



**Institut Teknologi Sepuluh Nopember
Surabaya**

JURUSAN TEKNIK FISIKA - FTI



PENYELESAIAN PERSAMAAN ALJABAR LINIER

Seri: Matematika Rekayasa 1

Oleh: Aulia Siti Aisjah

PAL

$$a_{11}x_1 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + \cdots + a_{2n}x_n = b_2$$

.....

$$a_{m1}x_1 + \cdots + a_{mn}x_n = b_m.$$

Vektor kolom

$$\mathbf{v} = [v_1 \quad \cdots \quad v_n],$$

Vektor baris



$$\mathbf{A} = [a_{jk}] =$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}.$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix},$$

$$\mathbf{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$$

matriks



Penjumlahan

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}.$$

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix},$$



Perkalian skalar

$$\mathbf{v} = \begin{bmatrix} 0.4 \\ -13 \end{bmatrix}, \quad \longrightarrow \quad 10\mathbf{v} = \begin{bmatrix} 4 \\ -130 \end{bmatrix}.$$

Perkalian matriks $\rightarrow \mathbf{C} = \mathbf{A} \times \mathbf{B}$

$$c_{jk} = \sum_{m=1}^n a_{jm} b_{mk}$$



- (a) $A + B = B + A$
- (b) $(A + B) + C = A + (B + C)$
- (c) $A + 0 = A$
- (d) $A + (-A) = 0.$



(a) $c(\mathbf{A} + \mathbf{B}) = c\mathbf{A} + c\mathbf{B}$

(b) $(c + k)\mathbf{A} = c\mathbf{A} + k\mathbf{A}$

(c) $c(k\mathbf{A}) = (ck)\mathbf{A}$

(d) $1\mathbf{A} = \mathbf{A}.$



$$(a) \quad (k\mathbf{A})\mathbf{B} = k(\mathbf{A}\mathbf{B}) = \mathbf{A}(k\mathbf{B})$$

$$(b) \quad \mathbf{A}(\mathbf{B}\mathbf{C}) = (\mathbf{A}\mathbf{B})\mathbf{C}$$

$$(c) \quad (\mathbf{A} + \mathbf{B})\mathbf{C} = \mathbf{A}\mathbf{C} + \mathbf{B}\mathbf{C}$$

$$(d) \quad \mathbf{C}(\mathbf{A} + \mathbf{B}) = \mathbf{C}\mathbf{A} + \mathbf{C}\mathbf{B}$$



Invers matriks

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}.$$

$$\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix},$$

$$\det \mathbf{A} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$



Transpose sebuah matriks

$$\mathbf{A}^T = [a_{kj}] = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \cdot & \cdot & \cdots & \cdot \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}.$$

$$\mathbf{A} = \begin{bmatrix} 5 & -8 & 1 \\ 4 & 0 & 0 \end{bmatrix}, \quad \text{then} \quad \mathbf{A}^T = \begin{bmatrix} 5 & 4 \\ -8 & 0 \\ 1 & 0 \end{bmatrix}.$$

- (a) $(\mathbf{A}^T)^T = \mathbf{A}$
- (b) $(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$
- (c) $(c\mathbf{A})^T = c\mathbf{A}^T$
- (d) $(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T.$



Triangular matrix.

If U_1, U_2 are upper triangular and L_1, L_2 are lower triangular, which of the following are triangular?

$$U_1 + U_2, \quad U_1 U_2, \quad U_1^2, \quad U_1 + L_1, \quad U_1 L_1, \\ L_1 + L_2$$



PAL – Augmented matrix

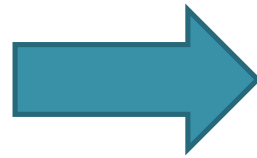
9.1

Materi

$$\tilde{\mathbf{A}} = \left[\begin{array}{ccc|c} a_{11} & \cdots & a_{1n} & b_1 \\ \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdots & \cdot & \cdot \\ a_{m1} & \cdots & a_{mn} & b_m \end{array} \right]$$

PAL – contoh soal

$$\begin{aligned}x_1 - x_2 + x_3 &= 0 \\ -x_1 + x_2 - x_3 &= 0 \\ 10x_2 + 25x_3 &= 90 \\ 20x_1 + 10x_2 &= 80.\end{aligned}$$



Augmented Matrix $\tilde{\mathbf{A}}$

Pivot 1 \longrightarrow

Eliminate \longrightarrow

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & 10 & 25 & 90 \\ 20 & 10 & 0 & 80 \end{array} \right]$$

Baris (1) + baris (0)

Baris (3) - 20 x baris (0)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 30 & -20 & 80 \end{array} \right]$$



Pivot 10 \longrightarrow

Eliminate 30 \longrightarrow

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 30 & -20 & 80 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

PAL

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 30 & -20 & 80 \end{array} \right]$$

$$x_1 - x_2 + x_3 = 0$$

$$0 = 0$$

$$10x_2 + 25x_3 = 90$$

$$30x_2 - 20x_3 = 80.$$

Pivot 10 \longrightarrow

Eliminate 30 \longrightarrow

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 30 & -20 & 80 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Baris (2) -3 x baris (0)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 0 & -95 & -190 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x_1 - x_2 + x_3 = 0$$

