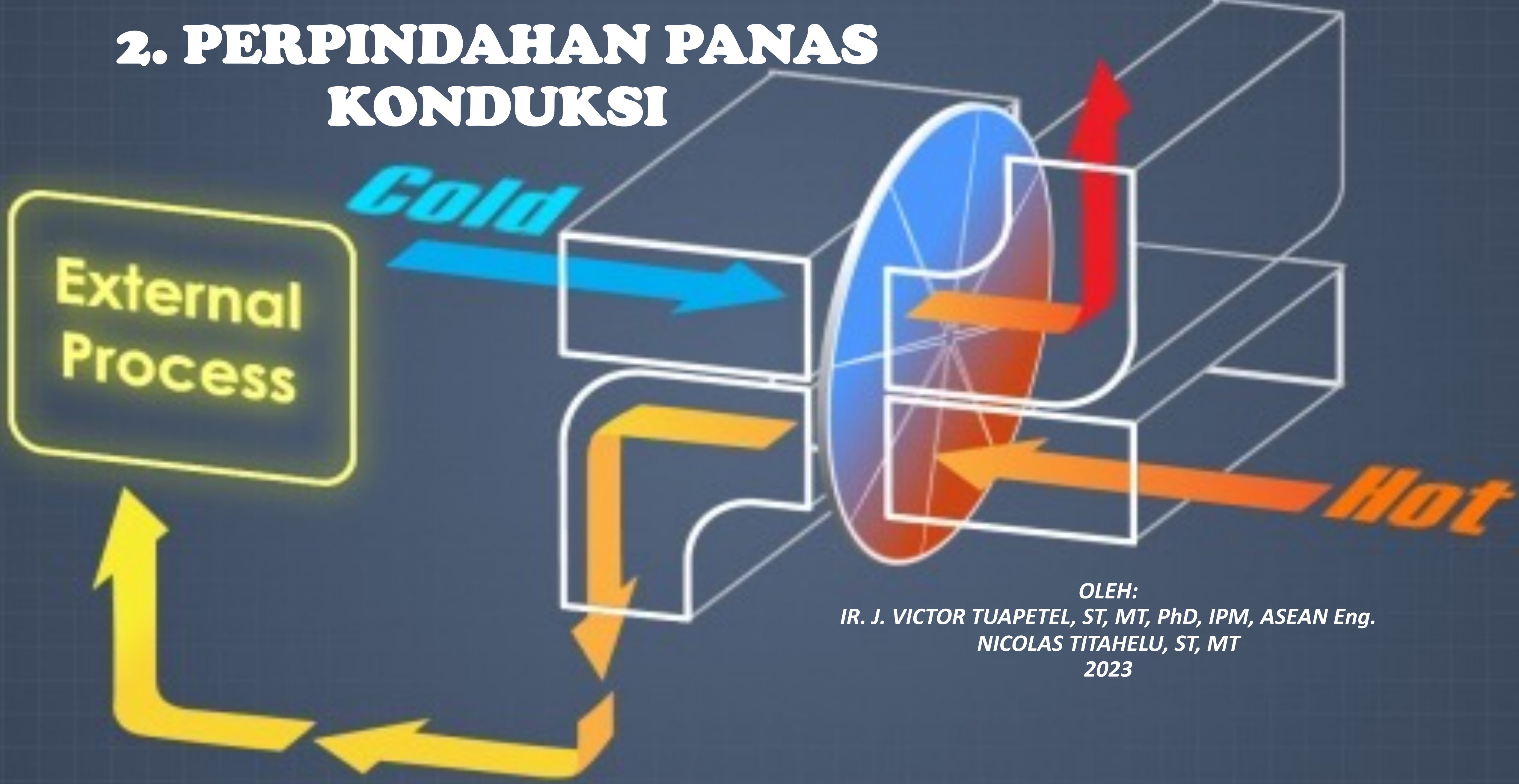
2. PERPINDAHAN PANAS KONDUKSI



OLEH:
IR. J. VICTOR TUAPETEL, ST, MT, PhD, IPM, ASEAN Eng.
NICOLAS TITAHELU, ST, MT
2023

Konduksi Steady State Satu Dimensi

1. Persamaan Umum Perpindahan Panas Konduksi dalam Koordinat Kartesius

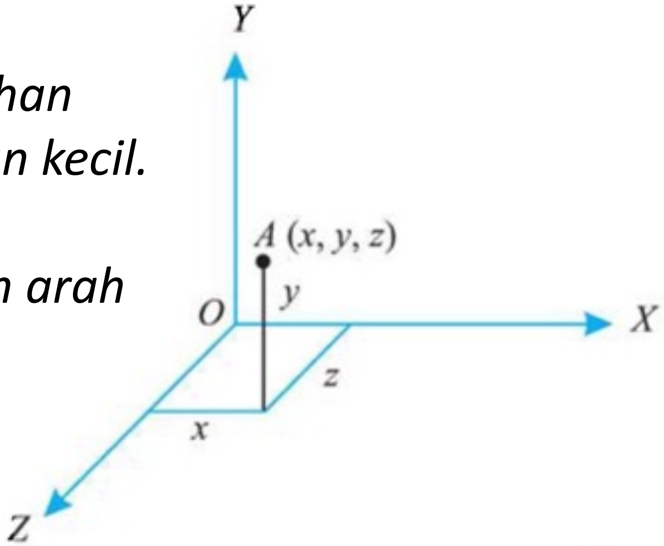
t = temperatur pada sisi kiri ABCD, diasumsikan sama untuk keseluruhan permukaan karena luas permukaan kecil.

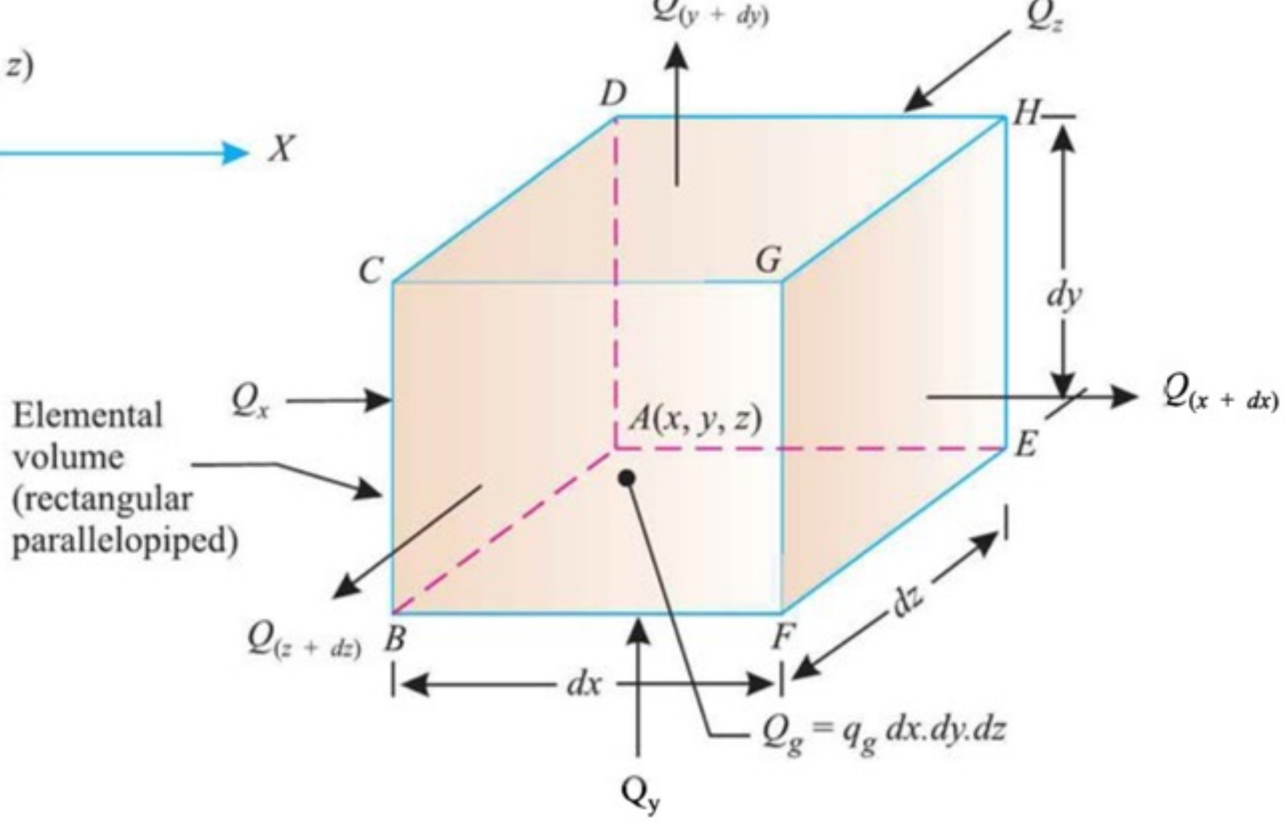
dt/dx = perubahan temperatur dalam arah X.

$\left(\frac{\partial t}{\partial x}\right)dx$ *= perubahan temperatur melalui jarak dx*

$t + \left(\frac{\partial t}{\partial x}\right)dx$ *= temperatur pada sis kanan EFGH.*

k_x, k_y, k_z *= konduktivitas termal pada arah sumbu X, Y dan Z*





- Jika sifat-sifat material adalah sama disebut *material isotropik (isotropic)* dan jika tidak sama disebut *material anisotropik (anisotropic)*.
- q_g = panas yang dibangkitkan per satuan volume per satuan waktu.

  (di dalam volume atur ada sumber panas karena adanya aliran listrik pada motor listrik atau generator, atau fisi nuklir, dll).
- ρ = massa jenis atau densitas material.
- c = panas spesifik material.
- Kesetimbangan energi untuk elemen volume:

  Q = jumlah aliran panas dalam suatu arah.

