\sin t+\sin t=0$$

$$\frac{d^2(\cos t)}{dt^2}+\cos t=-\cos t+\cos t=0$$

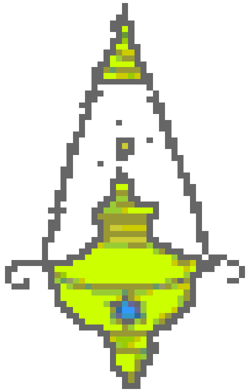
dan $y=\sin t+\cos t$ Juga sbg solusi:

cek

$$\frac{d^2(\sin t+\cos t)}{dt^2}+\sin t+\cos t$$

$$=-\sin t-\cos t+\sin t+\cos t=0$$

Approximately Linear – Swinging pendulum example



- The accurate **non-linear** equation for a swinging pendulum is:

$$\frac{d^2\theta}{dt^2} + \omega^2 \sin\theta = 0$$

- But for small angles of swing this can be **approximated** by the linear ODE:

$$\frac{d^2\theta}{dt^2} + \omega^2 \theta = 0$$

contoh

$$\frac{dv}{dt} = g$$

$$v(0) = v_0$$

- Order 1
- Linear
- nonhomogen
- Masalah Kondisi awal

$$\frac{d^2M}{dx^2} = w$$

$$M(0) = 0$$

dan

$$M(l) = 0$$

- Order 2
- Linear
- Nonhomogeneous
- Masalah nila batas

contoh

$$\frac{d^2\theta}{dt^2} + \omega^2 \sin\theta = 0$$

$$\theta(0) = \theta_0, \quad \frac{d\theta}{dt}(0) = 0$$

- Order 2
- Nonlinear
- Homogen
- Masalah nilai awal

$$\frac{d^2\theta}{dt^2} + \omega^2 \theta = 0$$

$$\theta(0) = \theta_0, \quad \frac{d\theta}{dt}(0) = 0$$

- Order 2
- Linear
- Homogen
- Masalah nilai awal

Metode Penyelesaian – Langsung

- Bentuk umum

$$\frac{dy}{dt} = f(t)$$

$$\frac{d^2y}{dt^2} = f(t)$$

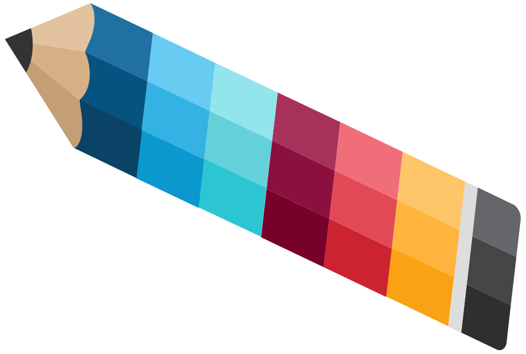
$$\vdots$$

$$\frac{d^ny}{dt^n} = f(t)$$

1–8 CALCULUS

Solve the ODE by integration or by remembering a differentiation formula.

1. $y' + 2 \sin 2\pi x = 0$
2. $y' + xe^{-x^2/2} = 0$
3. $y' = y$
4. $y' = -1.5y$
5. $y' = 4e^{-x} \cos x$
6. $y'' = -y$
7. $y' = \cosh 5.13x$
8. $y''' = e^{-0.2x}$



Soal Latihan

Tugas → NRP Genap no: 10, 14, dan sslah satu 16 – 19
NRP Ganjil, No: 9, 11 dan salah satu dr 15 - 19

9–15 VERIFICATION. INITIAL VALUE PROBLEM (IVP)

(a) Verify that y is a solution of the ODE. **(b)** Determine from y the particular solution of the IVP. **(c)** Graph the solution of the IVP.

9. $y' + 4y = 1.4, \quad y = ce^{-4x} + 0.35, \quad y(0) = 2$
10. $y' + 5xy = 0, \quad y = ce^{-2.5x^2}, \quad y(0) = \pi$
11. $y' = y + e^x, \quad y = (x + c)e^x, \quad y(0) = \frac{1}{2}$
12. $yy' = 4x, \quad y^2 - 4x^2 = c \ (y > 0), \quad y(1) = 4$
13. $y' = y - y^2, \quad y = \dfrac{1}{1 + ce^{-x}}, \quad y(0) = 0.25$
14. $y' \tan x = 2y - 8, \quad y = c \sin^2 x + 4, \quad y(\frac{1}{2}\pi) = 0$
15. Find two constant solutions of the ODE in Prob. 13 by inspection.
16. **Singular solution.** An ODE may sometimes have an additional solution that cannot be obtained from the general solution and is then called a *singular solution.* The ODE $y'^2 - xy' + y = 0$ is of this kind. Show by differentiation and substitution that it has the general solution $y = cx - c^2$ and the singular solution $y = x^2/4$. Explain Fig. 6.

19. **Free fall.** In dropping a stone or an iron ball, air resistance is practically negligible. Experiments show that the acceleration of the motion is constant (equal to $g = 9.80 \text{ m/sec}^2 = 32 \text{ ft/sec}^2$, called the **acceleration of gravity**). Model this as an ODE for $y(t)$, the distance fallen as a function of time t. If the motion starts at time $t = 0$ from rest (i.e., with velocity $v = y' = 0$), show that you obtain the familiar law of free fall

$$y = \tfrac{1}{2}gt^2.$$

20. **Exponential decay. Subsonic flight.** The efficiency of the engines of subsonic airplanes depends on air pressure and is usually maximum near 35,000 ft. Find the air pressure $y(x)$ at this height. *Physical information.* The rate of change $y'(x)$ is proportional to the pressure. At 18,000 ft it is half its value $y_0 = y(0)$ at sea level. *Hint.* Remember from calculus that if $y = e^{kx}$, then $y' = ke^{kx} = ky$. Can you see without calculation that the answer should be close to $y_0/4$?

Tugas diupload pd Myclassroom

3 Oktober 2020,
jam 24.00