friction factor is obtained from Fig. 8.14. Then the head loss is computed from Eqs. 8.32 and 8.38, and Eq. 8.28 is solved for pressure drop. The resulting trial value is compared to the system requirement.

If the trial value of Δp is too large, calculations are repeated for a larger assumed value of D. If the trial value of Δp is less than the criterion, a smaller assumed value of D should be checked.

In choosing a pipe diameter, it is logical to work with values that are available commercially. Pipe is manufactured in a limited number of standard sizes. Some data for standard pipe sizes are given in Table 8.4. For data on extra strong or double extra strong pipes, consult a handbook, e.g. [7]. Pipe larger than 12 in. nominal diameter is produced in multiples of 2 in. up to a nominal diameter of 36 in. and in multiples of 6 in. for still larger sizes.

Table 8.4 Standard Sizes for Carbon Steel, Alloy Steel, and Stainless Steel Pipe (Data from [7])

Nominal Pipe Size (in.)	Inside Diameter (in.)	Nominal Pipe Size (in.)	Inside Diameter (in.)
	0.269	$2\frac{1}{2}$	2.469
8	0.364	3	3.068
4 3 8		1 1 to X 4 .	4.026
<u>3</u>	0.493	5	5.047
$\frac{1}{2}$	0.622	6	6.065
3 4 1 To prove the co	0.824		7.981
- 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1.042	10	10.020
$1\frac{1}{2}$	1.610 2.067	12	12.000

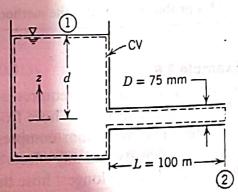
Example 8.5

A 100 m length of smooth horizontal pipe is attached to a large reservoir. What depth, d, must be maintained in the reservoir to produce a volume flow rate of 0.03 m³/sec of water? The inside diameter of the smooth pipe is 75 mm. The loss coefficient, K, for the square-edged inlet is 0.5. The water discharges to the atmosphere.

EXAMPLE PROBLEM 8.5

GIVEN: Water flow at 0.03 m³/sec through a 75 mm diameter pipe, with L = 100 m, attached to a constant-level reservoir. Inlet loss coefficient, K = 0.5.

FIND: Reservoir depth, d, to maintain the flow.



SOLUTION:

Computing equation:

uation:
$$\left(\frac{p_1}{\rho} \div \alpha_1 \frac{\bar{V}_1^2}{2} \div g z_1 \right) - \left(\frac{p_2}{\rho} \div \alpha_2 \frac{\bar{V}_2^2}{2} \div g z_2 \right) = h_{l_T} = h_l \div h_{l_m}$$
 (8.28)

where

$$h_l = \int \frac{L}{D} \frac{\overline{V}^2}{2}$$
 and $h_{l_m} = K \frac{\overline{V}^2}{2}$

For the given problem, $p_1 = p_2 = p_{\text{aim}}$, $\bar{V}_1 \simeq 0$, $\bar{V}_2 = \bar{V}$, and $z_2 \simeq 1.0$. If it is assumed that $z_2 = 0$, then $z_1 = d$. Simplifying Eq. 8.28 gives

$$gd - \frac{\bar{V}^2}{2} = \int \frac{L}{D} \frac{\bar{V}^2}{2} + K \frac{\bar{V}^2}{2}$$

Then

$$d = \frac{1}{g} \left[\int \frac{L}{D} \frac{\bar{V}^2}{2} \div K \frac{\bar{V}^2}{2} \div \frac{\bar{V}^2}{2} \right] = \frac{\bar{V}^2}{2g} \left[\int \frac{L}{D} + K + 1 \right]$$

Since
$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2}$$
, then

$$d = \frac{8Q^2}{\pi^2 D^2 g} \left[f \frac{L}{D} + K + 1 \right]$$

Assuming water at 20 C, $\rho = 999 \text{ kg/m}^3$, and $\mu = 1.0 \times 10^{-3} \text{ kg/m} \cdot \text{sec}$. Thus

$$Re = \frac{\rho \bar{V}D}{\mu} = \frac{4\rho Q}{\pi \mu D}$$

$$Re = \frac{4}{\pi} \times \frac{999 \text{ kg}}{\text{m}^3} \times \frac{0.03 \text{ m}^3}{\text{sec}} \times \frac{\text{m} \cdot \text{sec}}{1.0 \times 10^{-3} \text{ kg}} \times \frac{1}{0.075 \text{ m}} = 5.09 \times 10^5$$