  Q' = $(Q.d\tau)$ = aliran panas total (flux) pada arah tersebut (pada waktu $d\tau$).
- Kuantitas panas mengalir ke dalam elemen dari sisi ABCD selama interval waktu $d\tau$ dalam arah X:

  Heat influx:
  $$Q'_x = -k_x\,(dy.dz)\,\frac{\partial t}{\partial x}\cdot d\tau$$
- Panas yang keluar dari volume atur pada sisi EFGH,

  Heat efflux:
  $$Q'_{(x+dx)} = Q'_x + \frac{\partial}{\partial x}(Q'_x)\,dx$$

- Akumulasi panas dalam elemen karena aliran panas dalam arah X, Y dan Z:

$$dQ'_x = Q'_x - \left[Q'_x + \frac{\partial}{\partial x}(Q'_x)\,dx\right]$$

$$= -\frac{\partial}{\partial x}(Q'_x)\,dx$$

$$= -\frac{\partial}{\partial x}\left[-k_x\,(dy.dz)\,\frac{\partial t}{\partial x}\,.\,d\tau\right]dx$$

$$= \frac{\partial}{\partial x}\left[k_x \frac{\partial t}{\partial x}\right] dx.dy.dz.d\tau$$

$$dQ'_y = \frac{\partial}{\partial y}\left[k_y \frac{\partial t}{\partial y}\right] dx.dy.dz.d\tau$$

$$dQ'_z = \frac{\partial}{\partial z}\left[k_z \frac{\partial t}{\partial z}\right] dx.dy.dz.d\tau$$

- Akumulasi panas dalam elemen karena konduksi panas dari semua arah koordinat:

$$= \frac{\partial}{\partial x}\left[k_x \frac{\partial t}{\partial x}\right] dx.dy.dz.d\tau + \frac{\partial}{\partial y}\left[k_y \frac{\partial t}{\partial y}\right] dx.dy.dz.d\tau + \frac{\partial}{\partial z}\left[k_z \frac{\partial t}{\partial z}\right] dx.dy.dz.d\tau$$

$$= \left[\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right)\right] dx.dy.dz.d\tau$$

- Panas total yang dibangkitkan di dalam elemen:

$$\rho\,(dx.dy.dz)\,c.\frac{\partial t}{\partial \tau}\;.\;d\tau$$

- Panas tersimpan dalam elemen:

Panas yang tersimpan di dalam benda = massa benda x panas spesifik material benda x peningkatan temperatur benda.

$$\left[\frac{\partial}{\partial x}\left(k_x\frac{\partial t}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial t}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial t}{\partial z}\right)\right]dx.dy.dz.d\tau + q_g\;(dx.dy.dz.)d\tau = \rho\,(dx.dy.dz)\;c.\frac{\partial t}{\partial \tau}\;.\;d\tau$$

Dividing both sides by $dx.dy.dz.d\tau$, we have

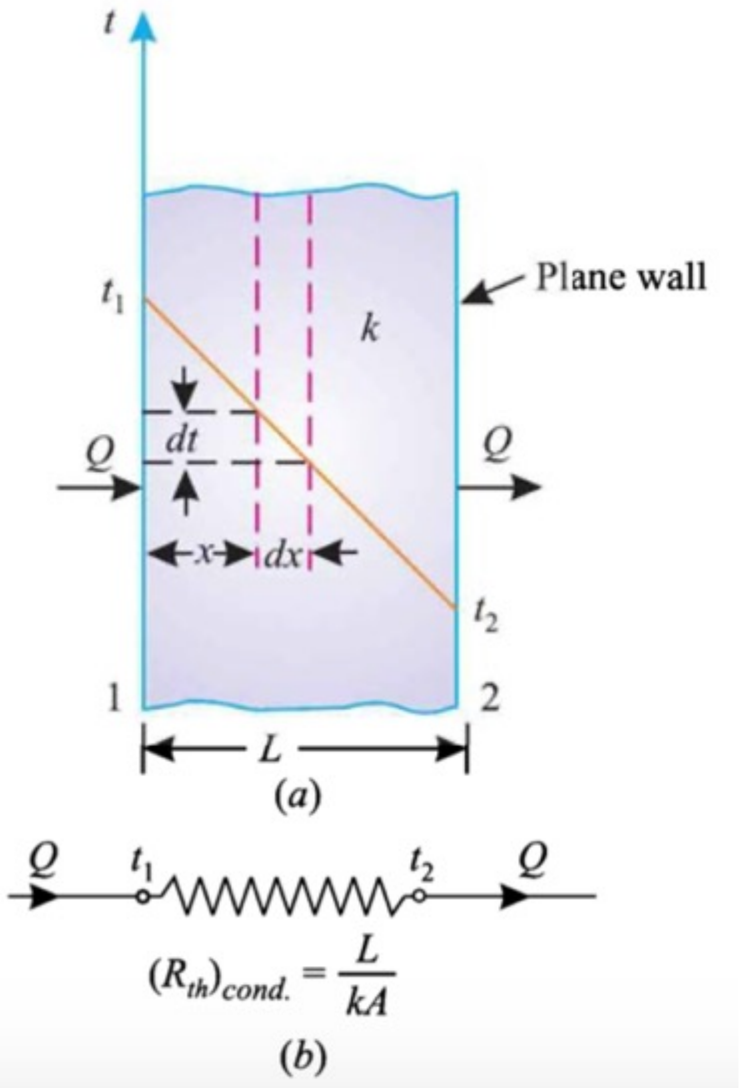
$$\frac{\partial}{\partial x}\left(k_x\frac{\partial t}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial t}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial t}{\partial z}\right)+q_g=\rho.c.\frac{\partial t}{\partial \tau}$$

or, using the vector operator ∇, we get

$$\nabla\,.(k\nabla t)+q_g=\rho.c.\frac{\partial t}{\partial \tau}$$

Konduksi Panas melalui Bidang Datar dan Dinding Komposit (Berlapis)

- Konduksi Panas melalui Dinding Datar



L = ketebalan dinding
A = luas penampang melintamg didinding.
k = konduktivitas termal dinding material.
t_1, t_2 = temperatur masing-masing permukaan dinding.

Persamaan umum konduksi panas:

$$\frac{\partial^2 t}{dx^2} + \frac{\partial^2 t}{dy^2} + \frac{\partial^2 t}{dz^2} + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}$$

- Panas melalui dinding datar dapat dihitung dengan menggunakan Hukum Fourier:

$$Q = -kA\frac{dt}{dx} \qquad \text{(where, } \frac{dt}{dx} = \text{Temperature gradient)}$$

$$\frac{dt}{dx} = \frac{d}{dx}\left[\left(\frac{t_2 - t_1}{L}\right)x + t_1\right] = \frac{t_2 - t_1}{L}$$

$$Q = -kA\frac{(t_2 - t_1)}{L} = \frac{kA\,(t_1 - t_2)}{L}$$

$$Q = \frac{(t_1 - t_2)}{(\frac{L}{kA})} = \frac{(t_1 - t_2)}{(R_{th})_{kond}}$$

dimana :

$(R_{th})_{kond}$ = tahanan termal untuk konduksi panas dimana equivalen dengan tahanan listrik pada gambar b.

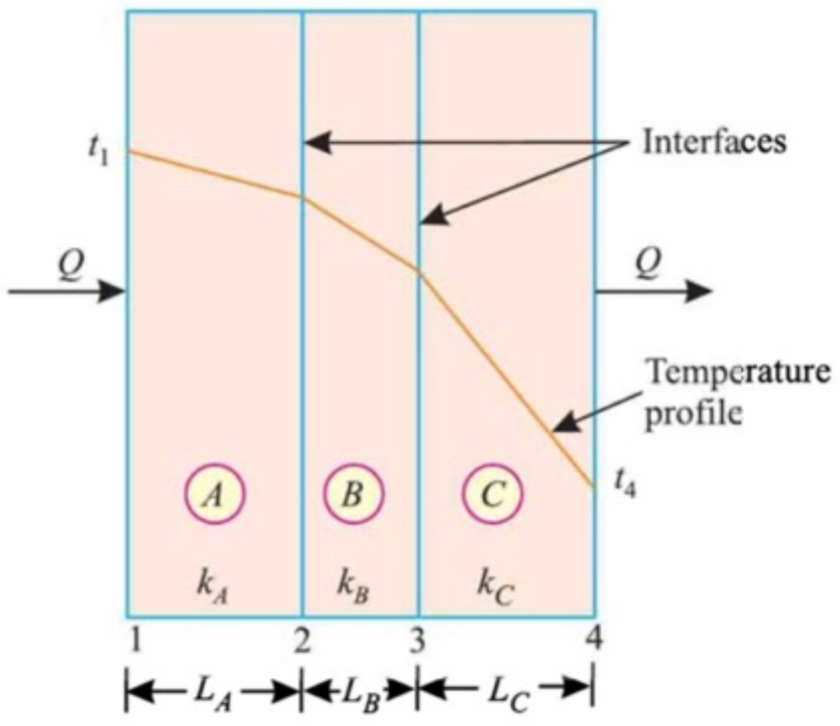
- $(R_{th})_{kond} = L / kA$
- Berat dinding, W = $\rho\ A\ L$
- Maka:

$$W = \rho\ A.\ (R_{th})_{cond.}\ kA = (\rho.k)A^2.(R_{th})_{cond.}$$

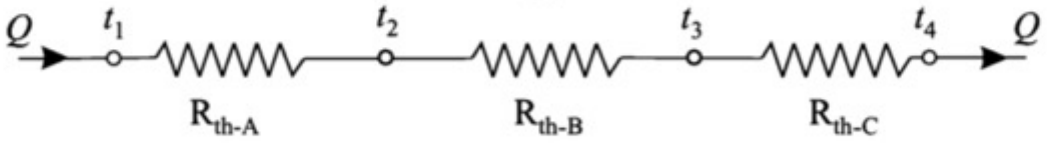
Konduksi Panas melalui Dinding Komposit/Berlapis

- L_A, L_B, L_C = tebal slab atau lapisan A, B, C
- k_A, k_B, k_C = konduktivitas termal A, B, C
- t_1, t_4 ($t_1 > t_4$) = temperatur permukaan dinding 1 dan 4
- t_2, t_3 = temperatur pada interface 2 dan 3

$$Q = \frac{k_A . A\,(t_1 - t_2)}{L_A} = \frac{k_B\ . A\,(t_2 - t_3)}{L_B} = \frac{k_C\ . A\,(t_3 - t_4)}{L_C}$$



(a)



$$R_{th.A} = \frac{L_A}{k_A . A}, \quad R_{th.B} = \frac{L_B}{k_B . A}, \quad R_{th.C} = \frac{L_C}{k_C . A}$$

(b)

$$t_1 - t_2 = \frac{Q \, . L_A}{k_A \, . \, A}$$

$$t_2 - t_3 = \frac{Q \, . L_B}{k_B \, . \, A}$$

$$t_3 - t_4 = \frac{Q \, . L_c}{k_C \, . \, A}$$

$$(t_1 - t_4) = Q \left[\frac{L_A}{k_A \, . \, A} + \frac{L_B}{k_B \, . \, A} + \frac{L_C}{k_C \, . \, A} \right]$$

