

Application of the fuzzy topsis multi-attribute decision making method to determine scholarship recipients

I Irvanizam¹

¹Department of Informatics, Syiah Kuala University, 5 Syech Abdurrauf Street, Darussalam Banda Aceh 23111, Aceh, Indonesia

Email: irvanizam.zamanhuri@unsyiah.ac.id

Abstract. Some scholarships have been routinely offered by Ministry of Research, Technology and Higher Education of the Republic of Indonesia for students at Syiah Kuala University. In reality, the scholarship selection process is becoming subjective and highly complex problem. Multi-Attribute Decision Making (MADM) techniques can be a solution in order to solve scholarship selection problem. In this study, we demonstrated the application of a fuzzy TOPSIS as an MADM technique by using a numerical example in order to calculate a triangular fuzzy number for the fuzzy data onto a normalized weight. We then use this normalized value to construct the normalized fuzzy decision matrix. We finally use the fuzzy TOPSIS to rank alternatives in descending order based on the relative closeness to the ideal solution. The result in terms of final ranking shows slightly different from the previous work.

1. Introduction

The national education system has a crucial role in creating a good learning and learning process [1]. Therefore, learners or students have to improve their potentials by doing some efforts and plans. The students are expected to become human beings who have strong religious faith, strong personality, healthy, knowledgeable, responsible, capable, creative, independent, and become citizens of a democratic. This is in line with what mentioned in the Act, Law No. 20 in Article 3, 2003 about The National Education System of Indonesia (Undang-Undang No. 20 Tahun 2003). In order to support this system, Ministry of Research, Technology and Higher Education of the Republic of Indonesia has been routinely offering some scholarships for students at Syiah Kuala University. One of those scholarships is including the Academic Achievement (AA) Scholarship.

The scholarship selection process is highly complex problem [2]. Typically, several eligible documents submitted by many scholarship recipient candidates or students have to be individually checked and evaluated by a scholarship committee. The committee then assign and select a list of the scholarship recipients based on their prejudices and personal preferences. As a result, the review process of assessing candidate becomes subjective and among vagueness [2].

Some researchers have developed decision making systems using Multi-Attribute Decision Making (MADM) techniques in order to solve problems in decision making. In their application systems, the MADM techniques were focused on how the experts or the decision makers assigned the weighting value of criteria based on their references. The experts gave numeric values in order to make the computation easier.

There are several methods that can be used to solve the MADM problem. One of those methods is Technique for order preference by similarity to an ideal solution (TOPSIS). This method has been developed by Hwang and Yoon [3]. The method selects the chosen alternative which is the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution [4]. As the decision makers give assessment values of criteria, the results have subjectivity and amount of vagueness [2,4]. In this case, this method cannot always be used. However, many years ago, several researchers have attempted to minimize this subjectivity and vagueness by applying fuzzy set theory to multiple attribute evaluations [5]. Typically, the overall alternative values are presented by a fuzzy number which is later called the fuzzy utility. Then, the alternatives are ranked based on the comparison of their fuzzy utilities. In [6], a fuzzy MADM problem was transformed into a crisp value using centroid defuzzification. Tsaur et al. then use TOPSIS to solve the non-fuzzy MADM problem. In [7], a fuzzy MADM problem was also converted to a crisp and solved it using TOPSIS. Meanwhile, Chen and Tzeng in [8] used fuzzy integral to solve MADM problem after they converted a fuzzy MADM into a crisp MADM.

Additionally, TOPSIS method is suitable to solve a decision making problem using quantity multiplication operation of triangular fuzzy number [1]. Yuan-guang in [9] has proved that the TOPSIS method can be implemented effectively with less information and the result is reasonable and objective. Therefore, in this study, a TOPSIS method is used in fuzzy multiple-attribute decision making to determine the right scholarship recipients by demonstrating a numeric example.

The rest of this paper is structurally organized as follows: in the second section, The previous research is described in brief. In section 3, some basic concepts of the TOPSIS are briefly described. Section 4 demonstrates the calculation of TOPSIS through a numerical example. In the last section, some conclusions are presented by showing comparison results of the TOPSIS and the previous research in [2].

