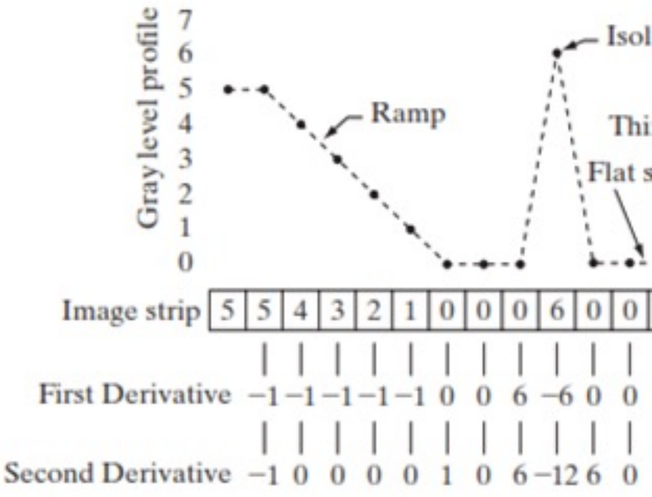
Pengolahan Citra Digital – TIK19504 - Pertemuan - 9

Neighborhood Operation : Filtering (Bagian 2) Linear Filtering



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Linear spatial filtering

- response :
  - a sum of products of the filter coefficients and the corresponding image pixels under the mask

- example 3x3 mask :

$$
\begin{aligned}
R(x,y) = \; & w(-1,-1)f(x-1, y-1) + \\
& w(-1,0)f(x-1,y) \quad + \dots + \\
& w(0,0)f(x,y) \quad\quad + \dots + \\
& w(1,0)f(x+1,y) \quad + \\
& w(1,1)f(x+1,y+1)
\end{aligned}
$$

Linear spatial filtering

Note :

- ✓ coefficient $w(0, 0)$ coincides with image value $f(x, y)$
- ✓ the mask is centered at (x, y)

Mask of size mxn,

- o assume $m=2a+1$ and $n=2b+1$, a and b => nonnegative integers
- o masks of odd sizes
- o smallest meaningful size being 3x3
- o (exclude a 1x1 mask)

Linear spatial filtering

- linear filtering of an image f of size MxN with a filter mask of size mxn:

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)$$

  - where a=(m-1)/2 and b=(n-1)/2.
  - x=0,1,2,…, M-1 and y=0,1,2,…, N-1.

- the mask processes all pixels in the image

Convolution

- linear filtering = frequency domain concept = convolution
- linear spatial filtering = "convolving a mask with an image"
- filter masks = convolution masks / convolution kernel
- response, R, of an m*n mask at any point (x, y) :

$$
\begin{aligned}
R &= w_1 z_1 + w_2 z_2 + \ldots + w_{mn} z_{mn} \\
&= \sum_{i=1}^{mn} w_i z_i
\end{aligned}
$$

w's	: mask coefficients
z's	: the values of the image gray levels corresponding to those coefficients
mn	: total number of coefficients in the mask

Smoothing Spatial Filters

Smoothing filters : for blurring and for noise reduction

Blurring :

- ✓used in preprocessing steps
- ✓removal of small details from an image prior to (large) object extraction,
- ✓bridging of small gaps in lines or curves.

Noise reduction :

- by blurring with a linear filter and also by nonlinear filtering.

Smoothing Linear Filters

- The output (response) of a smoothing linear spatial filter
  - average of the pixels contained in the neighborhood of the filter mask.
  - averaging filters.
  - lowpass filters.
- idea :
  - ✓ replacing the value of every pixel in an image by the average of the gray levels in the neighborhood defined by the filter mask,

- reduced “sharp” transitions in gray levels.
- random noise = sharp transitions => smoothing is noise reduction.