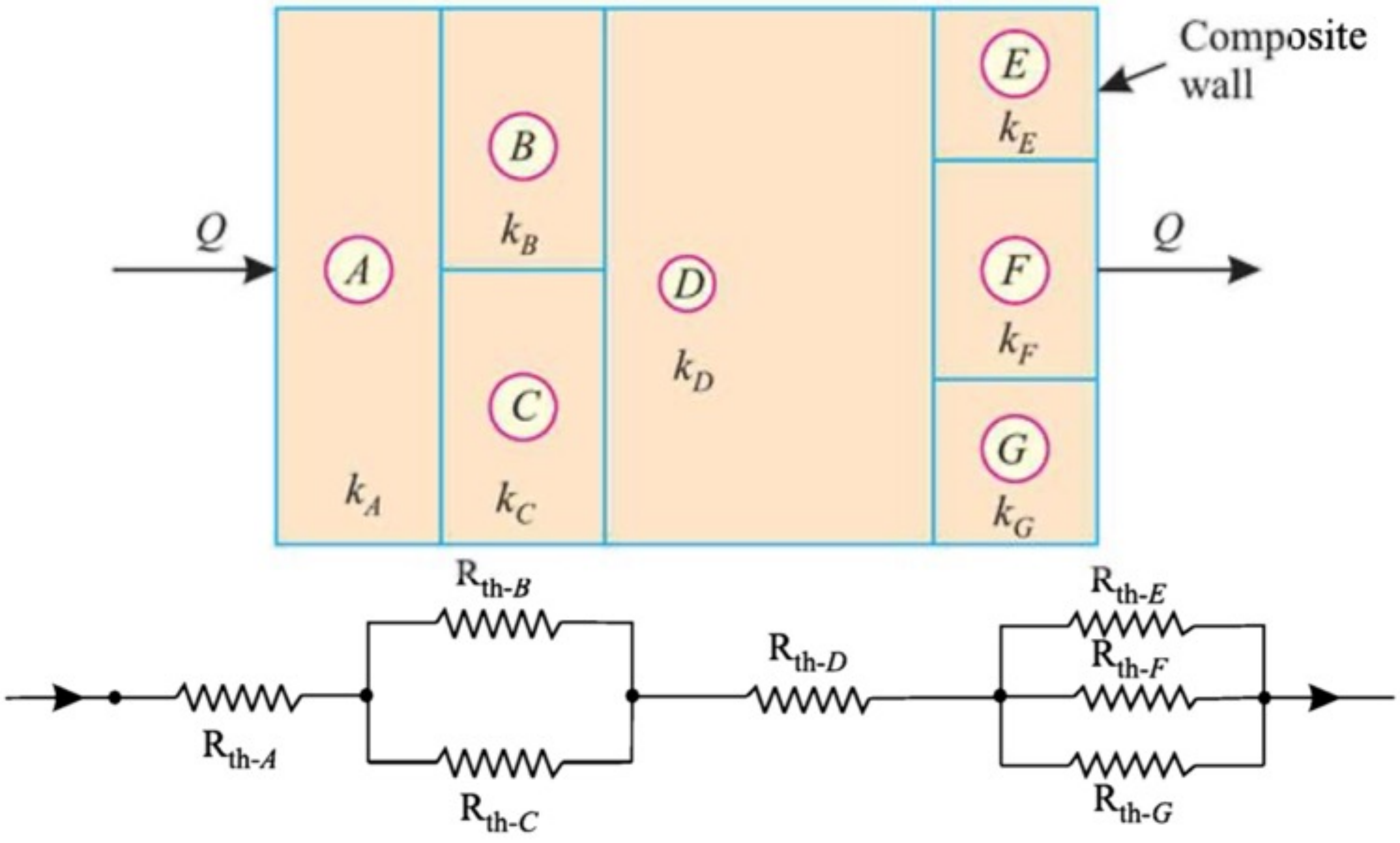
$$Q = \frac{A \, (t_1 - t_4)}{\left[\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} \right]}$$

$$Q = \frac{(t_1 - t_4)}{\left[\frac{L_A}{k_A \, . \, A} + \frac{L_B}{k_B \, . \, A} + \frac{L_C}{k_C \, . \, A} \right]} = \frac{(t_1 - t_4)}{[R_{th-A} + R_{th-B} + R_{th-C}]}$$

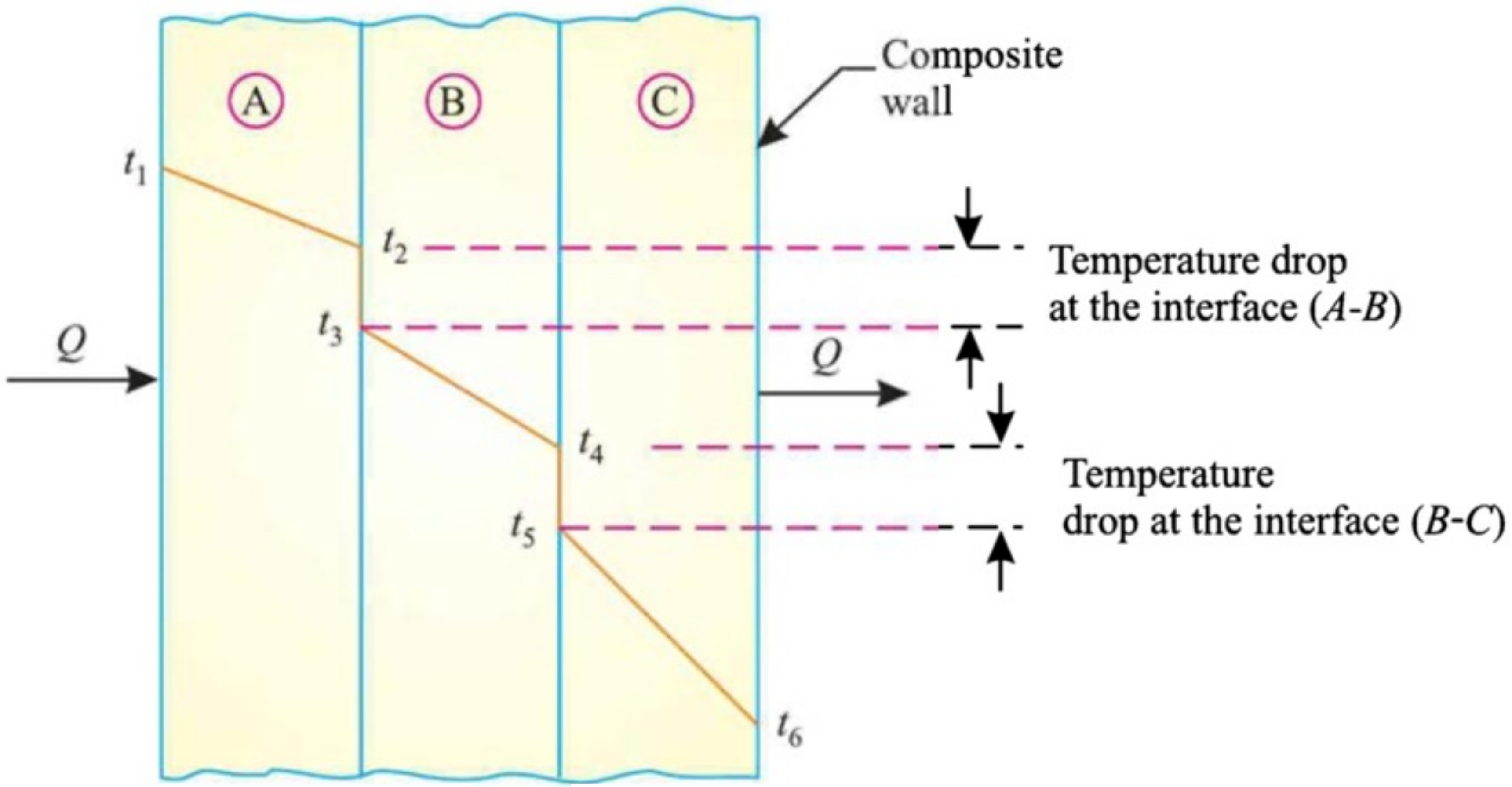
$$Q = \frac{[t_1 - t_{(n+1)}]}{\sum_1^n \frac{L}{kA}}$$

$$Q = \frac{\Delta \, t_{\text{overall}}}{\sum R_{th}}$$

Perpindahan panas satu dimensi melalui dinding komposit yang tersusun seri dan paralel, serta analogi aliran listrik pada hambatan.

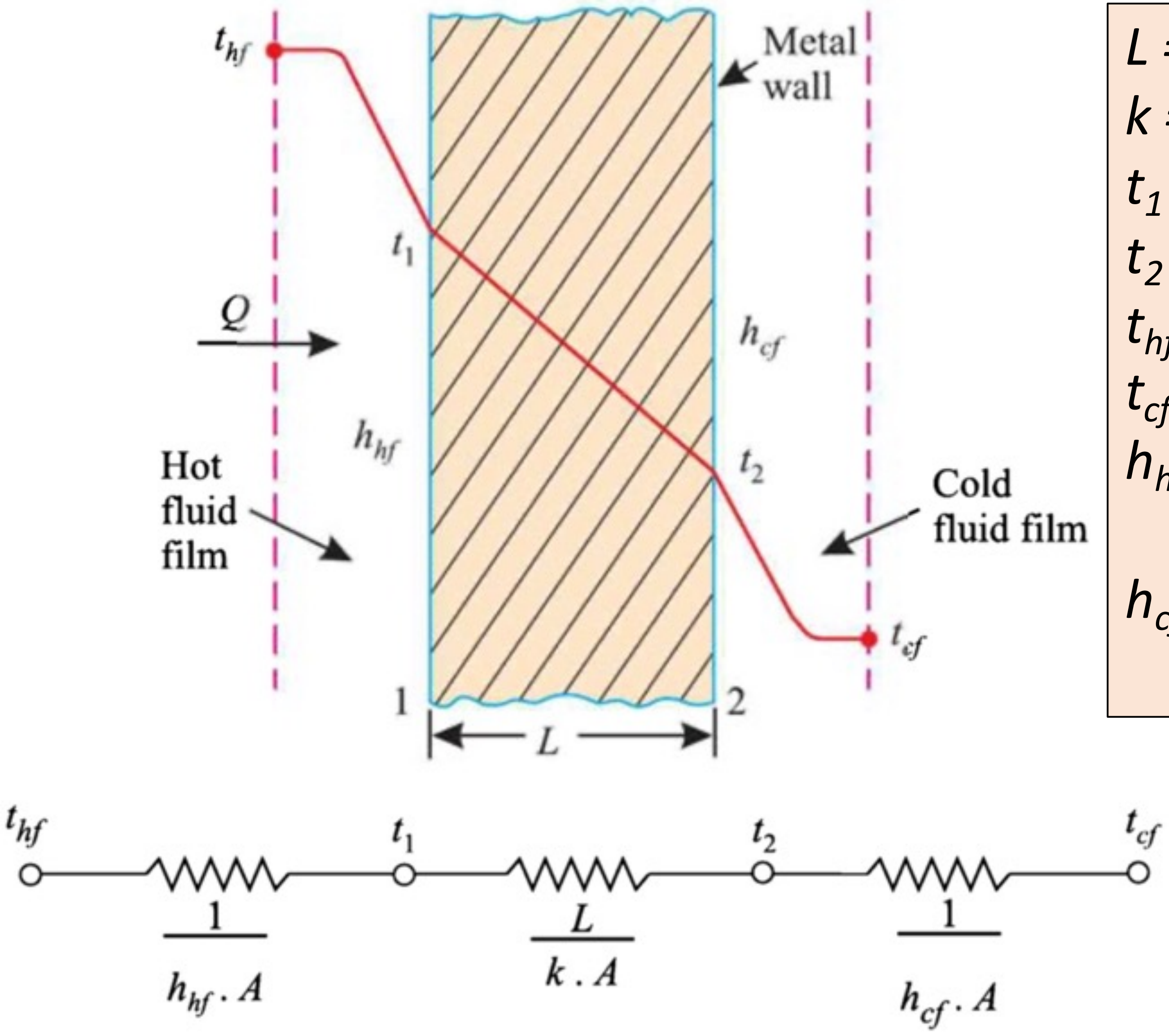


Tahanan kontak termal / Thermal contact resistance



$$(R_{th-AB})_{cont.} = \frac{(t_2 - t_3)}{Q / A} \qquad (R_{th-BC})_{cont.} = \frac{(t_4 - t_5)}{Q / A}$$