2. The previous research

Some researches related to MADM problems for scholarship selection have been conducted by scholars. For instance, the decision making system, developed by Uyun and Riadi [1], used Unified Modeling Language (UML) to select the candidates for academic and non-academic scholarships through three requirement criteria. The paper was focused on how to build the system using UML and showed a comparison between TOPSIS and Weighted Product methods. However, this our study has four criteria, uses triangular number for fuzzy data, and emphasizes on how to determine the scholarship recipients using a numerical example.

3. Methodology

This section describes the concept of fuzzy set theory and fuzzy TOPSIS. The fuzzy set theory is used to handle the fuzziness of decision maker's judgment whereas the fuzzy TOPSIS is used to rank the best alternative and to select the scholarship recipients.

3.1. Fuzzy set theory

Fuzzy set theory is one of the most preferred theories in decision making problem. Basically, this theory is an extended theory of ordinary set theory that was introduced by Zadeh [10] for handling uncertainty and imprecise information correlated with another. In some literatures, triangular fuzzy numbers (TFNs) are one of the fuzzy number forms that can be used in order to capture the vagueness of the criteria related to the study case. In this study, a TFN is used. A triangular fuzzy number $\tilde{a} (a_1, a_2, a_3)$ will be used to consider the fuzziness of the parent income criteria. The membership function $\mu(x, a_1, a_2, a_3)$ of the triangular fuzzy number may be defined as in equation (1) [11].

$$\mu_{\tilde{a}}(x, a_1, a_2, a_3) = \begin{cases} 0, & x \leq a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 < x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 < x \leq a_3 \\ 0, & a_3 < x \end{cases} \quad (1)$$

3.2. Technique for order preference by similarity to ideal solution (TOPSIS)

The TOPSIS method is an MADM approach that was firstly introduced by Hwang and Yoon, and later Chen and Hwan developed a fuzzy TOPSIS. The TOPSIS attempts to find the alternative, which is the shortest distance from the positive ideal solution (FIS) and longest distance from negative ideal solution (FIS). This method will rank the alternatives in descending order based on the closeness coefficient representing the distances to the PIS and NIS.

In general, the TOPSIS method procedure follows these steps:

Step 1: Construct a fuzzy decision matrix D with m alternatives and n criteria that can be concisely presented as in equation (2).

$$D = \begin{matrix} & C_1 & C_2 & C_3 & \dots & C_n \\ A_1 & \left(\begin{matrix} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \dots & x_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \dots & x_{mn} \end{matrix} \right) \end{matrix} \quad (2)$$

where A_i ($i=1,2,3,\dots,m$) is possible alternative, C_j ($j=1,2,3,\dots,n$) is possible criterion, and x_{ij} is the performance rating of the i -th alternative, A_i , with respect to the j -th criterion, C_j .

Step 2: Build normalized decision matrix R . Each element of the matrix D can be normalized using equation (4).

$$R = \begin{matrix} & C_1 & C_2 & C_3 & \dots & C_n \\ A_1 & \left(\begin{matrix} r_{11} & r_{12} & r_{13} & \dots & r_{1n} \\ r_{21} & r_{22} & r_{23} & \dots & r_{2n} \\ r_{31} & r_{32} & r_{33} & \dots & r_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & r_{m3} & \dots & r_{mn} \end{matrix} \right) \end{matrix} \quad (3)$$

$$r_{ij} = x_{ij} * \left(\sum_{i=1}^m x_{ij}^2 \right)^{-1/2} \quad (4)$$

Step 3: Construct weighted normalized matrix V . In this study, the matrix V has given weight $W = [w_1, w_2, w_3, \dots, w_n]$ which is either crisp or a triangular fuzzy number. Each element of the matrix V can be calculated using equation (6).

$$V = \begin{matrix} & C_1 & C_2 & C_3 & \dots & C_n \\ A_1 & \left(\begin{matrix} v_{11} & v_{12} & v_{13} & \dots & v_{1n} \\ v_{21} & v_{22} & v_{23} & \dots & v_{2n} \\ v_{31} & v_{32} & v_{33} & \dots & v_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ v_{m1} & v_{m2} & v_{m3} & \dots & v_{mn} \end{matrix} \right) \end{matrix} \quad (5)$$

$$v_{ij} = W_j * r_{ij} \quad (6)$$

Step 4: Determine the positive ideal solution (PIS), A^+ and the negative ideal solution (NIS), A^- . The PIS and NIS can be computed based on normalized weighted rating using equation (7) and (8) respectively.

$$PIS = A^+ = \left\{ \underset{i}{Max} v_{ij}; j \in J \right\} = \{v_1^+, v_2^+, v_3^+, \dots, v_m^+\} \quad (7)$$

$$NIS = A^- = \left\{ \underset{i}{Min} v_{ij}; j \in J \right\} = \{v_1^-, v_2^-, v_3^-, \dots, v_m^-\} \quad (8)$$

where J is associated with benefit criteria.