For smooth pipe, from Fig. 8.14, f = 0.0131. Then

$$d = \frac{8Q^2}{\pi^2 D^4 g} \left[f \frac{L}{D} + K + 1 \right]$$

$$= \frac{8}{\pi^2} \times \frac{(0.03)^2}{\text{sec}^2} \times \frac{1}{(0.075)^4 \text{ m}^4} \times \frac{\text{sec}^2}{9.81 \text{ m}} \left[(0.0131) \frac{100 \text{ m}}{0.075 \text{ m}} + 0.5 + 1 \right]$$

$$d = 44.6 \text{ m}$$

(This problem illustrates the method for calculating the total head loss.)

Example 8.6

A compressed air drill requires an air supply of 0.25 kg sec at a gage pressure of 650 kPa at the drill. The hose from the air compressor to the drill is 40 mm inside changes in density and any effects due to hose curvature, Air leaves the compressor at 40 C. Calculate the longest hose that may be used.

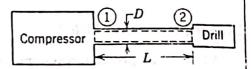
EXAMPLE PROBLEM 8.6

GIVEN: Air flow through a line of length, L, and diameter, D = 40 mm.

$$p_1 = 690 \text{ kPa}$$
 $p_2 = 650 \text{ kPa}$

$$T_1 = 40 \text{ C}$$
 $\dot{m} = 0.25 \text{ kg/sec}$ $\rho \approx \text{constant}$

FIND: Allowable length of hose.



SOLUTION:

Computing equation:

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} = h_l + h_{l_m}$$
 (8.28)

where

$$h_l = \int \frac{L}{D} \frac{\bar{V}^2}{2} \qquad h_{l_m} = K \frac{\bar{V}^2}{2}$$

For $\rho = c$, then $\overline{V}_1 = \overline{V}_2$, since $A_1 = A_2$. Since p_1 and p_2 are given, neglect minor losses. Assume that $\alpha_1 = \alpha_2$. Neglect changes in elevation; assume $z_1 = z_2$. Then Eq. 8.28 can be written

$$\frac{p_1 - p_2}{\rho} = \int \frac{L}{D} \frac{\bar{V}^2}{2}$$
 or $L = \frac{(p_1 - p_2)}{\rho} \frac{2D}{f \bar{V}^2}$

The density is

$$\rho = \rho_1 = \frac{p_1}{RT_1} = \frac{7.91 \times 10^5}{\text{m}^2} \times \frac{\text{kg} \cdot \text{K}}{287 \text{ N} \cdot \text{m}} \times \frac{1}{313 \text{ K}} = 8.81 \text{ kg/m}^3$$

From continuity

$$\bar{V} = \frac{\dot{m}}{\rho A} = \frac{4\dot{m}}{\pi \rho D^2} = \frac{4}{\pi} \times \frac{0.25 \text{ kg}}{\text{sec}} \times \frac{\text{m}^3}{8.81 \text{ kg}} \times \frac{1}{(0.04)^2 \text{ m}^2} = 22.6 \text{ m/sec}$$

For air at 40 C, $\mu = 1.8 \times 10^{-5}$ kg/m·sec, so

$$Re = \frac{\rho \bar{V}D}{\mu} = \frac{8.81 \text{ kg}}{\text{m}^3} \times \frac{22.6 \text{ m}}{\text{sec}} \times \frac{0.04 \text{ m}}{\text{sec}} \times \frac{\text{m} \cdot \text{sec}}{1.8 \times 10^{-5} \text{ kg}} = 4.42 \times 10^5$$

Assume smooth pipe: then from Fig. 8.14, f = 0.0134. Substituting gives

$$L = \frac{(p_1 - p_2)}{\rho} \frac{2D}{f \bar{V}^2}$$

$$= \frac{0.40 \times 10^5 \text{ N}}{\text{m}^2} \times \frac{2}{\text{ N}} \times \frac{0.04 \text{ m}}{\text{m}^3} \times \frac{\text{m}^3}{8.81 \text{ kg}} \times \frac{1}{0.0134} \times \frac{\text{sec}^2}{(22.6)^2 \text{ m}^2} \times \frac{\text{kg} \cdot \text{m}}{\text{N} \cdot \text{sec}^2}$$

$$L = 53.1 \text{ m}$$

This problem illustrates the method of solving for an unknown pipe length. Note that the relative change in density for this problem, $\Delta \rho/\rho \simeq \Delta p/p$, is only about 5 percent. Thus the assumption of incompressible flow is reasonable.