Koefisien Perpindahan Panas Keseluruhan
(The Overall Heat-Transfer Coefficient)



L = tebal dinding logam.

k = konduktivitas termal material dinding

t_1 = *temperatur permukaan 1*

t_2 = *temperatur permukaan 2*

t_{hf} = *temperatur fluida panas (hot fluid)*

t_{cf} = *temperatur fluida dingin (cold fluid)*

h_{hf} = *koefisien perpindahan panas dari fluida panas ke dinding logam*

h_{cf} = *koefisien perpindahan panas dari dinding logam ke fluida dingin*

$$Q = h_{hf} A (t_{hf} - t_1)$$

$$Q = \frac{k.A (t_1 - t_2)}{L}$$

$$Q = h_{cf} A (t_2 - t_{cf})$$

$$t_{hf} - t_1 = \frac{Q}{h_{hf} . A}$$

$$t_1 - t_2 = \frac{QL}{k . A}$$

$$t_2 - t_{cf} = \frac{Q}{k_{cf} . A}$$

$$t_{hf} - t_{cf} = Q \left[\frac{1}{h_{hf} . A} + \frac{L}{k . A} + \frac{1}{h_{cf} . A} \right]$$

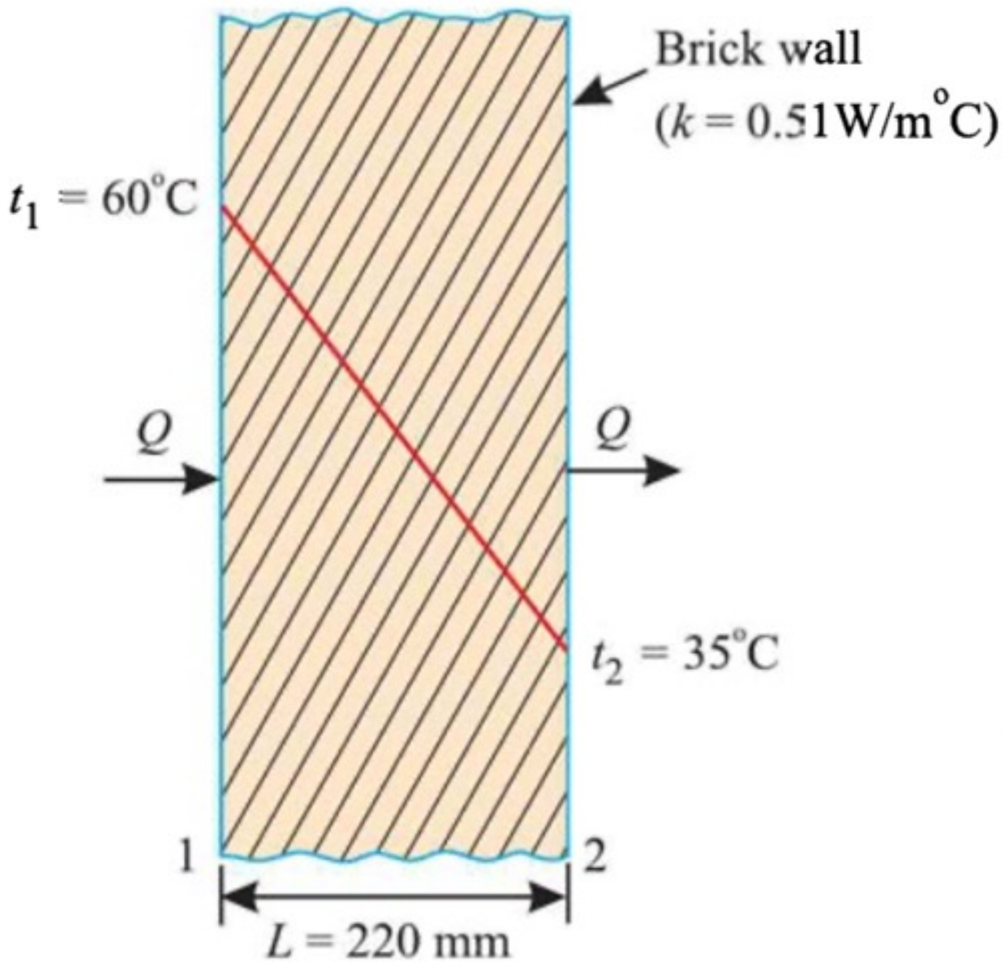
$$Q = \frac{A (t_{hf} - t_{cf})}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}}$$

$$Q = U.A (t_{hf} - t_{cf}) = \frac{A (t_{hf} - t_{cf})}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}}$$

$$U = \frac{1}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}}$$

Contoh 2.1:

Temperatur permukaan bagian dalam suatu dinding datar batu bata 60 ^oC dan permukaan luar 35 ^oC. Hitunglah laju perpindahan panas per m^2 dari luas permukaan dinding dimana tebal dinding 220 mm. Konduktivitas termal batu bata 0,51 W/m^oC.



Penyelesaian:

t_1 = 60 ^oC, t_2 = 35 ^oC, L = 220 mm = 0,22 m,

k =0,51 W/m^oC

$$q = \frac{Q}{A} = \frac{k\,(t_1 - t_2)}{L}$$

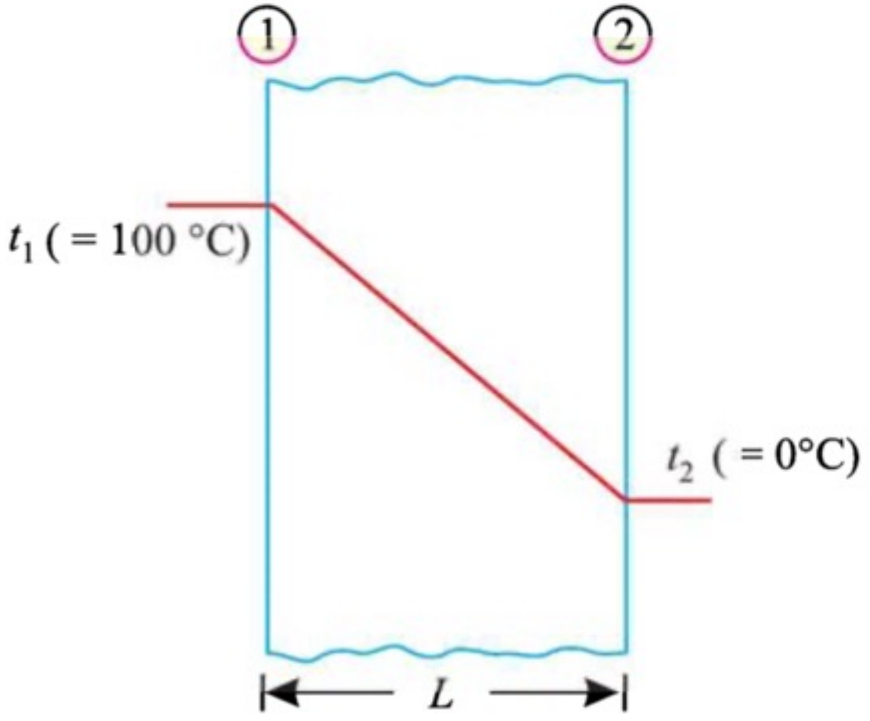
$$q = \frac{0.51 \times (60 - 35)}{0.22} = \mathbf{57.95\ W/m^2}$$

Contoh 2.2:

Tebal slab *L = 0,25 m*. Temperatur salah satu permukaan 100 °C dan permukaan yang lain 0 °C. Hitunglah *net flux* melalui slab yang terbuat dari tembaga murni dimana konduktivitas termal 387,6 W/m °C.

Penyelesaian:

L = 0,25 m; t_1 = 100 °C, t_2 = 0 °C; k = 387,6 W/m °C.



$$Q = -kA\,\frac{dt}{dx}$$

$$q = \frac{Q}{A} = -\,k\cdot\frac{(t_2 - t_1)}{L}$$

$$= -\,387.6 \times \frac{(0 - 100)}{0.25}$$

$$= \mathbf{1.55 \times 10^5\ W/m^2}$$

Contoh 2.3:

Suatu dinding reaktor, tebal 320 mm dimana terdiri dari lapisan bagian dalam batu bata tahan api / *fire brick* (k = 0,84 W/m^{o}C) dan insulasi (k = 0,16 W/m^{o}C). Reaktor beroperasi pada temperatur 1325 ^{o}C dan temperatur lingkungan 25 ^{o}C.

(i) Hitunglah tebal *fire brick* dan insulasi yang mana memberikan kerugian panas minimum.

(ii) Hitunglah kerugian panas dengan asumsi awal bahwa material insulasi mempunyai temperatur maksimum 1200 ^{o}C.

