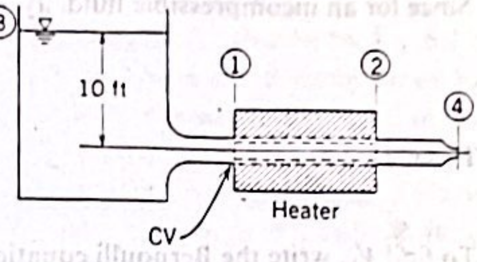
Example 6.8

Water flows steadily from a large open reservoir through a short length of pipe and a nozzle with cross-sectional area, $A = 0.864$ in.^{2} A well-insulated 10 kW heater surrounds the pipe. The flow is assumed to be steady, frictionless, and incompressible. Find the temperature rise of the fluid.

EXAMPLE PROBLEM 6.8

GIVEN: Water flows from a large reservoir through the system shown and discharges to atmospheric pressure. The heater is 10 kW. $A_4 = 0.864 \text{ in.}^2$



FIND: The temperature rise of the fluid between points ① and ②.

SOLUTION:

Basic equations:
$$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

$$0 = \overbrace{\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall}^{=0(1)} + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

$$\dot{Q} + \overbrace{\dot{W}_s}^{=0(4)} + \overbrace{\dot{W}_{\text{shear}}}^{=0(4)} = \overbrace{\frac{\partial}{\partial t}\int_{CV} e\rho \, d\forall}^{=0(1)} + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho \vec{V} \cdot d\vec{A}$$

Assumptions:
(1) Steady flow
(2) Frictionless flow
(3) Incompressible flow
(4) No shaft work, no shear work
(5) Flow along a streamline

Under the assumptions listed, the first law of thermodynamics for the CV shown becomes

$$\dot{Q} = \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho \vec{V} \cdot d\vec{A}$$

$$= \int_{A_1}\left(u + pv + \frac{V^2}{2} + gz\right)\rho \vec{V} \cdot d\vec{A} + \int_{A_2}\left(u + pv + \frac{V^2}{2} + gz\right)\rho \vec{V} \cdot d\vec{A}$$

For uniform properties at ① and ②

$$\dot{Q} = -|\rho A_1 V_1|\left(u_1 + p_1 v + \frac{V_1^2}{2} + gz_1\right) + |\rho V_2 A_2|\left(u_2 + p_2 v + \frac{V_2^2}{2} + gz_2\right)$$

From conservation of mass $|\rho A_1 V_1| = |\rho A_2 V_2| = \dot{m}$

$$\dot{Q} = \dot{m}\left[u_2 - u_1 + \left(\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2\right) - \left(\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1\right)\right]$$

For frictionless, incompressible, steady flow, along a streamline,

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Therefore,

$$\dot{Q} = \dot{m}(u_2 - u_1)$$

Since for an incompressible fluid, $u_2 - u_1 = c(T_2 - T_1)$, then

$$T_2 - T_1 = \frac{\dot{Q}}{\dot{m}c}$$

From continuity,

$$\dot{m} = \rho A_4 V_4$$

To find V_4, write the Bernoulli equation between the free surface, ③, and ④.

$$\frac{p_3}{\rho} + \frac{V_3^2}{2} + gz_3 = \frac{p_4}{\rho} + \frac{V_4^2}{2} + gz_4$$

Since $p_3 = p_4$ and $V_3 \approx 0$, then

$$V_4 = \sqrt{2g(z_3 - z_4)} = \sqrt{2 \times 32.2 \frac{\text{ft}}{\text{sec}^2} \times 10 \text{ ft}} = 25.4 \text{ ft/sec}$$

and

$$\dot{m} = \rho A_4 V_4 = 1.94 \frac{\text{slug}}{\text{ft}^3} \times 0.864 \text{ in.}^2 \times \frac{\text{ft}^2}{144 \text{ in.}^2} \times 25.4 \frac{\text{ft}}{\text{sec}}$$

$$\dot{m} = 0.296 \text{ slug/sec}$$

Assuming no heat loss to the surroundings, we obtain

$$T_2 - T_1 = \frac{\dot{Q}}{\dot{m}c} = 10 \text{ kW} \times \frac{3413 \text{ Btu}}{\text{kW} \cdot \text{hr}} \times \frac{\text{hr}}{3600 \text{ sec}} \times \frac{\text{sec}}{0.296 \text{ slug}} \times \frac{\text{slug}}{32.2 \text{ lbm}} \times \frac{\text{lbm} \cdot \text{R}}{1 \text{ Btu}}$$

$$T_2 - T_1 = 0.995 \text{ R}$$

$T_2 - T_1$

This problem illustrates that in general the first law of thermodynamics and the Bernoulli equation are independent equations.