INTEGRAL TENTU

Integral Tentu

$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$$

Contoh: Hitunglah $\int_1^3 (3x^2 - x + 1)\,dx$

Penyelesaian:

$$\int_1^3 (3x^2 - 4x + 1)\,dx = [x^3 - 2x^2 + x]_1^3 = (3^3 - 2.3^2 + 3) - (1^3 - 2.1^2 + 1) = 12$$

Contoh Penyelesaian Integral Tentu

Contoh 2:

$$\int_0^3 (x^2 + 3)\,dx$$

$$= \left[\frac{1}{3}x^3 + 3x\right]_0^3$$

$$= (\frac{1}{3}\cdot 3^3 + 3\cdot 3) - (\frac{1}{3}\cdot 0^3 + 3\cdot 0)$$

$$= (9 + 9) - 0$$

$$= 18$$

Contoh 3:

$$\int_0^2 (4x^3 - 2x + 5)\,dx$$

$$= \left[x^4 - x^2 + 5x\right]_0^2$$

$$= (2^4 - 2^2 + 5\cdot 2) - (0^4 - 0^2 + 5\cdot 0)$$

$$= (16 - 4 + 10) - 0$$

$$= 22$$

Contoh 4

$$\int_{-1}^{2} (2x^2 - 3)^2 \, dx$$

$$= \int_{-1}^{2} (4x^4 - 12x^2 + 9) \, dx$$

$$= \left[\frac{4}{5}x^5 - 4x^3 + 9x \right]_{-1}^{2}$$

$$= (\frac{4}{5} \cdot 2^5 - 4 \cdot 2^3 + 9 \cdot 2) - (\frac{4}{5} \cdot (-1)^5 - 4 \cdot (-1)^3 + 9 \cdot (-1))$$

$$= (\frac{128}{5} - 32 + 18) - (-\frac{4}{5} + 4 - 9)$$

$$= (25\frac{3}{5} - 14) - (-5\frac{4}{5})$$

$$= 31\frac{2}{5} - 14$$

$$= 17\frac{2}{5}$$

Contoh 5

$$\int_{1}^{3} (5x^2 - 6x + \frac{12}{x^2}) \, dx$$

$$= \int_{1}^{3} (5x^2 - 6x + 12x^{-2}) \, dx$$

$$= \left[\frac{5}{3}x^3 - 3x^2 - 12x^{-1} \right]_{1}^{3}$$

$$= \left[\frac{5}{3}x^3 - 3x^2 - \frac{12}{x} \right]_{1}^{3}$$

$$= (\frac{5}{3} \cdot 3^3 - 3 \cdot 3^2 - \frac{12}{3}) - (\frac{5}{3} \cdot 1^3 - 3 \cdot 1^2 - \frac{12}{1})$$

$$= (45 - 27 - 4) - (\frac{5}{3} - 3 - 12)$$

$$= 14 - (-13\frac{1}{3})$$

$$= 27\frac{1}{3}$$

contoh 6:

$$\int_1^4 (x\sqrt{x}+\frac{1}{x\sqrt{x}})^2\,dx$$

$$=\int_1^4 (x\sqrt{x})^2+2+\frac{1}{(x\sqrt{x})^2}\,dx$$

$$=\int_1^4 (x\sqrt{x})^2+2+(x\sqrt{x})^{-2}dx$$

$$=\int_1^4 x^3+2+x^{-3}\,dx$$

$$=\left[\frac{1}{4}x^4+2x-\frac{1}{2}x^{-2}\right]_1^4$$

$$=\left[\frac{1}{4}x^4+2x-\frac{1}{2x^2}\right]_1^4$$

$$=(\frac{1}{4}\cdot 4^4+2\cdot 4-\frac{1}{2\cdot 4^2})-(\frac{1}{4}\cdot 1^4+2\cdot 1-\frac{1}{2\cdot 1^2})$$

$$=(64+8-\frac{1}{32})-(\frac{1}{4}+2-\frac{1}{2})$$

$$=71\frac{31}{32}-1\frac{3}{4}$$

$$=70\frac{7}{32}$$

contoh 7: Integral Fungsi Trigonometri

$$\int_0^\pi \sin 2x+\cos x\,dx$$

$$=\left[-\frac{1}{2}\cos 2x+\sin x\right]_0^\pi$$

$$=\left(-\frac{1}{2}\cos 2\pi+\sin\pi\right)-\left(-\frac{1}{2}\cos 0+\sin 0\right)$$

$$=\left(-\frac{1}{2}\cdot 1+0\right)-\left(-\frac{1}{2}\cdot 1+0\right)$$

$$=-\frac{1}{2}+\frac{1}{2}$$

$$=0$$

contoh 8:

$$\int_0^{\frac{\pi}{3}} (2\sin x + 6\cos 3x - 1)\, dx$$

$$= \left[-2\cos x + \frac{6}{3}\sin 3x - x\right]_0^{\frac{\pi}{3}}$$

$$= \left[-2\cos x + 2\sin 3x - x\right]_0^{\frac{\pi}{3}}$$

$$= \left(-2\cos\frac{\pi}{3} + 2\sin 3\cdot\frac{\pi}{3} - \frac{\pi}{3}\right) - \left(-2\cos 0 + 2\sin 0 - 0\right)$$

$$= \left(-2\cos\frac{\pi}{3} + 2\sin\pi - \frac{\pi}{3}\right) - \left(-2\cos 0 + 2\sin 0 - 0\right)$$

$$= \left(-2\cdot\frac{1}{2} + 2\cdot 0 - \frac{\pi}{3}\right) - \left(-2\cdot 1 + 2\cdot 0 - 0\right)$$

$$= \left(-1 - \frac{\pi}{3}\right) - (-2)$$

$$= \left(-1 - \frac{\pi}{3}\right) + 2$$

$$= 2 - \frac{\pi}{3}$$