Penyelesaian:

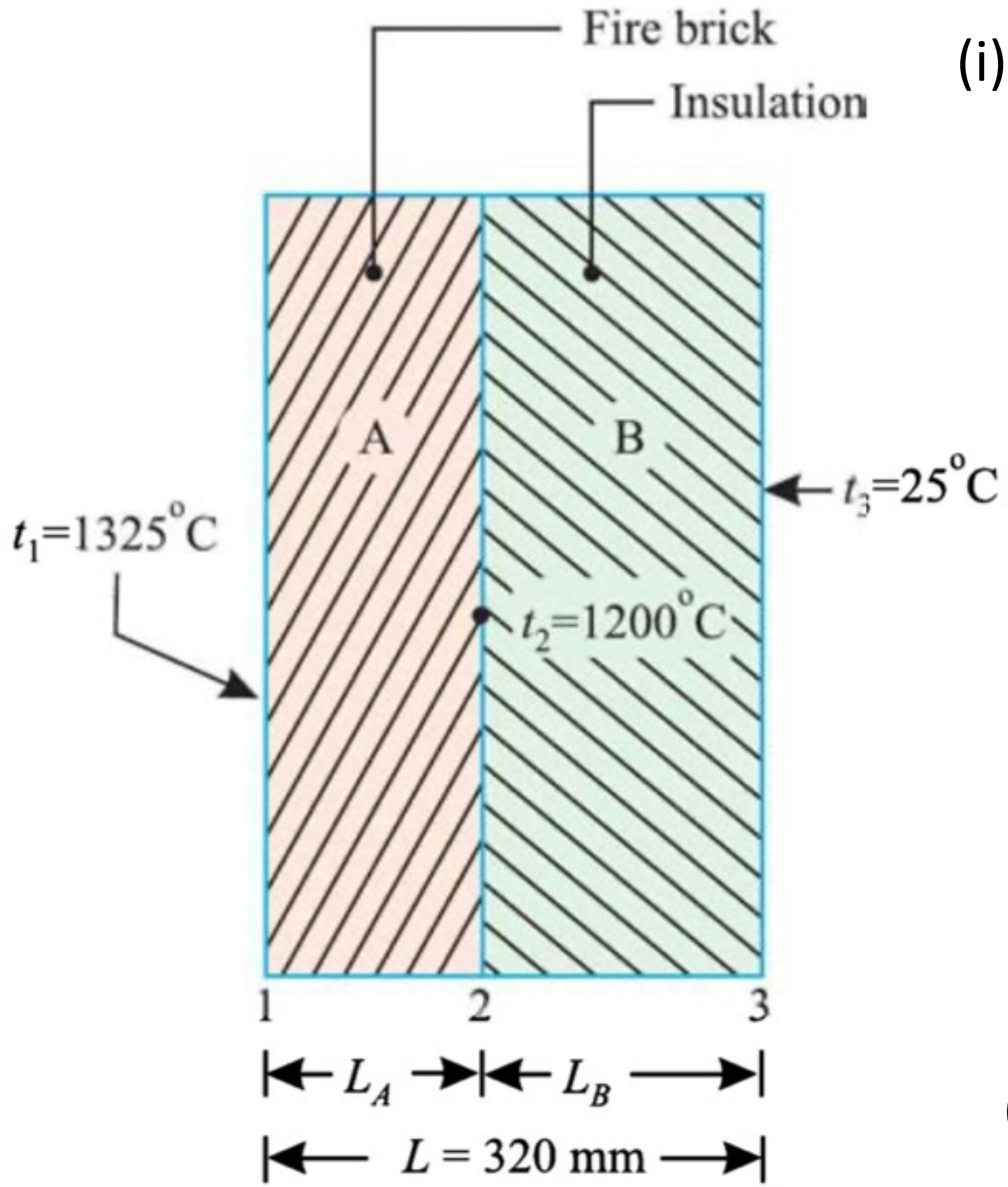
$$t_1 = 1325°C;\ t_2 = 1200°C,\ t_3 = 25°C$$

$$L_A + L_B = L = 320 \text{ mm or } 0.32 \text{ m}$$

$$L_B = (0.32 - L_A);$$

$$k_A = 0.84 \text{ W/m°C};$$

$$k_B = 0.16 \text{ W/m°C}.$$



(i)
$$q = \frac{t_1 - t_3}{L_A / k_A + L_B / k_B} = \frac{t_1 - t_2}{L_A / k_A} = \frac{t_2 - t_3}{L_B / k_B}$$

$$\frac{(1325 - 25)}{L_A / 0.84 + L_B / 0.16} = \frac{(1325 - 1200)}{L_A / 0.84}$$

$$\frac{1300}{1.190\, L_A + 6.25\,(0.32 - L_A)} = \frac{105}{L_A}$$

$$\frac{1300}{1.190\, L_A + 2 - 6.25\, L_A} = \frac{105}{L_A}$$

$$\frac{1300}{2 - 5.06\, L_A} = \frac{105}{L_A}$$

$$1300\, L_A = 105\,(2 - 5.06\, L_A)$$

$$1300\, L_A = 210 - 531.3\, L_A$$

$$L_A = \frac{210}{(1300 + 531.3)} = 0.1146 \text{ m or } \mathbf{114.6\ mm}$$

$$L_B = 320 - 114.6 = \mathbf{205.4\ mm}$$

(ii)
$$q = \frac{t_1 - t_2}{L_A / k_A} = \frac{1325 - 1200}{0.1146 / 0.84} = \mathbf{916.23\ W/m^2}$$

Contoh 2.4:

Suatu dinding *furnace* terbuat dari lapisan bagian dalam *silica brick* (k = 1,7 W/m°C) dengan ketebalan 120 mm dan dilapisi dengan *magnesite brick* (k = 5,8 W/m°C), tebal 240 mm. Temperatur permukaan bagian dalam dinding *silica brick* 725 °C dan permukaan luar dinding *magnesite brick* 110 °C. Tahanan kontak termal pada *interface* 0,0035 °C/W per satuan luas. *Hitunglah:*

(i) Laju perpindahan panas per satuan luas dinding,

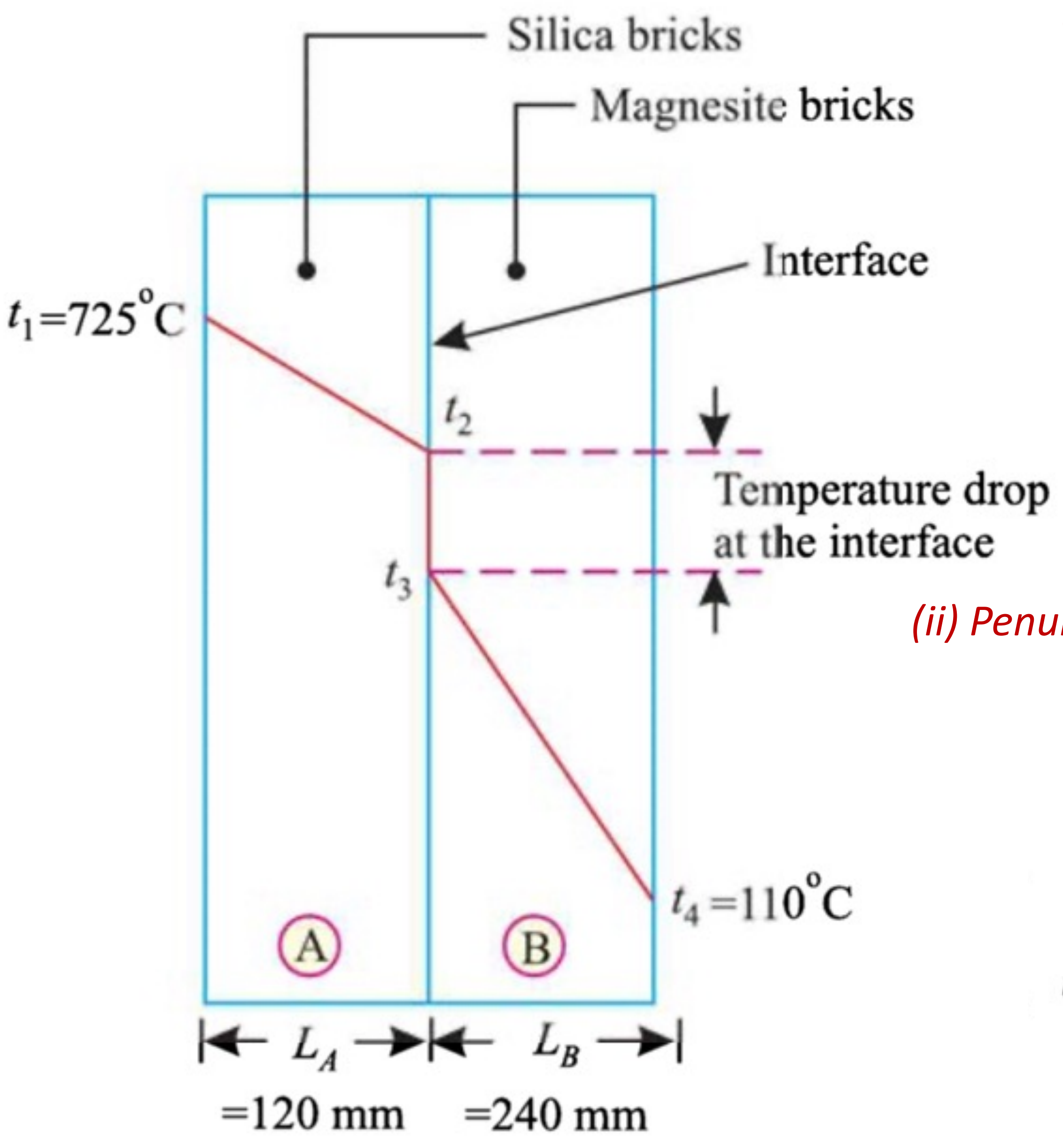
(ii) Penurunan temperatur (temperature drop) pada interface.