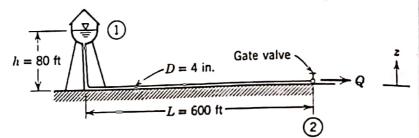
Example 8.7

A fire protection system is supplied from a water tower and standpipe 80 ft tall. The longest pipe in the system is 600 ft long, and is made of cast iron about 20 years old. The pipe contains one gate valve; other minor losses may be neglected. The pipe diameter is 4 in. Determine the maximum rate of flow through this pipe, in gallons per minute.

EXAMPLE PROBLEM 8.7

GIVEN: Fire protection system, as shown.

FIND: Q, gpm.



SOLUTION:

Computing equations: $\left(\frac{p_1}{\rho} + \alpha_1 \frac{\overline{V}_2^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\overline{V}_2^2}{2} + gz_2\right) = h_{l_T}$ (8.28) $h_{l_T} = \int \frac{L}{D} \frac{\bar{V}_2^2}{2} + h_{l_m} = \int \frac{(L + L_e)}{D} \frac{\bar{V}_2^2}{2}$

Assumptions: (1) $p_1 = p_2 = p_{atm}$ (2) $\overline{V}_1 \simeq 0$, and $\alpha_2 \simeq 1.0$

For a fully open gate valve, from Table 8.3, $L_e/D = 8$. Then

$$h_{l_T} = f \frac{L}{D} \frac{\bar{V}_2^2}{2} + 8f \frac{\bar{V}_2^2}{2} = g(z_1 - z_2) - \frac{\bar{V}_2^2}{2}$$

ОΓ

$$\frac{\bar{V}_{2}^{2}}{2} \left[f \left(\frac{L}{D} + 8 \right) + 1 \right] = g(z_{1} - z_{2})$$

Solving for \bar{V}_2 , we obtain

$$\bar{V}_2 = \left[\frac{2y(z_1 - z_2)}{\int (L/D + 8) + 1} \right]^{1/2}$$

To be conservative, assume the standpipe is the same diameter as the horizontal pipe. Then

$$\frac{L}{D} = \frac{600 \text{ ft} + 80 \text{ ft}}{4 \text{ in.}} \times \frac{12 \text{ in.}}{\text{ft}} = 2040$$

Also

$$z_1 - z_2 = h = 80 \text{ ft}$$

Since \bar{V}_2 is not known, we cannot compute Re. But we can assume a value of friction factor in the fully rough flow region. From Fig. 8.15, e/D = 0.0025 for cast iron pipe. Since the pipe is quite old, choose e/D = 0.005. Then, from Fig. 8.14, guess $f \simeq 0.03$. Then a first approximation to \bar{V}_2 is

Now check the value assumed for f.

$$Re = \frac{\rho \bar{V}D}{\mu} = \frac{\bar{V}D}{v} = \frac{9.08}{\text{sec}} \times \frac{\text{ft}}{3} \times \frac{\text{sec}}{1.2 \times 10^{-5} \text{ ft}^2} = 2.52 \times 10^5$$

For e/D = 0.005, f = 0.031 from Fig. 8.14. Using this value, we obtain

Thus convergence is satisfactory. The volume flow rate is

$$Q = \bar{V}_2 A = \bar{V}_2 \frac{\pi D^2}{4} = \frac{8.94 \text{ ft}}{\text{sec}} \times \frac{\pi}{4} \left(\frac{1}{3}\right)^2 \text{ ft}^2 \times \frac{7.48 \text{ gal}}{\text{ft}^3} \times \frac{60 \text{ sec}}{\text{min}}$$

$$Q = 350 \text{ gpm}$$

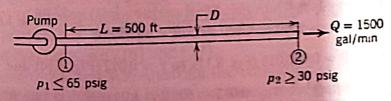
This problem illustrates the procedure for solving pipe flow problems in which the flow rate is unknown. Note that the velocity and, hence, the flow rate, is essentially proportional to $1/\sqrt{f}$. Doubling the value of e/D to account for aging reduced the flow rate by about 10 percent.