- However :
  - edges (which almost always are desirable features of an image) = sharp transitions
  - averaging filters => undesirable side effect = blur edges

Smoothing (averaging) filter masks

- ❑Example : 3*3 smoothing (averaging) filter masks.
- ❑The constant multiplier in front of each mask = the sum of the values of its coefficients => required to compute an average.

$$\frac{1}{9} \times \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

Averaging filter

- Use : reduction of “irrelevant” detail in an image.
- “irrelevant” = pixel regions that are small with respect to the size of the filter mask.
- 3*3 smoothing filters :
  - ✓Note that, instead of being 1/9, the coefficients of the filter are all 1’s.
- idea : computationally more efficient to have coefficients valued 1.
  - ✓At the end of the filtering process the entire image is divided by 9.
- An m*n mask => a normalizing constant = 1/mn.
- A spatial averaging filter in which all coefficients are equal => a box filter.

Weighted average filter

- pixels are multiplied by different coefficients,
  - giving more importance (weight) to some pixels at the expense of others.
- the pixel at the center of the mask is multiplied by a higher value than any other,
  - giving this pixel more importance in the calculation of the average.
- The other pixels are inversely weighted as a function of their distance from the center of the mask.
- The diagonal terms are further away from the center than the orthogonal neighbors
  - weighed less than these immediate neighbors of the center pixel.
- The basic strategy :
  - weighing the center point the highest
  - reducing the value of the coefficients as a function of increasing distance from the origin
  - an attempt to reduce blurring in the smoothing process.
  - picked other weights to accomplish the same general objective.
- However, the sum of all the coefficients in the mask is equal to 16,
  - an attractive feature for computer implementation because it has an integer power of 2

Weighted average filter

$$g(x, y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t)}$$

- filtering an M*N image with a weighted averaging filter m*n
  - (m and n odd)
  - x=0, 1, 2,…, M-1 and y=0, 1, 2,…, N-1
- The denominator in equation = the sum of the mask coefficients
  - a constant
  - computed only once.
- this scale factor is applied to all the pixels of the output image after the filtering process is completed.

Sharpening Spatial Filters

- principal objective : highlight fine detail in an image or to enhance detail that has been blurred,
  - either in error or as a natural effect of a particular method of image acquisition.
- uses : applications ranging from electronic printing and medical imaging to industrial inspection and autonomous guidance in military systems.
- image blurring => pixel averaging/integration in a neighborhood
- sharpening => spatial differentiation
- various ways of sharpening
  - by digital differentiation
- the strength of the response of a derivative operator
  - proportional to the degree of discontinuity of the image at the point
  - image differentiation enhances edges and other discontinuities (such as noise)
  - deemphasizes areas with slowly varying gray-level values.

Sharpening Spatial Filters

- based on first- and second-order derivatives
- behavior of these derivatives
  - in areas of constant gray level (flat segments),
  - at the onset and end of discontinuities (step and ramp discontinuities),
  - and along gray-level ramps
- can be used to model noise points, lines, and edges in an image.
- derivatives of a digital function are defined in terms of differences

First Derivative and Second Derivative

- first derivative :
  - (1) must be zero in flat segments (areas of constant gray-level values);
  - (2) must be nonzero at the onset of a gray-level step or ramp;
  - (3) must be nonzero along ramps
- second derivative :
  - (1) must be zero in flat areas;
  - (2) must be nonzero at the onset and end of a gray-level step or ramp;
  - (3) must be zero along ramps of constant slope
- the shortest distance over which that gray-level change can occur is between adjacent pixels

First Derivative and Second Derivative

- A basic definition of the first-order derivative of a one-dimensional function f(x) is the difference