Step 5: Calculate the separation measures using the Euclidean distance. The separation of each alternative from PIS, D_i^+ can be calculated using equation (9). Similarly, the separation of each alternative from NIS, D_i^- can be calculated using equation (10).

$$D_i^+ = \left\{ \sum_{j=1}^n (v_{ij} - v_j^+)^2 \right\}^{1/2}, 1 \leq i \leq m \quad (9)$$

$$D_i^- = \left\{ \sum_{j=1}^n (v_{ij} - v_j^-)^2 \right\}^{1/2}, 1 \leq i \leq m \quad (10)$$

Step 6: Compute the relative closeness to the ideal solution. The closeness of alternative A_i with respect to A^+ can be calculated using equation (11).

$$RC_i^+ = \frac{D_i^-}{D_i^+ + D_i^-}, \quad 1 \leq i \leq m \quad (11)$$

since $D_i^- \geq 0$ and $D_i^+ \geq 0$, then clearly, $RC_i^+ \in [0, 1]$.

Step 7: Rank the preference order. We can rank alternatives in descending order based on RC_i^+ value.

4. Numerical example

In this section, we demonstrate a numerical example, taken from [2], to illustrate the fuzzy TOPSIS algorithm for solving the scholarship selection problem with a triangular fuzzy number. We assume that Syiah Kuala University desires to select seven of ten students in order to distribute the AA scholarship. After having the requirement documents, the decision maker has identified four benefit criteria which are: Grade Point Average (C_1), the number of credit (C_2), the number of dependents parent (C_3) and parent incomes (C_4). The decision maker used the linguistic variable as shown in table 1 to assess the importance of each criterion. This linguistic variable is converted into triangular fuzzy numbers to construct the fuzzy decision matrix and determine the weight fuzzy number of each criterion, as shown in table 2.

Table 1. Linguistic variables for the importance weight of each criterion.

Variable	A triangular fuzzy number
Very low (VL)	(0.0; 0.0; 0.1)
Low (L)	(0.1; 0.2; 0.2)
Medium low (ML)	(0.2; 0.3; 0.4)
Medium (M)	(0.4; 0.5; 0.6)
Medium high (MH)	(0.5; 0.6; 0.7)
High (H)	(0.6; 0.8; 0.8)
Very high (VH)	(0.8; 0.9; 1.0)

Table 2. Importance weight of the criteria from a decision maker for the Academic Achievement (AA) Scholarship.

Criteria	Fuzzy number			
	Linguistic Variable	Weight	Defuzzied value	Normalized weight
C ₁	VH	(0.8; 0.9; 1.0)	0.9	0.45
C ₂	M	(0.4; 0.5; 0.6)	0.5	0.25
C ₃	ML	(0.2; 0.3; 0.4)	0.3	0.15
C ₄	ML	(0.2; 0.3; 0.4)	0.3	0.15

All criteria used crisp data except for the 4-th criterion. The parent income criterion, C_4 had suitability degree with several alternatives decision: $T(\text{suitability}) = \{S, M, RG, B, VG\}$. The membership function of each element was represented by using triangular fuzzy numbers as shown in table 3.