Example 8.8

Spray heads in an agricultural spraying system are to be supplied with water through 500 ft of drawn aluminum tubing from an engine-driven pump. In its most efficient operating range, the pump output is 1500 gpm at a discharge pressure not exceeding 65 psig. For satisfactory operation, the sprinklers must operate at 30 psig or higher pressure. Minor losses and elevation changes may be neglected. Determine the smallest standard pipe size that can be used.

EXAMPLE PROBLEM 8.8

GIVEN: Water supply system, as shown.



FIND: Smallest standard D.

SOLUTION:

 Δp , L, and Q are known. D is unknown, so iteration will be required to determine the minimum. minimum standard diameter that satisfies the pressure drop constraint at the given flow rate. The maximum allowable pressure drop is

tble pressure drop is
$$\Delta p_{\text{max}} = p_{1 \text{ max}} - p_{2 \text{ min}} = (65 - 30) \text{ psi} = 35 \text{ psi}$$

Computing equations:

ns:

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^4}{\sqrt{2}} + g_{-1}^4\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^4}{\sqrt{2}} + g_{-2}^4\right) = h_{l_T}$$

$$= 0(3)$$

$$h_{l_T} = h_l + h_m^{l_m} = f \frac{L}{D} \frac{\bar{V}_2^2}{2}$$
(8.28)

Steady flow Assumptions:

(2) Incompressible flow

(3)
$$h_{l_{\pi}} = h_{l}$$
, i.e. $h_{l_{m}} = 0$

$$(4) \quad z_1 = z_2$$

(3)
$$h_{l_T} = h_{l_1}$$
, i.e. $h_{l_m} = 0$
(4) $z_1 = z_2$
(5) $\overline{V}_1 = \overline{V}_2 = \overline{V}$; $\alpha_1 \simeq \alpha_2$

Then

$$\Delta p = p_1 - p_2 = \int \frac{L}{D} \frac{\rho \bar{V}^2}{2}$$

Since trial values of D are to be assumed, it is convenient to substitute $\bar{V} = Q/A = 4Q/\pi D^2$ so

$$\Delta p = f \frac{L}{D} \frac{\rho}{2} \left(\frac{4Q}{\pi D^2} \right)^2 = \frac{8f L \rho Q^2}{\pi^2 D^5}$$
 (1)

The Reynolds number is needed to find f. In terms of Q,

$$Re = \frac{\rho \bar{V}D}{\mu} = \frac{\bar{V}D}{v} = \frac{4Q}{\pi D^2} \frac{D}{v} = \frac{4Q}{\pi v D}$$

Finally, Q must be converted to cubic feet per second.

$$Q = \frac{1500}{\text{min}} \times \frac{\text{min}}{60 \text{ sec}} \times \frac{\text{ft}^3}{7.48 \text{ gal}} = 3.34 \text{ ft}^3/\text{sec}$$

For an initial guess, take nominal 4 in. (4.026 in. i.d.) pipe:

$$Re = \frac{4Q}{\pi vD} = \frac{4}{\pi} \times \frac{3.34 \text{ ft}^3}{\text{sec}} \times \frac{\text{sec}}{1.2 \times 10^{-5} \text{ ft}^2} \times \frac{1}{4.026 \text{ in.}} \times \frac{12 \text{ in.}}{\text{ft}} = 1.06 \times 10^6$$