Penyelesaian:

L_A = 120 mm = 0,12 m; L_B = 240 mm = 0,24 m; k_A = 1,7 W/m°C,

k_B = 5,8 W/m°C; Tahanan kontak termal, $(R_{th})_{kont}$ = 0,0035 °C/W

t_1 = 725 °C; t_4 = 110 °C



(i) Laju perpindahan panas per satuan luas dinding, q:

$$q = \frac{\Delta t}{\Sigma R_{th}} = \frac{\Delta t}{R_{th-A} + (R_{th})_{cont.} + R_{th-B}}$$

$$= \frac{(t_1 - t_4)}{L_A / k_A + 0.0035 + L_B / k_B}$$

$$= \frac{(725 - 110)}{0.12/1.7 + 0.0035 + 0.24/5.8}$$

$$= \frac{615}{0.0706 + 0.0035 + 0.0414}$$

$$= 5324.67 \text{ W/m}^2$$

(ii) Penurunan temperatur (temperature drop) pada interface ($t_2 - t_3$):

$$q = \frac{t_1 - t_2}{L_A / k_A} = \frac{t_3 - t_4}{L_B / k_B}$$

$$5324.67 = \frac{(725 - t_2)}{0.12/1.7}$$

$$t_2 = 725 - 5324.67 \times \frac{0.12}{1.7} = 349.14°C$$

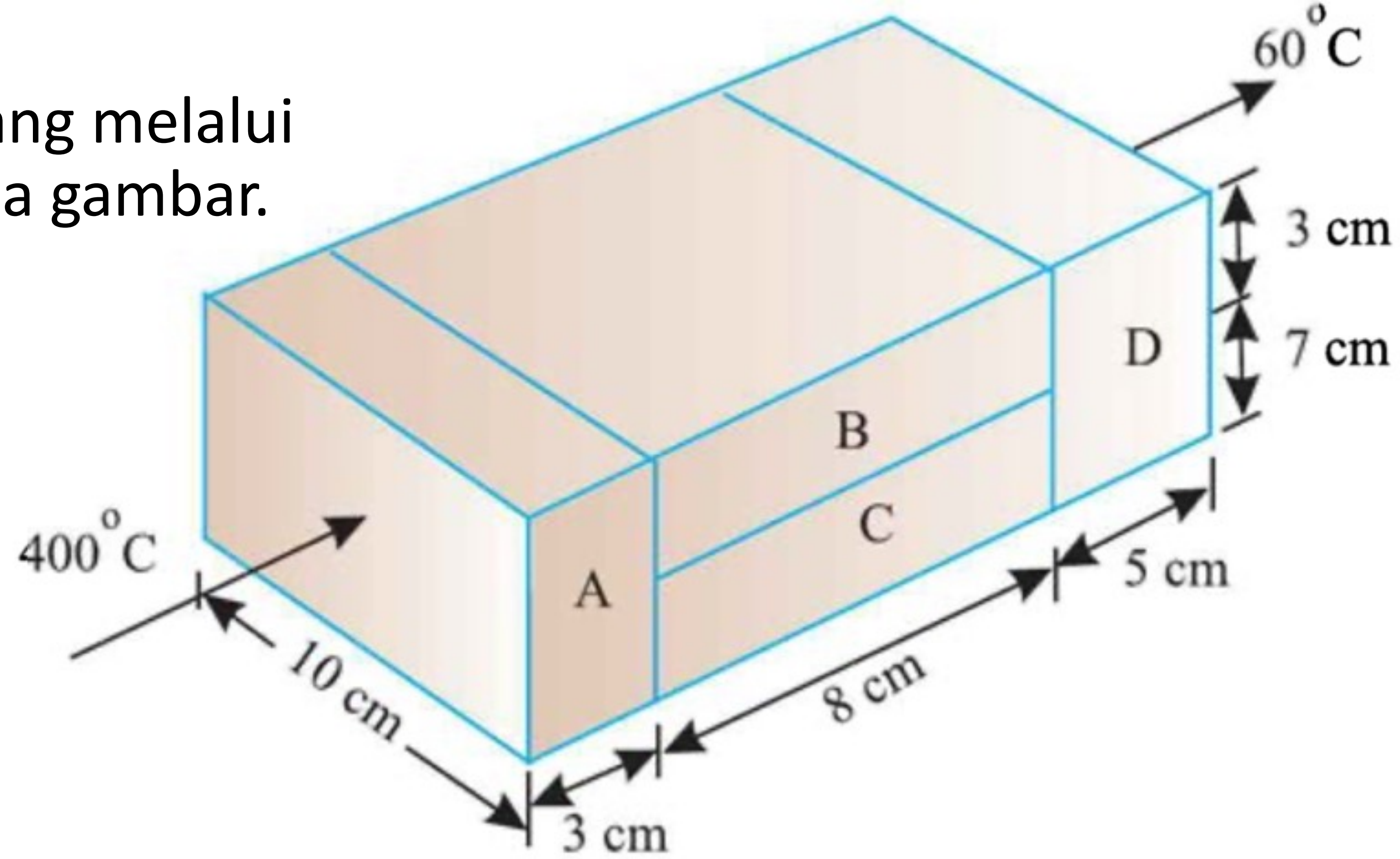
$$5324.67 = \frac{(t_3 - 110)}{0.24/5.8}$$

$$t_3 = 110 + 5324.67 \times \frac{0.24}{5.8} = 330.33°C$$

Maka :
$t_2 - t_3$ = 349,14 – 330,33
= 18,81 °C

Contoh 2.5:

Hitunglah laju aliran panas yang melalui dinding komposit seperti pada gambar. Asumsi aliran satu dimensi.



$k_A = 150$ W/m°C,

$k_B = 30$ W/m°C,

$k_C = 65$ W/m°C,

$k_D = 50$ W/m°C

Penyelesaian:

Thickness :

$L_A = 3$ cm $= 0.03$ m; $L_B = L_C = 8$ cm $= 0.08$ m; $L_D = 5$ cm $= 0.05$ m

Areas :

$$A_A = 0.1 \times 0.1 = 0.01 \text{m}^2 ; \qquad A_B = 0.1 \times 0.03 = 0.003 \text{m}^2$$

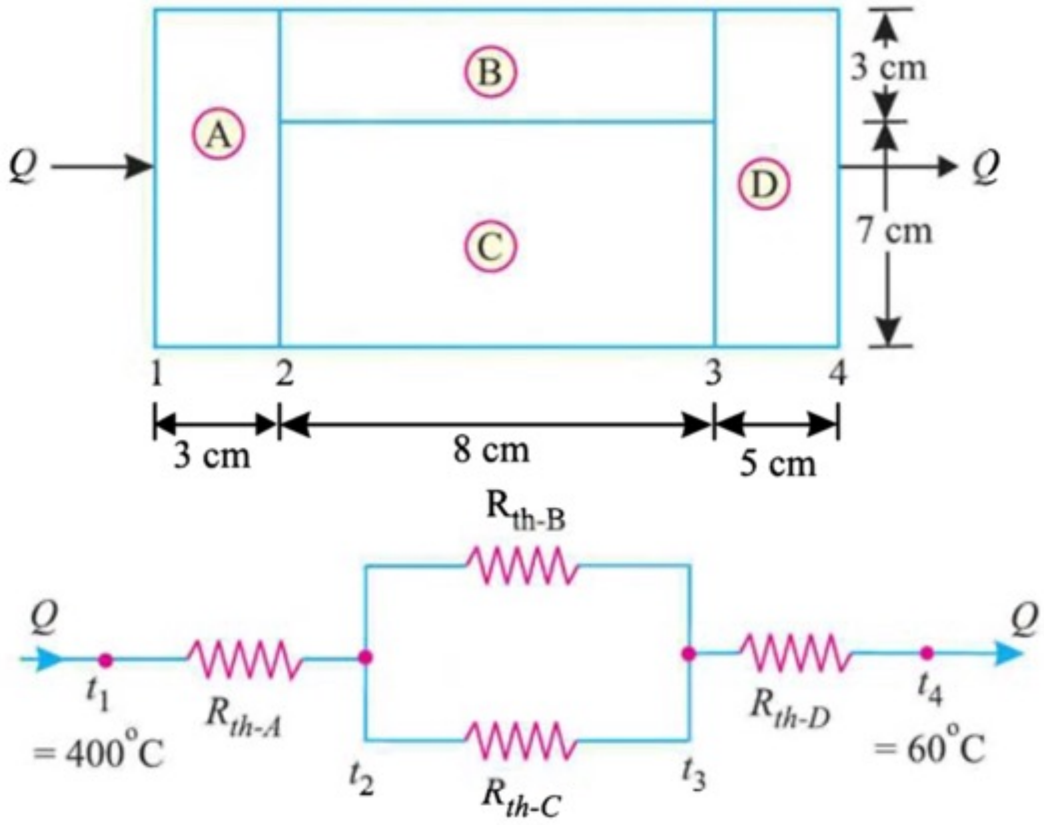
$$A_C = 0.1 \times 0.07 = 0.007 \text{m}^2 ; \quad A_D = 0.1 \times 0.1 = 0.01 \text{m}^2$$

Heat flow rate, Q :

The thermal resistances are given by,

$$R_{th-A} = \frac{L_A}{k_A\,A_A} = \frac{0.03}{150 \times 0.01} = 0.02$$

$$R_{th-B} = \frac{L_B}{k_B\,A_B} = \frac{0.08}{30 \times 0.003} = 0.89$$



$$R_{th-C} = \frac{L_C}{k_C\,A_C} = \frac{0.08}{65 \times 0.007} = 0.176$$

$$R_{th-D} = \frac{L_D}{k_D\,A_D} = \frac{0.05}{50 \times 0.01} = 0.1$$

$$\frac{1}{(R_{th})_{eq}} = \frac{1}{R_{th-B}} + \frac{1}{R_{th-C}} = \frac{1}{0.89} + \frac{1}{0.176} = 6.805$$

$$(R_{th})_{eq.} = \frac{1}{6.805} = 0.147$$

$$(R_{th})_{\text{total}} = R_{th-A} + (R_{th})_{\text{eq.}} + R_{th-D} = 0.02 + 0.147 + 0.1 = 0.267$$

$$Q = \frac{(\Delta t)_{overall}}{(R_{th})_{total}} = \frac{(400 - 60)}{0.267} = \mathbf{1273.4\ W}$$

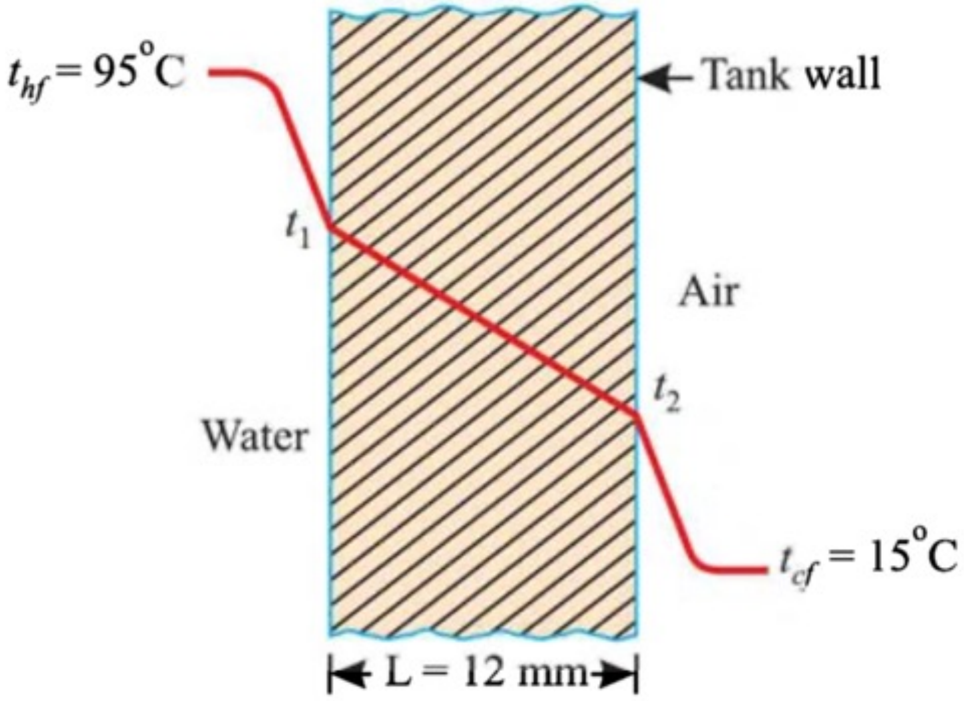
Contoh 2.6:

Suatu dinding tanki baja lunak *(mild steel)* mempunyai tebal 12 mm dimana di dalam tanki terdapat air yang bertemperatur 95 °C. Konduktivitas termal *mild steel* 50 W/m°C, dan koefisien perpindahan panas dalam tanki 2850 $W/m^2{}^oC$ dan di luar tanki 10 $W/m^2{}^oC$. Temperatur atmosfir 15 °C.

