



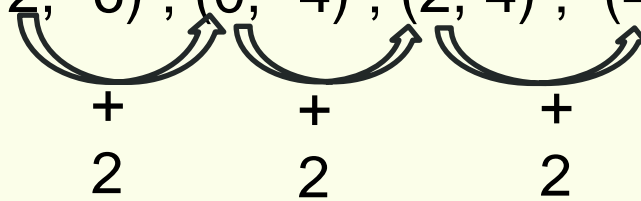
**D. Interpolasi Newton
Gregory Maju-
Mundur**

Interpolasi Newton Gregory Maju-Mundur hanya dapat digunakan jika "x" memiliki selisih/beda yang tetap.

Contoh:

Diketahui data-data berikut.

$(-2, -6) ; (0, -4) ; (2, 4) ; (4, 20).$



Soal seperti di atas dapat dikerjakan dengan interpolasi

Newton Gregory Maju-Mundur

1. Interpolasi Newton Gregory Maju

Rumus:

$$\begin{aligned} f(x) \approx P_n(x) &= f(x_0) + \Delta f(x_0) \cdot s + \frac{\Delta^2 f(x_0)}{2!} \cdot s(s-1) + \frac{\Delta^3 f(x_0)}{3!} \cdot s(s-1)(s-2) \\ &+ \frac{\Delta^4 f(x_0)}{4!} \cdot s(s-1)(s-2)(s-3) + \dots \\ &+ \frac{\Delta^n f(x_0)}{n!} \cdot s(s-1) \dots (s-(n-1)) \end{aligned}$$

dengan $s = \frac{x - x_0}{h}$ dan $h = x_{i+1} - x_i$

Jika diketahui 2 titik, maka $f(x) \approx P_1(x) = f(x_0) + \Delta f(x_0) \cdot s$

Contoh:

Diketahui data- data berikut: $(-2, -6)$; $(0, -4)$, $(2, 4)$; $(4, 20)$.

Tentukan: (a) $f(x)$ dan (b) $f(1)$

Penyelesaian:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
-2	-6			
		2		
0	-4		6	
		8		2
2	4		8	
		16		
4	20			



$\rightarrow f(x_0)$

$\rightarrow \Delta f(x_0)$

$\rightarrow \Delta^2 f(x_0)$

$\rightarrow \Delta^3 f(x_0)$



$$h = 2$$
$$s = \frac{x + 2}{2}$$



$$P_3(x) = f(x_0) + \Delta f(x_0) \cdot s + \frac{\Delta^2 f(x_0)}{2!} \cdot s(s-1) + \frac{\Delta^3 f(x_0)}{3!} \cdot s(s-1)(s-2)$$

$$= -6 + 2 \cdot \left(\frac{x+2}{2}\right) + \frac{6}{2} \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) + \frac{2}{6} \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) \left(\left(\frac{x+2}{2}\right) - 2\right)$$

$$= -6 + (x+2) + 3 \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) + \frac{1}{3} \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) \left(\left(\frac{x+2}{2}\right) - 2\right)$$

$$f(x) \approx P_3(x)$$

$$\approx -6 + (x+2) + 3 \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) + \frac{1}{3} \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) \left(\left(\frac{x+2}{2}\right) - 2\right)$$


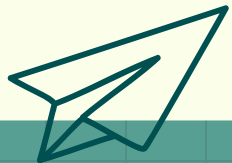



$$f(x) \approx P_3(x)$$

$$\approx -6 + (x + 2) + 3 \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) + \frac{1}{3} \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) \left(\left(\frac{x+2}{2}\right) - 2\right)$$

$$f(1) \approx -6 + (1 + 2) + 3 \cdot \left(\frac{1+2}{2}\right) \left(\left(\frac{1+2}{2}\right) - 1\right) + \frac{1}{3} \cdot \left(\frac{1+2}{2}\right) \left(\left(\frac{1+2}{2}\right) - 1\right) \left(\left(\frac{1+2}{2}\right) - 2\right)$$

$$\approx -6 + 3 + 3 \cdot \left(\frac{3}{2}\right) \left(\frac{1}{2}\right) + \frac{1}{3} \cdot \left(\frac{3}{2}\right) \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right)$$