For drawn tubing, e/D = 0.000,016 (Fig. 8.15), so $f \simeq 0.012$ (Fig. 8.14), and

$$\Delta p = \frac{8f L\rho Q^2}{\pi^2 D^5} = \frac{8}{\pi^2} \times \frac{0.012}{\times} \times \frac{500 \,\text{ft}}{\times} \times \frac{1.94 \,\text{slug}}{\text{ft}^3} \times \frac{(3.34)^2}{\text{sec}^2}$$

$$\times \frac{1}{(4.026)^5 \,\text{in.}^5} \times \frac{1728 \,\text{in.}^3}{\text{ft}^3} \times \frac{\text{lbf} \cdot \text{sec}^2}{\text{slug} \cdot \text{ft}}$$

$$\Delta p = 172 \,\text{lbf/in.}^2 > \Delta p_{\text{max}}$$

Since this value is too large, try D = 6 in. (actually 6.065 in. i.d.):

$$Re = \frac{4}{\pi} \times \frac{3.34 \text{ ft}^3}{\text{sec}} \times \frac{\text{sec}}{1.2 \times 10^{-5} \text{ ft}^2} \times \frac{1}{6.065 \text{ in.}} \times \frac{12 \text{ in.}}{\text{ft}} = 7.01 \times 10^5$$

For drawn tubing, $\epsilon/D = 0.000,010$ (Fig. 8.15), so $f \approx 0.013$ (Fig. 8.14), and

$$\Delta p = 24.0 \, \text{lbf/in.}^2 < \Delta p_{\text{max}}$$

Since this value is less than the allowable pressure drop, we should check a 5 in. (nominal) pipe. With an actual i.d. of 5.047 in.,

$$Re = \frac{4}{\pi} \times \frac{3.34 \text{ ft}^3}{\text{sec}} \times \frac{\text{sec}}{1.2 \times 10^{-5} \text{ ft}^2} \times \frac{1}{5.047 \text{ in.}} \times \frac{12 \text{ in.}}{\text{ft}} = 8.43 \times 10^5$$

For drawn tubing, e/D = 0.000.012 (Fig. 8.15), so $f \simeq 0.012$ (Fig. 8.14), and

$$\Delta p = \frac{8}{\pi^2} \times \frac{0.012}{\text{x}} \times \frac{500 \text{ ft}}{\text{x}} \times \frac{1.94 \text{ slug}}{\text{ft}^3} \times \frac{(3.34)^2}{\text{sec}^2} \frac{\text{ft}^6}{\text{sec}^2}$$

$$\times \frac{1}{(5.047)^5 \text{ in.}^5} \times \frac{(12)^3 \text{ in.}^3}{\text{ft}^3} \times \frac{\text{lbf} \cdot \text{sec}^2}{\text{slug} \cdot \text{ft}}$$

$$\Delta p = 55.5 \, \text{lbf/in.}^2 > \Delta p_{\text{max}}$$

Thus the criterion for pressure drop is satisfied for a minimum nominal diameter of 6 in.

D

pipe.

This problem illustrates the procedure for solving pipe flow problems when the diameter is unknown. Note from Eq. 1 that the pressure drop in turbulent pipe flow is proportional to f/D^5 . The variation of f is small, so Δp at constant flow rate is approximately proportional to $1/D^5$.

We have solved each of Example Problems 8.7 and 8.8 by direct iteration. Several specialized forms of friction factor versus Reynolds number diagrams have been introduced to solve problems of this type without the need for iteration. For examples of these specialized diagrams, see [16] and [17].

Example Problems 8.9 and 8.10 illustrate the evaluation of minor loss coefficients and the application of a diffuser to reduce exit kinetic energy from a flow system.

Example 8.9

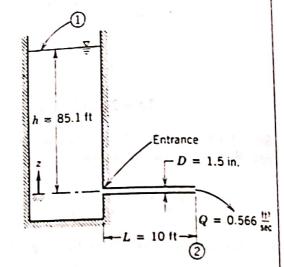
Reference 18 reports results of measurements made to determine entrance losses for flow from a reservoir to a pipe with various degrees of entrance rounding. A copper pipe 10 ft long with 1.5 in. i.d. was used for the tests. The pipe discharged to atmosphere.

For a square-edged entrance a discharge of 0.566 ft³/sec was measured when the ror a square-edged entrance a discnarge of o.555 reservoir level was 85.1 ft above the pipe centerline. From these data evaluate the loss coefficient for a square-edged entrance.

EXAMPLE PROBLEM 8.9

GIVEN: Pipe with square-edged entrance discharging from reservoir as shown.

FIND: Kentrance.



SOLUTION:

Apply the energy equation for steady, incompressible pipe flow.

