

## Tranformasi Laplace

Fungsi  $F(t)$  dengan domain himpunan bilangan real positif sehingga nilai  $\int_0^{\infty} e^{-st} f(t) dt$ , dapat di tentukan dimana  $S$  suatu parameter tranformasi laplace fungsi  $(t)$  ditulis  $L\{f(t)\}$ .

di definikan sebagai :

$$L\{f(t)\} = F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

Beberapa dalil/rumus :

$$1) L\{k\} = \frac{K}{s} \quad (s < 0)$$

$$\begin{aligned} L\{k\} &= \int_0^{\infty} e^{-st} k dt \\ &= K \int_0^{\infty} e^{-st} dt \\ &= -\frac{K}{s} (e^{-st}) = \frac{K}{s} \end{aligned}$$

$$2) L\{t^n\} = \frac{n!}{s^{n+1}}$$

$$3) L\{\cos kt\} = \frac{s}{s^2+k^2}$$

$$4) L\{\sin kt\} = \frac{k}{s^2+k^2}$$

$$5) L\{e^{at}\} = \frac{1}{s-a}$$

$$6) L\{\arccos at\} = \frac{a}{s^2-a^2}$$

$$7) L\{\arcsin at\} = \frac{s}{s^2-a^2}$$

$$8) L\{f(t)\} = F(s)$$

$$L\{f(t)\} = sF(s) - F(0)$$

$$9) L\{f^{(n)}(t)\} = s^n \cdot F(s) - s^{n-1} F(0) - s^{n-2} F'(0) - \dots - F^{(n-1)}(0)$$

$$10) L\{e^{at} f(t)\} = F(s-a)$$

$$11) L \{ e^{a,t} \cdot t^n \} = \frac{T(n+1)}{(s-a)^{n+1}}$$

$$12) L \{ e^{at} \cdot \sin k t \} = \frac{k}{(s-a)^2+k^2}$$

$$13) L \{ e^{at} \cdot \cos k t \} = \frac{s-a}{(s-a)^2+k^2}$$

$$14) L \{ t^n \cdot F(t) \} = (-1)^{(n)} f^{(n)}$$

Contoh :

$$1) \text{ Tentukan } L \{ 4t^2 - 3\cos 2t + 5e^{-t} \}$$

Jawab :

$$\begin{aligned} L \{ 4t^2 - 3\cos 2t + 5e^{-t} \} &= 4L \{ t^2 \} - 3L \{ \cos 2 t \} + 5L \{ e^{-t} \} \\ &= 4 \frac{2+1}{2^2+1} - 3 \frac{s}{s^2+2^2} + 5 \frac{1}{s-(-1)} \\ &= \frac{12}{s^3} - \frac{3s}{s^2+4} - \frac{5}{s+1} \end{aligned}$$

$$2) \text{ Tentukan } \{ e^{4t} \cos 5t \}$$

Jawab :

$$\begin{aligned} L \{ e^{4t} \cos st \} &= \frac{s-a}{(s-a)^2+t^2} \\ &= \frac{s-4}{(s-4)^2+5^2} \\ &= \frac{s-4}{(s^2-8s+16)+25} \\ &= \frac{s-4}{(s^2-8s+41)} \end{aligned}$$

$$3) \text{ Tentukan } e^{2t} \{ 3\sin 4t - 4\cos 4t \}$$

Jawab :

$$L \{ e^{at} \} \{ 3l \{ \sin 4 t \} - 4 \{ \cos 4 t \} \} = \frac{1}{s-2} \left\{ 3 \cdot \frac{4}{s^2+4^2} - 4 \frac{s}{s^2+4^2} \right\}$$