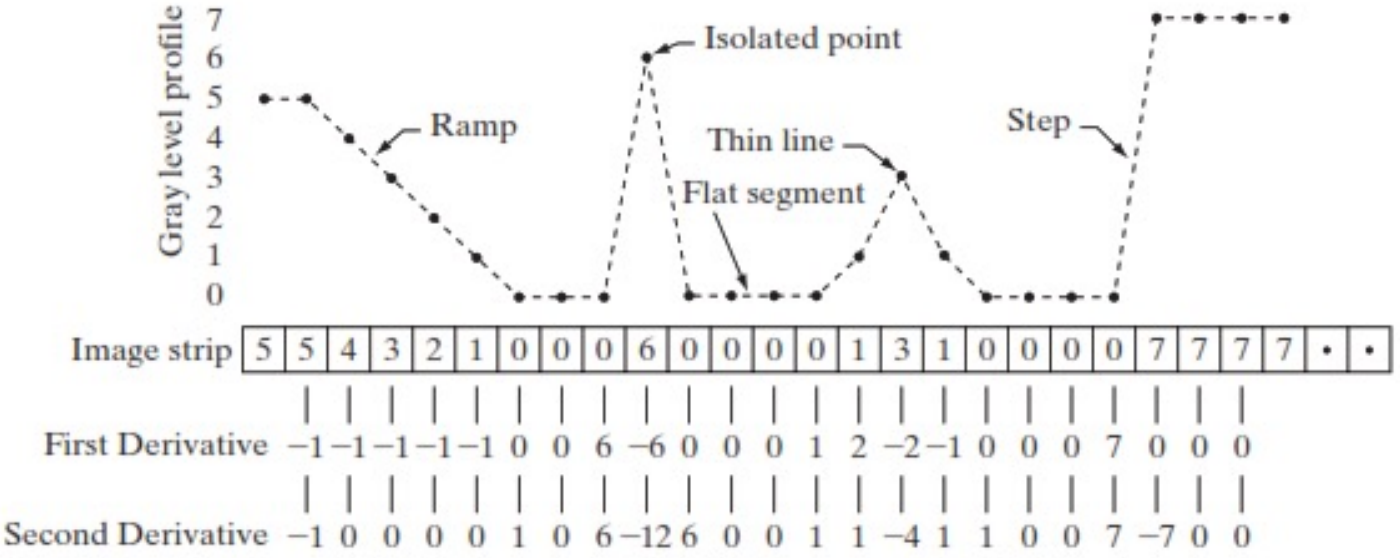
$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- a second-order derivative as the difference

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Visualization

- a simple image that contains various solid objects, a line, and a single noise point



Use of First Derivatives for Enhancement—The Gradient

- implemented using the magnitude of the gradient.
- for a function f(x,y), the gradient of f at coordinates (x, y) is defined as the two dimensional column vector :

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

- magnitude of this vector :

$$\begin{aligned} \nabla f &= \operatorname{mag}(\nabla \mathbf{f}) \\ &= \left[G_x^2 + G_y^2\right]^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2} \end{aligned}$$

- approximate the magnitude of the gradient by using absolute values

$$\nabla f \approx |G_x| + |G_y|$$

Roberts and Sobel Operators

- the simplest approximations to a first-order derivative that satisfy the conditions stated in that section are
  - $G_x=(z_8-z_5)$ and $G_y = (z_6-z_5)$
  - Roberts [1965] : use cross differences
  - $G_x = (z_9 - z_5)$ and $G_y = (z_8 - z_6)$
- Roberts cross-gradient operators :

$$\nabla f = \left[(z_9 - z_5)^2 + (z_8 - z_6)^2\right]^{1/2}$$

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

- Sobel operators :

$$\nabla f \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Use of Second Derivatives for Enhancement– The Laplacian

- isotropic filters:
  - whose response is independent of the direction of the discontinuities in the image to which the filter is applied.
  - rotation invariant, in the sense that rotating the image and then applying the filter gives the same result as applying the filter to the image first and then rotating the result.
- (Rosenfeld and Kak [1982]) :
  - the simplest isotropic derivative operator is the Laplacian
  - for a function (image) f(x,y) of two variables :

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Use of Second Derivatives for Enhancement– The Laplacian

- partial second-order derivative in the x-direction

$$\frac{\partial^2 f}{\partial^2 x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

- in the y-direction

$$\frac{\partial^2 f}{\partial^2 y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

- summing these two components

$$\nabla^2 f = \left[f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)\right] - 4f(x, y)$$

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1