Hitunglah:

(i) Laju aliran panas per m^2 dari luas permukaan tanki.

(ii) Temperatur permukaan luar tanki.



Penyelesaian:

L = 12 mm = 0,012 m; t_{hf} = 95 °C, t_{cf} = 15 °C; k =50 W/m°C; h_{hf} = 2850 $W/m^2{}^oC$; h_{cf} = 10 $W/m^2{}^oC$

(*i*) Rate of heat loss per m² of the tank surface area, *q*:

Rate of heat loss per m^2 of tank surface,

$$q = UA\,(t_{hf} - t_{cf})$$

The overall heat transfer coefficient, U is found from the relation,

$$\frac{1}{U} = \frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}} = \frac{1}{2850} + \frac{0.012}{50} + \frac{1}{10}$$

$$= 0.0003508 + 0.00024 + 0.1 = 0.1006$$

$$\therefore \quad U = \frac{1}{0.1006} = 9.94 \text{ W/m}^2\,°\text{C}$$

$$\therefore \quad q = 9.94 \times 1 \times (95 - 15) = \mathbf{795.2\ W/m^2\ (Ans.)}$$

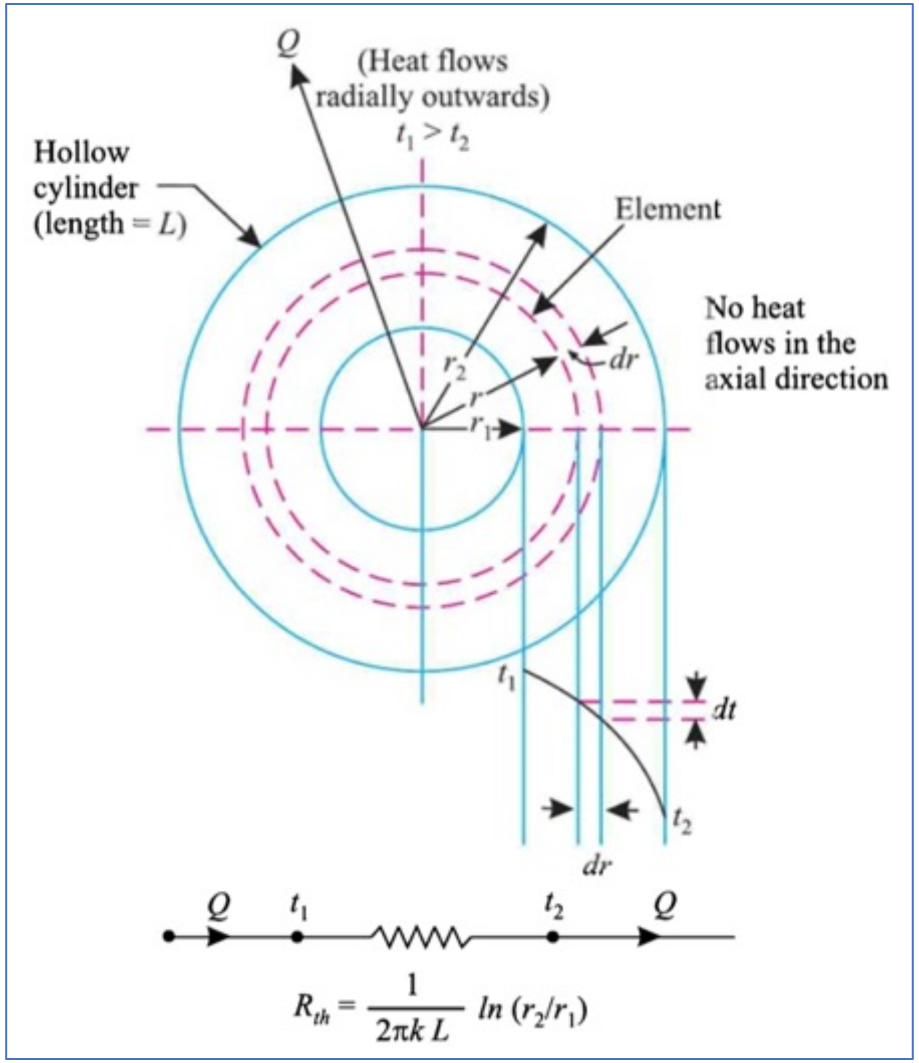
(*ii*) Temperature of the outside surface of the tank, t_2:

We know that, $$q = h_{cf} \times 1 \times (t_2 - t_{cf})$$

or, $$795.2 = 10\,(t_2 - 15)$$

or, $$t_2 = \frac{795.2}{10} + 15 = \mathbf{94.52°C}$$

PERPINDAHAN PANAS KONDUKSI MELALUI PIPA (SILINDER) HOLLOW DAN KOMPOSIT



r_1, r_2 = radius dalam dan luar

t_1, t_2 = temperatur permukaan dalam dan luar.

k = koefisien konduktivitas termal

$$Q = -kA \frac{dt}{dr}$$

$$= -kA \frac{d}{dr}\left[t_1 + \frac{t_1 - t_2}{\ln(r_2/r_1)} \ln(r_1) - \frac{(t_1 - t_2)}{\ln(r_2/r_1)} \ln(r)\right]$$

$$= -k(2\pi r . L)\left[\frac{-(t_1 - t_2)}{r \ln(r_2/r_1)}\right]$$

$$= 2\pi k L \frac{(t_1 - t_2)}{\ln(r_2/r_1)} = \frac{(t_1 - t_2)}{\frac{\ln(r_2/r_1)}{2\pi k L}} \left[= \frac{\Delta t}{R_{th}}\right] \quad \left(\text{where, } R_{th} = \frac{\ln(r_2/r_1)}{2\pi k L}\right)$$

$$Q = \frac{(t_1 - t_2)}{\frac{\ln(r_2/r_1)}{2\pi k L}}$$

Metode alternatif:

$$Q = -kA \cdot \left(\frac{dt}{dr} \right)$$

$$= -k.2\pi r.L \frac{dt}{dr} \text{ per unit time}$$

$$Q.\frac{dr}{r} = -k.2\pi L.dt$$

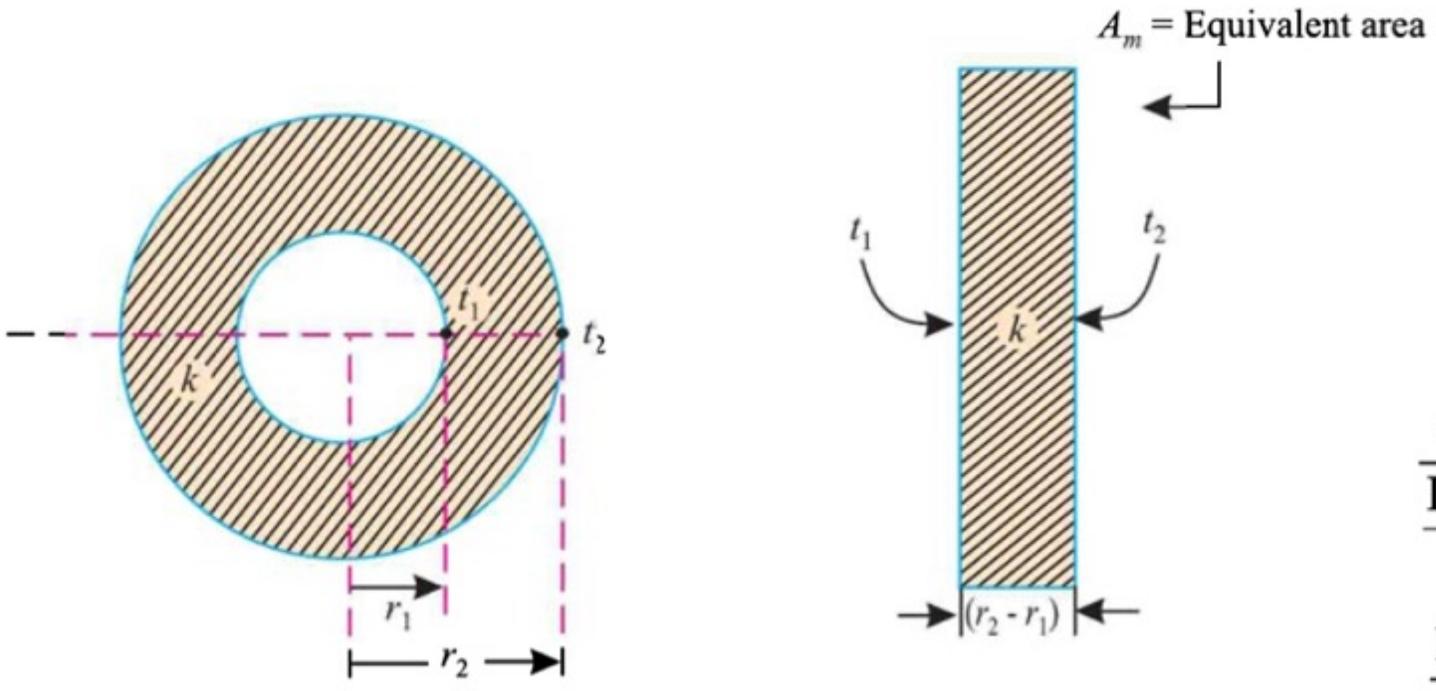
$$Q \int_{r_1}^{r_2} \frac{dr}{r} = -k.2\pi L \int_{t_1}^{t_2} dt$$

$$Q[\text{In}(r)]_{r_1}^{r_2} = -k.2\pi L[t]_{t_1}^{t_2}$$

$$Q.\ln(r_2/r_1) = -k.2\pi L(t_2 - t_1) = k.2\pi L(t_1 - t_2)$$

$$Q = \frac{k.2\pi L(t_1 - t_2)}{\ln(r_2/r_1)} = \frac{(t_1 - t_2)}{\left[\frac{\ln(r_2/r_1)}{2\pi k L} \right]}$$

