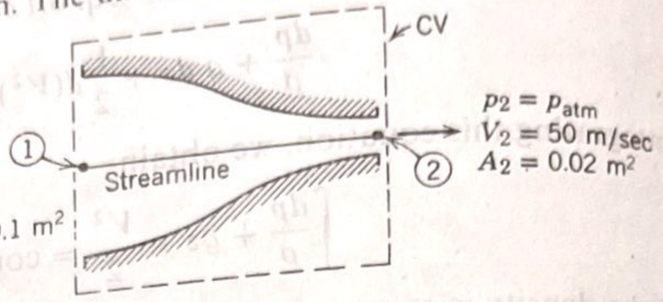
reference frame (see the discussion of translating frames in Section 4-4.2). The procedure is illustrated in Example Problem 6.5.

Example 6.2

Air flows steadily and at low speed through a horizontal nozzle, discharging to the atmosphere. At the nozzle inlet, the area is 0.1 m^2. At the nozzle exit, the area is 0.02 m^2. The flow is essentially incompressible, and frictional effects are negligible. Determine the gage pressure required at the nozzle inlet to produce an outlet speed of 50 m/sec.

EXAMPLE PROBLEM 6.2

GIVEN: Flow through a nozzle, as shown. The air flow is steady, incompressible, and frictionless.



FIND: $p_1 - p_{atm}$

SOLUTION:

Basic equations:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

$$0 = \overset{=0(1)}{\frac{\partial}{\partial t} \int_{CV} \rho \, d\forall} + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

Assumptions:
(1) Steady flow
(2) Incompressible flow
(3) Frictionless flow
(4) Flow along a streamline
(5) $z_1 = z_2$
(6) Uniform flow at sections ① and ②

At the exit, $M_2 = V_2/c = 50$ m/sec/340 m/sec $= 0.147$. This is less than 0.3, so the flow may be treated as incompressible.

Apply the Bernoulli equation along a streamline between points ① and ② to evaluate p_1. Then

$$p_1 - p_{atm} = p_1 - p_2 = \frac{\rho}{2}(V_2^2 - V_1^2)$$

Apply the continuity equation to determine V_1,

$$0 = \{-|\rho V_1 A_1|\} + \{|\rho V_2 A_2|\} \qquad \text{or} \qquad V_1 A_1 = V_2 A_2$$

so that

$$V_1 = V_2 \frac{A_2}{A_1} = \frac{50 \text{ m}}{\text{sec}} \times \frac{0.02 \text{ m}^2}{0.1 \text{ m}^2} = 10 \text{ m/sec}$$

For air at standard conditions, $\rho = 1.23\ \text{kg/m}^3$.

$$p_1 - p_{atm} = \frac{\rho}{2}(V_2^2 - V_1^2)$$

$$= \frac{1}{2} \times 1.23\ \frac{\text{kg}}{\text{m}^3}\left[(50)^2\ \frac{\text{m}^2}{\text{sec}^2} - (10)^2\ \frac{\text{m}^2}{\text{sec}^2}\right]\frac{\text{N}\cdot\text{sec}^2}{\text{kg}\cdot\text{m}}$$

$$p_1 - p_{atm} = 1.48\ \text{kPa}$$

$p_1 - p_{atm}$

{This problem illustrates a typical application of the Bernoulli equation. Note that if the flow streamlines are straight at the nozzle inlet and exit, the pressure will be uniform at those sections.}

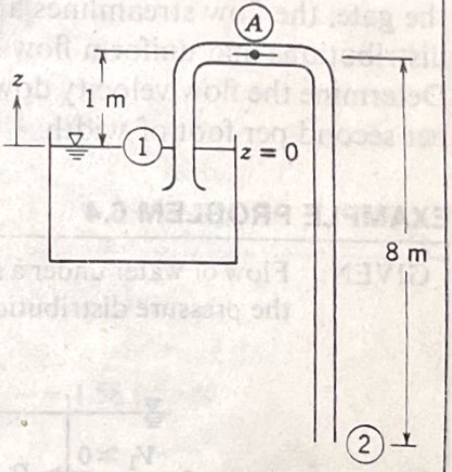
Example 6.3

A U tube acts as a water siphon. The bend in the tube is 1 m above the water surface; the tube outlet is 7 m below the water surface. If the flow is frictionless as a first approximation, and the fluid issues from the bottom of the siphon as a free jet at atmospheric pressure, determine (after listing the necessary assumptions) the velocity of the free jet and the absolute pressure of the fluid in the flow at the bend.

EXAMPLE PROBLEM 6.3

GIVEN: Water flowing through a siphon as shown.