$$10x_2 + 25x_3 = 90$$

$$-95x_3 = -190$$

$$0 = 0.$$

$X_3 = \dots \rightarrow X_2 \rightarrow X_1$

Eliminasi Gauss Backward



PAL Latihan Soal 2: Selesaikan PAL berikut

9.1

Materi

1. $4x - 6y = -11$

$$-3x + 8y = 10$$

3. $x + y - z = 9$

$$8y + 6z = -6$$

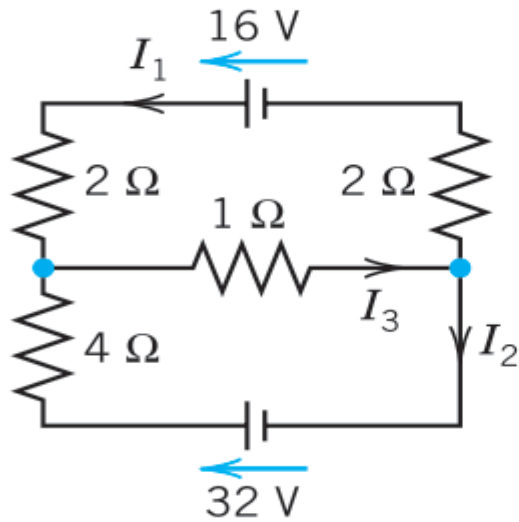
$$-2x + 4y - 6z = 40$$

2.
$$\begin{bmatrix} 3.0 & -0.5 & 0.6 \\ 1.5 & 4.5 & 6.0 \end{bmatrix}$$

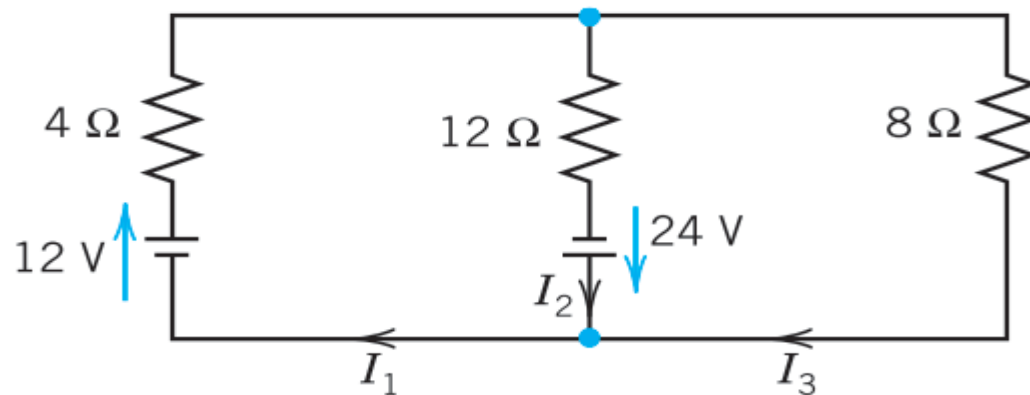
4.
$$\begin{bmatrix} 4 & 1 & 0 & 4 \\ 5 & -3 & 1 & 2 \\ -9 & 2 & -1 & 5 \end{bmatrix}$$



17.



18.



Upload Jawab soal latihan di myclassroom Tanggal, 6 Nov. 2020 Jam 24.00



PAL – Linier homogen

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = 0$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = 0$$

.....

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = 0$$

PAL

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

.....

$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n$$

$$x_1 = \frac{D_1}{D}, \quad x_2 = \frac{D_2}{D}, \cdots, \quad x_n = \frac{D_n}{D} \quad \text{(Cramer's rule)}$$

$$D = a_{1k}C_{1k} + a_{2k}C_{2k} + \cdots + a_{nk}C_{nk}, \quad \text{C = cofaktor}$$



PAL – METODE GAUS JORDAN

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

9.1

Materi

$$[\mathbf{A} \quad \mathbf{I}] = \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 3 & -1 & 1 & 0 & 1 & 0 \\ -1 & 3 & 4 & 0 & 0 & 1 \end{array} \right]$$

Row 2 + 3 Row 1

Row 3 - Row 1

$$\left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 7 & 3 & 1 & 0 \\ 0 & 2 & 2 & -1 & 0 & 1 \end{array} \right]$$

Row 3 - Row 2

$$\left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 7 & 3 & 1 & 0 \\ 0 & 0 & -5 & -4 & -1 & 1 \end{array} \right]$$

-Row 1

0.5 Row 2

-0.2 Row 3

Row 1 + 2 Row 3

Row 2 - 3.5 Row 3

$$\left[\begin{array}{ccc|ccc} 1 & -1 & -2 & -1 & 0 & 0 \\ 0 & 1 & 3.5 & 1.5 & 0.5 & 0 \\ 0 & 0 & 1 & 0.8 & 0.2 & -0.2 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0.6 & 0.4 & -0.4 \\ 0 & 1 & 0 & -1.3 & -0.2 & 0.7 \\ 0 & 0 & 1 & 0.8 & 0.2 & -0.2 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -0.7 & 0.2 & 0.3 \\ 0 & 1 & 0 & -1.3 & -0.2 & 0.7 \\ 0 & 0 & 1 & 0.8 & 0.2 & -0.2 \end{array} \right]$$



PAL – LATIHAN: cari invers matriks

Setiap mhs mengerjakan minimal 3 soal

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Max

Tanggal: 9 Nov

2020, jam 24.00

1.
$$\begin{bmatrix} 1.80 & -2.32 \\ -0.25 & 0.60 \end{bmatrix}$$

2.
$$\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

3.
$$\begin{bmatrix} 0.3 & -0.1 & 0.5 \\ 2 & 6 & 4 \\ 5 & 0 & 9 \end{bmatrix}$$

4.
$$\begin{bmatrix} 0 & 0 & 0.1 \\ 0 & -0.4 & 0 \\ 2.5 & 0 & 0 \end{bmatrix}$$

5.
$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 1 \end{bmatrix}$$

6.
$$\begin{bmatrix} -4 & 0 & 0 \\ 0 & 8 & 13 \\ 0 & 3 & 5 \end{bmatrix}$$



PAL

9.1

Materi