$$\approx -3 + \frac{9}{4} - \frac{1}{8}$$

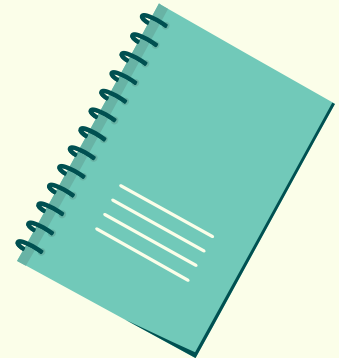
$$\approx -\frac{7}{8} = -0,875$$


2. Interpolasi Newton Gregory Mundur

Rumus:

$$\begin{aligned} f(x) \approx P_n(x) = & f(x_n) + \nabla f(x_n) \cdot s + \frac{\nabla^2 f(x_n)}{2!} \cdot s(s+1) + \frac{\nabla^3 f(x_n)}{3!} \cdot s(s+1)(s+2) \\ & + \frac{\nabla^4 f(x_n)}{4!} \cdot s(s+1)(s+2)(s+3) + \dots \\ & + \frac{\nabla^n f(x_n)}{n!} \cdot s(s+1) \dots (s+(n-1)) \end{aligned}$$

dengan $s = \frac{x-x_n}{h}$ dan $h = x_{i+1} - x_i$



contoh:

Diketahui data- data berikut: $(-2, -6)$; $(0, -4)$; $(2, 4)$; $(4, 20)$.

Tentukan: (a) $f(x)$ dan (b) $f(1)$

Penyelesaian: \circ

x	f(x)	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$
-2	-6			
		2		
0	-4		6	
		8		2
2	4		8	
		16		
4	20			

$$h = 2$$

$$s = \frac{x - 4}{2}$$

$$\nabla^3 f(x_3)$$

$$\nabla^2 f(x_3)$$

$$\nabla f(x_3)$$

$$f(x_3)$$



$$P_3(x) = f(x_3) + \nabla f(x_3) \cdot s + \frac{\nabla^2 f(x_3)}{2!} \cdot s(s+1) + \frac{\nabla^3 f(x_3)}{3!} \cdot s(s+1)(s+2)$$

$$= 20 + 16 \cdot \left(\frac{x-4}{2}\right) + \frac{8}{2} \cdot \left(\frac{x-4}{2}\right) \left(\left(\frac{x-4}{2}\right) + 1\right) + \frac{2}{6} \cdot \left(\frac{x-4}{2}\right) \left(\left(\frac{x-4}{2}\right) + 1\right) \left(\left(\frac{x-4}{2}\right) + 2\right)$$

$$= 20 + 8(x-4) + 4 \cdot \left(\frac{x-4}{2}\right) \left(\left(\frac{x-4}{2}\right) + 1\right) + \frac{1}{3} \cdot \left(\frac{x-4}{2}\right) \left(\left(\frac{x-4}{2}\right) + 1\right) \left(\left(\frac{x-4}{2}\right) + 2\right)$$


$$f(x) \approx P_3(x)$$

$$\approx 20 + 8(x-4) + 4 \cdot \left(\frac{x-4}{2}\right) \left(\left(\frac{x-4}{2}\right) + 1\right) + \frac{1}{3} \cdot \left(\frac{x-4}{2}\right) \left(\left(\frac{x-4}{2}\right) + 1\right) \left(\left(\frac{x-4}{2}\right) + 2\right)$$

$$f(1) \approx 20 + 8(1-4) + 4 \cdot \left(\frac{1-4}{2}\right) \left(\left(\frac{1-4}{2}\right) + 1\right) + \frac{1}{3} \cdot \left(\frac{1-4}{2}\right) \left(\left(\frac{1-4}{2}\right) + 1\right) \left(\left(\frac{1-4}{2}\right) + 2\right)$$



Latihan Soal



Diketahui data- data berikut:

$(0, -6)$; $(2, 4)$; $(4, 10)$.

Tentukan perkiraan dari $f(1)$



TERIMA KASIH