FIND: (a) Velocity of water leaving as a free jet.
(b) Pressure at point Ⓐ in the flow.



SOLUTION:

Basic equation: $$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Assumptions:
(1) Neglect friction
(2) Steady flow
(3) Incompressible flow
(4) Flow along a streamline
(5) Reservoir is large compared to pipe

Apply the Bernoulli equation between points ① and ②.

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Since $\text{area}_{reservoir} \gg \text{area}_{pipe}$, $V_1 \approx 0$. Also $p_1 = p_2 = p_{atm}$, so

$$gz_1 = \frac{V_2^2}{2} + gz_2 \qquad \text{and} \qquad V_2^2 = 2g(z_1 - z_2)$$

$$V_2 = \sqrt{2g(z_1 - z_2)} = \sqrt{2 \times 9.81\ \frac{\text{m}}{\text{sec}^2} \times 7\ \text{m}} = 11.7\ \text{m/sec}$$

V_2

To determine the pressure at location Ⓐ, we write the Bernoulli equation between ① and Ⓐ.

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A$$

Again $V_1 \approx 0$ and from conservation of mass $V_A = V_2$. Hence

$$\frac{p_A}{\rho} = \frac{p_1}{\rho} + gz_1 - \frac{V_2^2}{2} - gz_A = \frac{p_1}{\rho} + g(z_1 - z_A) - \frac{V_2^2}{2}$$

$$p_A = p_1 + \rho g(z_1 - z_A) - \rho\frac{V_2^2}{2}$$

$$= 1.01 \times 10^5 \frac{\text{N}}{\text{m}^2} + 999 \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{m}}{\text{sec}^2} \times (-1\text{ m}) \times \frac{\text{N} \cdot \text{sec}^2}{\text{kg} \cdot \text{m}}$$

$$- \frac{1}{2} \times 999 \frac{\text{kg}}{\text{m}^3} \times (11.7)^2 \frac{\text{m}^2}{\text{sec}^2} \times \frac{\text{N} \cdot \text{sec}^2}{\text{kg} \cdot \text{m}}$$

$$p_A = 22.8 \text{ kPa (abs)} \longleftarrow p_A$$

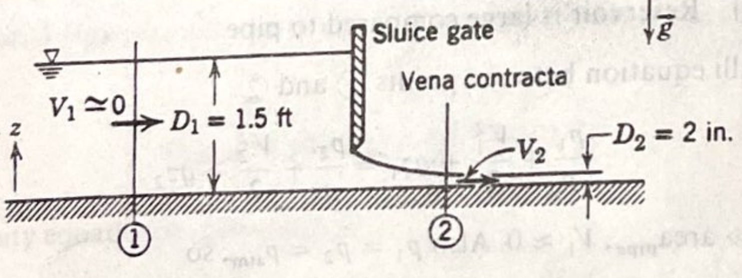
This problem illustrates a straightforward application of the Bernoulli equation with elevation changes included.

Example 6.4

Water flows under a sluice gate on a horizontal bed at the inlet to a flume. Above the gate, the water level is 1.5 ft and the velocity is negligible. At the vena contracta below the gate, the flow streamlines are straight and the depth is 2 in. Hydrostatic pressure distributions and uniform flow may be assumed at each section; friction is negligible. Determine the flow velocity downstream from the gate, and the discharge in cubic feet per second per foot of width.

EXAMPLE PROBLEM 6.4

GIVEN: Flow of water under a sluice gate. Flow is frictionless, uniform at each section, and the pressure distribution is hydrostatic at sections ① and ②.



FIND: (a) V_2.
(b) Q in ft^3/sec/ft of width.

SOLUTION:

The flow satisfies all conditions necessary to apply the Bernoulli equation. The question is, what streamline do we use?

Basic equation: $$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Assumptions:
(1) Steady flow
(2) Incompressible flow
(3) Frictionless flow
(4) Flow along a streamline
(5) Uniform flow at each section
(6) Hydrostatic pressure distribution

From assumption 6,

$$\frac{dp}{dz} = -\gamma \quad \text{so that,} \quad p = p_{atm} + \gamma(D - z) \quad \text{or} \quad \frac{p}{\rho} = \frac{p_{atm}}{\rho} + g(D - z)$$

