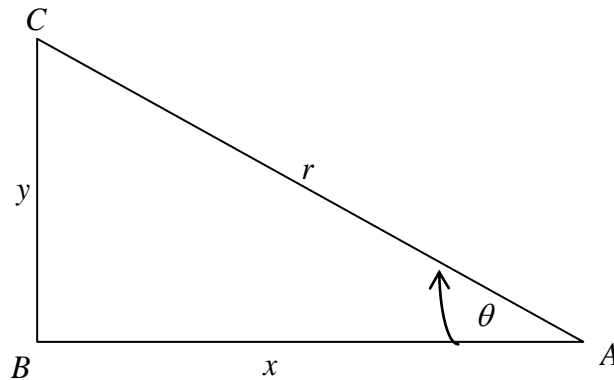


## FUNGSI TRIGONOMETRI

### Fungsi Trigonometri



Gambar 1

Pada gambar 1 di atas,  $\triangle ABC$  adalah segitiga yang salah satu sudutnya  $\theta$  dan siku-siku pada  $\angle CBA$ . Misal  $AB = x$ ,  $BC = y$  dan  $AC = r$ , berdasarkan segitiga

ABC yaitu:  $\frac{BC}{AC}$ ,  $\frac{AB}{AC}$ ,  $\frac{BC}{AB}$ ,  $\frac{AB}{AC}$ ,  $\frac{AC}{AB}$ ,  $\frac{AC}{BC}$

Karena  $\angle A = \theta$  maka perbandingan tersebut dinyatakan dengan:

$$1. \frac{BC}{AC} = \frac{y}{r} = \sin \theta$$

$$2. \frac{AB}{AC} = \frac{x}{r} = \cos \theta$$

$$3. \frac{BC}{AB} = \frac{y}{x} = \frac{BC/AC}{AB/AC} = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$4. \frac{AB}{BC} = \frac{x}{y} = \frac{AB/AC}{BC/AC} = \frac{\cos \theta}{\sin \theta} = \cot \theta$$

$$5. \frac{AC}{AB} = \frac{1}{AB/AC} = \frac{1}{x/r} = \frac{1}{\cos \theta} = \sec \theta$$

$$6. \frac{AC}{BC} = \frac{1}{BC/AC} = \frac{1}{y/r} = \frac{r}{y} = \frac{1}{\sin \theta} = \csc \theta$$

Karena  $\triangle ABC$  salah satu sudutnya siku-siku, sehingga menurut teorema Pythagoras berlaku:

$$AB^2 + BC^2 = AC^2$$

$$\Leftrightarrow x^2 + y^2 = r^2$$

Selanjutnya secara berurutan persamaan  $x^2 + y^2 = r^2$  dibagi  $x^2, y^2, r^2$  diperoleh persamaan baru

$$1. \frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$$

$$\Leftrightarrow \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

$$\Leftrightarrow (\cos \theta)^2 + (\sin \theta)^2 = 1$$

$$\Leftrightarrow \cos^2 \theta + \sin^2 \theta = 1 \dots\dots\dots(1)$$

$$2. \frac{x^2}{x^2} + \frac{y^2}{x^2} = \frac{r^2}{x^2}$$

$$\Leftrightarrow 1 + \left(\frac{y}{x}\right)^2 = \left(\frac{r}{x}\right)^2$$

$$\Leftrightarrow 1 + (\tan \theta)^2 = (\sec \theta)^2$$

$$\Leftrightarrow 1 + \tan^2 \theta = \sec^2 \theta \dots\dots\dots(2)$$

$$3. \frac{x^2}{y^2} + \frac{y^2}{y^2} = \frac{r^2}{y^2}$$

$$\Leftrightarrow \left(\frac{x}{y}\right)^2 + 1^2 = \left(\frac{r}{y}\right)^2$$

$$\Leftrightarrow (\cot \theta)^2 + 1 = (\csc \theta)^2$$

$$\Leftrightarrow \cot^2 \theta + 1 = \csc^2 \theta \dots\dots\dots(3)$$

Persamaan (1), (2), dan (3) dinamakan rumus-rumus identitas.

Selanjutnya berdasarkan perbandingan tersebut dapat dibuat beberapa rumus tentang fungsi trigonometri, berikut:

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha \dots\dots\dots(4)$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \dots\dots\dots(5)$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \dots\dots\dots(6)$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \dots\dots(7)$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \dots\dots\dots (8)$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \dots\dots\dots (9)$$

Beberapa rumus fungsi trigonometri yang lain adalah:

$$1. \sin(-\alpha) = -\sin \alpha$$

$$2. \cos(-\alpha) = \cos \alpha$$

$$3. \tan(-\alpha) = -\tan \alpha$$

$$4. \sin \alpha + \sin \beta = 2 \sin \left( \frac{\alpha + \beta}{2} \right) \cos \left( \frac{\alpha - \beta}{2} \right)$$

$$5. \cos \alpha + \cos \beta = 2 \cos \left( \frac{\alpha + \beta}{2} \right) \cos \left( \frac{\alpha - \beta}{2} \right)$$

$$6. \sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$7. \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2\cos^2 \alpha - 1 = 1 - 2\sin^2 \alpha$$

$$8. \sin \alpha \sin \beta = -\frac{1}{2}(\cos(\alpha + \beta) - \cos(\alpha - \beta))$$

$$9. \cos \alpha \cos \beta = \frac{1}{2}(\cos(\alpha + \beta) + \cos(\alpha - \beta))$$

$$10. \sin \alpha \cos \beta = \frac{1}{2}(\sin(\alpha + \beta) + \sin(\alpha - \beta))$$

$$11. \sin \left( \frac{x}{2} \right) = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$12. \cos \left( \frac{x}{2} \right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$