Table 3. Fuzzy linguistic variables and fuzzy number for the criterion C_4

Parent Income (IDR)	Fuzzy Set		
	Linguistic Variable	A triangular fuzzy number	Defuzzied value
0 - 1000000	SMALL (S)	(0.0; 0.1; 0.4)	0.15
1000001 - 2000000	MODERATE (M)	(0.1; 0.4; 0.5)	0.35
2000001 - 3000000	RATHER BIG (RG)	(0.4; 0.5; 0.6)	0.5
3000001 - 4000000	BIG (B)	(0.5; 0.6; 0.8)	0.625
4000001 - 5000000	VERY BIG (VG)	(0.6; 0.8; 1.0)	0.8

Table 4. The fuzzy decision matrix

Alternative	C ₁	C ₂	C ₃	C ₄	
				Income	Fuzzy Number
S ₁	3.68	98	4	1500000	0.35
S ₂	3.9	98	4	2500000	0.5
S ₃	3.9	98	3	1000000	0.15
S ₄	3.7	54	2	2000000	0.35
S ₅	3.59	54	1	2500000	0.5
S ₆	3.43	54	4	2000000	0.35
S ₇	3.19	85	4	1000000	0.15
S ₈	3.28	41	4	1000000	0.15
S ₉	3.8	41	8	1000000	0.15
S ₁₀	3	20	6	5000000	0.8

The data were first constructed into a fuzzy decision matrix, that shown as in table 4. The algorithm then built a normalized fuzzy decision matrix using equation (3). A weighted normalized matrix was then computed using equation (5) and shown as table 6. The next steps are to compute the PIS (A^+) and NIS (A^-), respectively and shown in table 5.

Next, the separation measure of PIS (D^+) and NIS (D^-) are computed using equation (9) and (10), respectively. The closeness of each alternative A_i is then calculated using equation (11). A larger value of the closeness shows that alternative A_i is preferred to the AA scholarship. Finally, the result of the fuzzy calculation is shown in table 6. The first student who will receive the AA scholarship is S_2 and then $S_1, S_3, S_{10}, S_7, S_9,$ and S_6 will be respectively. Meanwhile, the result from the previous work done by Irvanizam [2] is $S_3, S_2, S_1, S_7, S_9, S_4$ and S_6 . This indicates that by using this method, minor changes in first and sixth ranking take place. Therefore, the result of this method is slightly different to the one using Simple Additive Weighting (SAW).

Table 5. The positive ideal solution (PIS) and the negative ideal solution (NIS) for the AA scholarship

Alternative	The AA scholarship			
	C ₁	C ₂	C ₃	C ₄
Positive Ideal Solution (A^+)	0.1559	0.1112	0.0862	0.0949
Negative Ideal Solution (A^-)	0.1199	0.0227	0.0108	0.0178

Table 6. The weighted normalized fuzzy decision matrix for the AA scholarship and The result of fuzzy TOPSIS for the AA scholarship

Alternative	The AA scholarship							
	C ₁	C ₂	C ₃	C ₄	D ⁺	D ⁻	RC ⁺	Rank
S ₁	0.1471	0.1112	0.0431	0.0415	0.0692	0.1009	0.5932	2
S ₂	0.1559	0.1112	0.0431	0.0593	0.0559	0.1091	0.6612	1
S ₃	0.1559	0.1112	0.0323	0.0178	0.0941	0.0979	0.5101	3
S ₄	0.1479	0.0613	0.0215	0.0415	0.0979	0.0543	0.3569	9
S ₅	0.1435	0.0613	0.0108	0.0593	0.0980	0.0614	0.3853	8
S ₆	0.1371	0.0613	0.0431	0.0415	0.0869	0.0582	0.4012	7
S ₇	0.1275	0.0964	0.0431	0.0178	0.0940	0.0809	0.4626	5
S ₈	0.1311	0.0465	0.0431	0.0178	0.1123	0.0417	0.2707	10
S ₉	0.1519	0.0465	0.0862	0.0178	0.1007	0.0853	0.4585	6
S ₁₀	0.1199	0.0227	0.0646	0.0949	0.0979	0.0941	0.4899	4

5. Conclusion

In this study, we have demonstrated the fuzzy TOPSIS based on linguistic and crisp values for determining the scholarship recipients. The result in terms of final ranking shows slightly different from the previous work, done by Irvanizam [2]. However, this study still used a decision maker to assign the criteria weight so that this becomes the limitation of this study. The research still should be expanded on some study cases such as the use of triangular and trapezoidal fuzzy numbers for all criteria, the use of fuzzy multi-attribute group decision making (FMAGDM) and the use of a linear programming method.

Acknowledgments

This research was supported by the Department of Informatics, Syiah Kuala University and Database & Data Mining Information Retrieval Research Group (DMIR).

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