Computing equations: $\frac{p}{h} + \alpha_1 \frac{\overline{V}^2}{2} + gz_1 = \frac{p}{h} + \alpha_2 \frac{\overline{V}^2}{2} + gz_2 + h_{l_T}$

$$h_{l_T} = f \frac{L}{D} \frac{\vec{V}_2^2}{2} + K_{\text{entrance}} \frac{\vec{V}_2^2}{2}$$

Assumptions: (1) $p_1 = p_2 = p_{atm}$ (2) $\bar{V}_1 \approx 0$

Substituting for h_{I_1} and dividing by g gives $z_1 = h = \alpha_2 \frac{\bar{V}_2^2}{2a} + \int \frac{L}{D} \frac{\bar{V}_2^2}{2a} + K_{\text{entrance}} \frac{\bar{V}_2^2}{2a}$ or

$$K_{\text{entrance}} = \frac{2gh}{\bar{V}_2^2} - \int \frac{L}{D} - \alpha_2$$
 (1)

The average velocity is

$$\bar{V}_2 = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times \frac{0.566 \text{ ft}^3}{\text{sec}} \times \frac{1}{(1.5)^2 \text{ in.}^2} \times \frac{144 \text{ in.}^2}{\text{ft}^2} = 46.1 \text{ ft/sec}$$

Assume $T = 75 \,\text{F}$ (24 C), so $v = 8.8 \times 10^{-7} \,\text{m}^2/\text{sec}$ (Fig. A.3). Then

$$Re = \frac{\overline{VD}}{v} = \frac{46.1 \text{ ft}}{\text{sec}} \times \frac{1.5 \text{ in.}}{\text{sec}} \times \frac{\text{sec}}{8.8 \times 10^{-7} \text{ m}^2} \times \frac{\text{ft}}{12 \text{ in.}} \times \frac{(0.3048)^2 \text{ m}^2}{\text{ft}^2} = 6.08 \times 10^5$$

For drawn tubing, e/D = 0.000,04 (Fig. 8.15), so f = 0.013 (Fig. 8.14). From Fig. 8.11, n = 8.7

$$\frac{\bar{V}}{U} = \frac{2n^2}{(n+1)(2n+1)} = 0.848 \tag{8.23}$$

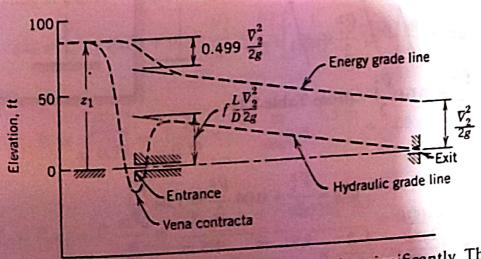
$$\alpha = \left(\frac{U}{\bar{V}}\right)^3 \frac{2n^2}{(3+n)(3+2n)} = 1.04 \tag{8.26}$$

Substituting into Eq. 1, we obtain

$$K_{\text{entrance}} = \frac{2 \times 32.2 \text{ ft}}{\text{sec}^2} \times \frac{85.1 \text{ ft}}{\text{ ft}} \times \frac{\text{sec}^2}{(46.1)^2 \text{ ft}^2} - \frac{(0.013) \frac{10 \text{ ft}}{1.5 \text{ in.}}}{1.5 \text{ in.}} \times \frac{12 \text{ in.}}{\text{ ft}} - \frac{1.04}{1.04}$$

 $K_{\text{entrance}} = 0.499$

This value compares favorably with that shown in Table 8.1. The hydraulic and energy grade lines are shown below. The large head loss in a square-edged entrance is due primarily to separation at the sharp inlet corner and formation of a vena contracta immediately downstream from the corner. The effective flow area reaches a minimum at the vena contracta, so the flow velocity is a maximum there. The flow expands again following the vena contracta to fill the pipe. The uncontrolled expansion following the vena contracta is responsible for most of the head loss. (See Example Problem 8.12.)



Rounding the inlet corner reduces the extent of separation significantly. This reduces the velocity increase through the vena contracta and consequently reduces the head loss due to the entrance. A "well-rounded" inlet almost eliminates flow separation; the flow pattern approaches that shown in Fig. 8.1. The added head loss in a well-rounded inlet compared to fully developed flow is the result of higher wall shear stresses in the entrance length.