Substituting this relation into the Bernoulli equation gives

$$\frac{p_{atm}}{\rho} + g(D_1 - z_1) + \frac{V_1^2}{2} + gz_1 = \frac{p_{atm}}{\rho} + g(D_2 - z_2) + \frac{V_2^2}{2} + gz_2$$

or

$$\frac{V_1^2}{2} + gD_1 = \frac{V_2^2}{2} + gD_2$$

This result implies that $V^2/2 + gD =$ constant, and the constant has the same value along *any* streamline for this flow. Solving for V_2 yields

$$V_2 = \sqrt{2g(D_1 - D_2) + V_1^2}$$

But $V_1^2 \approx 0$, so

$$V_2 = \sqrt{2g(D_1 - D_2)} = \sqrt{2 \times 32.2 \frac{\text{ft}}{\text{sec}^2}\left(1.5 \text{ ft} - 2 \text{ in.} \times \frac{\text{ft}}{12 \text{ in.}}\right)}$$

$$V_2 = 9.23 \text{ ft/sec}$$ V_2

For uniform flow, $Q = VA = VDw$, or

$$\frac{Q}{w} = VD = V_2D_2 = 9.23 \frac{\text{ft}}{\text{sec}} \times 2 \text{ in.} \times \frac{\text{ft}}{12 \text{ in.}} = 1.58 \text{ ft}^2/\text{sec}$$

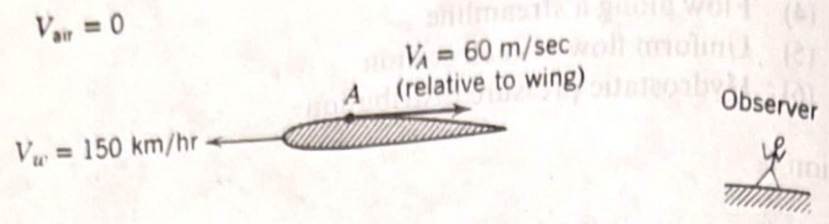
$$\frac{Q}{w} = 1.58 \text{ ft}^3/\text{sec per foot of width}$$ $\frac{Q}{w}$

Example 6.5

A Piper Cub flies at 150 km/hr in standard air at an altitude of 1000 m. At a certain point close to the wing, the air speed *relative* to the wing is 60 m/sec. Compute the pressure at this point.

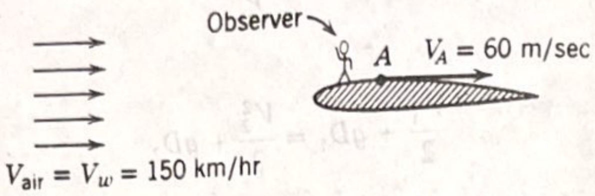
EXAMPLE PROBLEM 6.5

GIVEN: Aircraft in flight at 150 km/hr at 1000 m altitude in standard air.



FIND: Pressure, p_A, at point A.

SOLUTION:
Flow is unsteady when observed from a fixed frame, that is, by an observer on the ground. However, an observer *on* the wing sees the following steady flow:



At point A, $M_A = V_A/c = 60$ m/sec/336 m/sec $= 0.178$. This is less than 0.3, so the flow may be treated as incompressible. Thus the Bernoulli equation can be applied along a streamline in the moving observer's inertial reference frame.

Basic equation: $$\frac{p_{air}}{\rho} + \frac{V_{air}^2}{2} + gz_{air} = \frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A$$

Assumptions:
(1) Steady flow
(2) Incompressible flow ($V < 100$ m/sec)
(3) Frictionless flow
(4) Flow along a streamline
(5) Neglect Δz

Values for pressure and density may be found from Table A.3. Thus, at 1000 m,

$$\frac{p}{p_0} = 0.8870 \qquad \text{and} \qquad \frac{\rho}{\rho_0} = 0.9075$$

Consequently,

$$p = 0.8870 p_0 = 0.8870 \times 1.01 \times 10^5 \frac{\text{N}}{\text{m}^2} = 8.96 \times 10^4 \text{ N/m}^2$$

and

$$\rho = 0.9075 \rho_0 = 0.9075 \times 1.23 \frac{\text{kg}}{\text{m}^3} = 1.12 \text{ kg/m}^3$$

Solving for p_A, we obtain

$$p_A = p_{air} + \frac{\rho}{2}(V_{air}^2 - V_A^2)$$

$$= 8.96 \times 10^4 \frac{\text{N}}{\text{m}^2}$$

$$+ \frac{1}{2} \times 1.12 \frac{\text{kg}}{\text{m}^3}\left[\left(150 \frac{\text{km}}{\text{hr}} \times 1000 \frac{\text{m}}{\text{km}} \times \frac{\text{hr}}{3600 \text{ sec}}\right)^2 - (60)^2 \frac{\text{m}^2}{\text{sec}^2}\right]\frac{\text{N} \cdot \text{sec}^2}{\text{kg} \cdot \text{m}}$$

$$p_A = 88.6 \text{ kPa (abs)} \qquad p_A$$