LUAS RATA-RATA LOGARITMIK UNTUK HOLLOW SILINDER



(*a*) Hollow cylinder (*b*) Plane wall

$$Q = \frac{(t_1 - t_2)}{\dfrac{\ln(r_2/r_1)}{2\pi k L}}$$

$$Q = \frac{(t_1 - t_2)}{\dfrac{(r_2 - r_1)}{k A_m}}$$

$$\frac{(t_1 - t_2)}{\dfrac{\ln(r_2/r_1)}{2\pi k L}} = \frac{(t_1 - t_2)}{\dfrac{(r_2 - r_1)}{k A_m}}$$

$$\frac{\ln(r_2/r_1)}{2\pi k L} = \frac{(r_2 - r_1)}{k A_m}$$

$$A_m = \frac{2\pi L(r_2 - r_1)}{\ln(r_2/r_1)} = \frac{2\pi L r_2 - 2\pi L r_1}{\ln(2\pi L r_2 / 2\pi L r_1)}$$

$$A_m = \frac{A_0 - A_i}{\ln(A_0 - A_i)}$$

A_i, A_0 adalah luas permukaan inside dan outside

$\frac{A_0}{A_i} < 2,$ then we can take,

$A_{av.} = \frac{A_i + A_0}{2}$ which is within 4% of A_m (where, $A_{av.}$ = Average area)

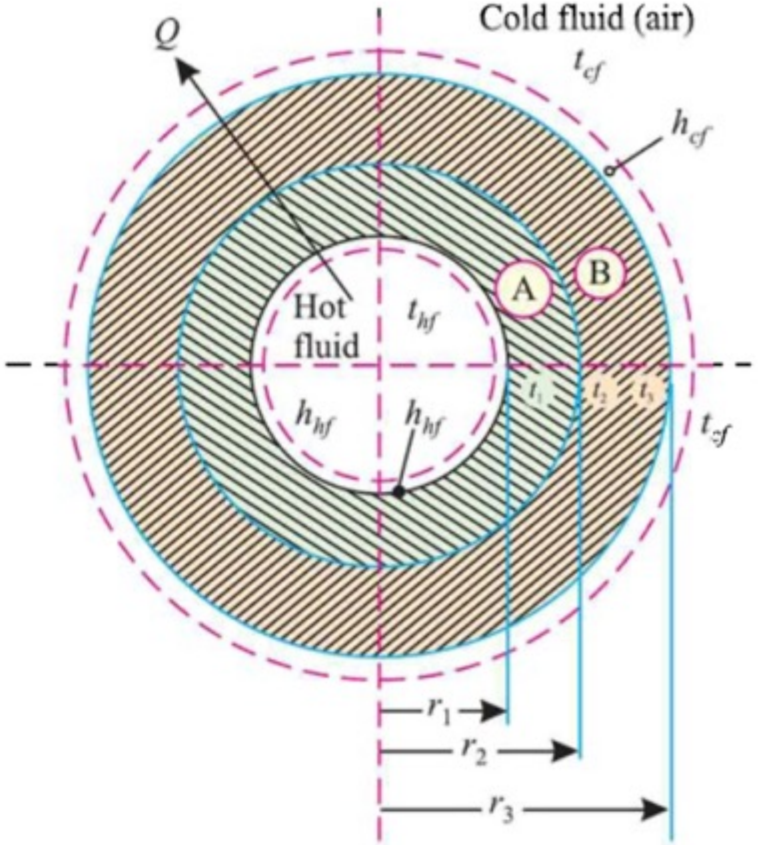
$$A_m = 2\pi r_m L = \frac{2\pi L (r_2 - r_1)}{\ln (r_2 / r_1)}$$

$$r_m = \frac{(r_2 - r_1)}{\ln (r_2 / r_1)}$$

PERPINDAHAN PANAS KONDUKSI PADA PIPA BERLAPIS (PIPA KOMPOSIT)

$$Q = h_{hf} \,.\, 2\pi r_1 \,.\, L(t_{hf} - t_1) = \frac{k_A \,.\, 2\pi L(t_1 - t_2)}{\ln(r_2 / r_1)}$$

$$= \frac{k_B \,.\, 2\pi L(t_2 - t_3)}{\ln(r_3 / r_2)} = h_{cf} \,.\, 2\pi r_3 \,.\, L(t_3 - t_{cf})$$



t_{hf} =temperatur fluida panas yang mengalir di dalam pipa.
t_{cf} = temperatur fluida dingin (temperature atmosfir)
k_A = konduktivitas termal lapisan dalam A.
k_B = konduktivitas termal lapisan luar B.
t_1, t_2, t_3 = temperatur titik 1, 2 dan 3
L = panjang pipa komposit
h_{hf}, h_{cf} = koefisien perpindahan panas di dalam dan di luar pipa

$$t_{hf} - t_1 = \frac{Q}{h_{hf} \,.\, r_1 \,.\, 2\pi L}$$

$$t_1 - t_2 = \frac{Q}{\dfrac{k_A \,.\, 2\pi L}{\ln(r_2 / r_1)}}$$

$$t_2 - t_3 = \frac{Q}{\dfrac{k_B \,.\, 2\pi L}{\ln(r_3 / r_2)}}$$

$$t_3 - t_{cf} = \frac{Q}{h_{cf} \,.\, r_3 \,.\, 2\pi L}$$

$$\frac{Q}{2\pi L}\left[\frac{1}{h_{hf}\,.\,r_1}+\frac{1}{\dfrac{k_A}{\ln(r_2/r_1)}}+\frac{1}{\dfrac{k_B}{\ln(r_3/r_2)}}+\frac{1}{h_{cf}\,.\,r_3}\right]=t_{hf}-t_{cf}$$

$$Q=\frac{2\pi L(t_{hf}-t_{cf})}{\left[\dfrac{1}{h_{hf}\,.\,r_1}+\dfrac{1}{\dfrac{k_A}{\ln(r_2/r_1)}}+\dfrac{1}{\dfrac{k_B}{\ln(r_3/r_2)}}+\dfrac{1}{h_{cf}\,.\,r_3}\right]}$$

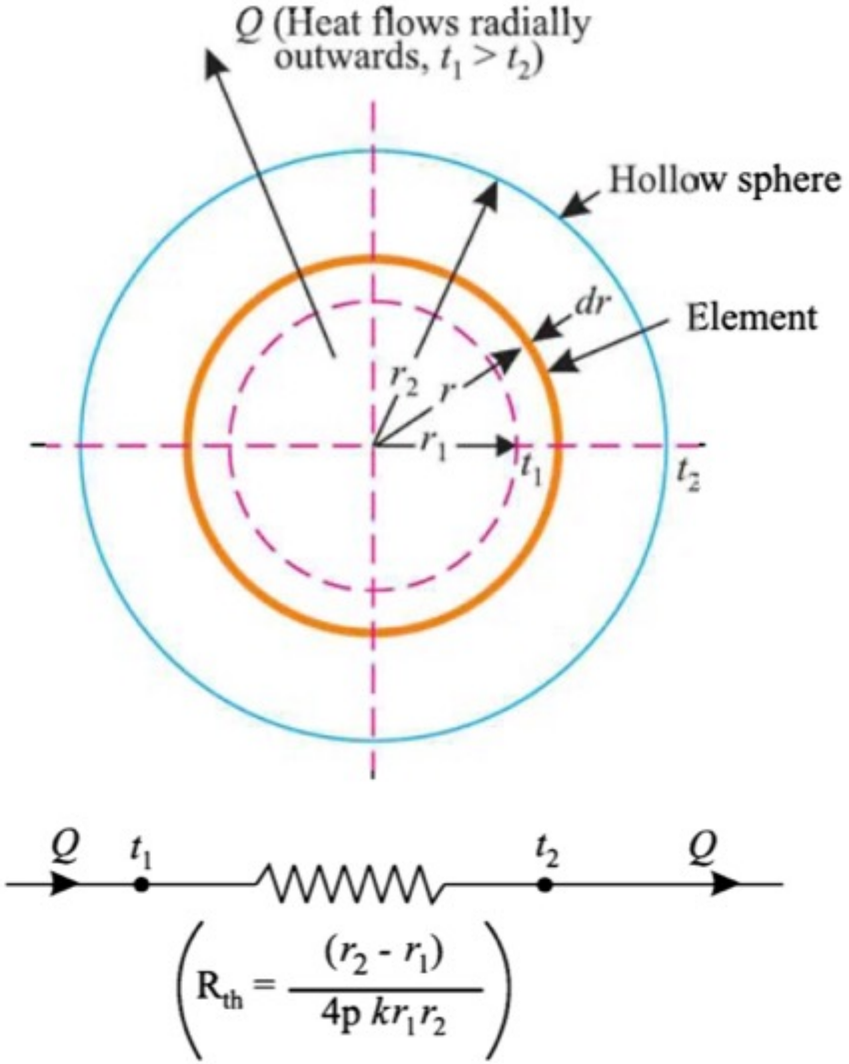
$$Q=\frac{2\pi L(t_{hf}-t_{cf})}{\left[\dfrac{1}{h_{hf}\,.\,r_1}+\dfrac{\ln(r_2/r_1)}{k_A}+\dfrac{\ln(r_3/r_2)}{k_B}+\dfrac{1}{h_{cf}\,.\,r_3}\right]}$$

$$Q=\frac{2\pi L(t_{hf}-t_{cf})}{\left[\dfrac{1}{h_{hf}\,.\,r_1}+\sum_{n=1}^{n=n}\dfrac{1}{k_n}\ln\left\{r_{(n+1)}/r_n\right\}+\dfrac{1}{h_{cf}\,.\,r_{(n+1)}}\right]}$$

$$Q=\frac{2\pi L\left[t_1-t_{(n+1)}\right]}{\sum_{n=1}^{n=n}\dfrac{1}{k_n}\ln\left[r_{(n+1)}/r_n\right]}$$

PERPINDAHAN PANAS KONDUKSI MELALUI BOLA BERONGGA DAN KOMPOSIT

PERPINDAHAN PANAS KONDUKSI MELALUI BOLA BERONGGA



$$Q = -kA\frac{dt}{dr}$$

$$= -k.4\pi r^2 . \frac{d}{dr}\left[t_1 + \frac{(t_1 - t_2)}{(1/r_2 - 1/r_1)}\left(\frac{1}{r_1} - \frac{1}{r}\right)\right]$$

$$= -k.4\pi r^2 . \frac{t_1 - t_2}{(1/r_2 - 1/r_1)} \times -\left(-\frac{1}{r^2}\right)$$

$$= -k.4\pi r^2 . \frac{(t_1 - t_2)}{\left(\frac{r_1 - r_2}{r_1 . r_2}\right)} \times \frac{1}{r^2}$$

$$= -4\pi k\frac{(t_1 - t_2) r_1 r_2}{(r_1 - r_2)} = \frac{4\pi k (t_1 - t_2) r_1 r_2}{(r_2 - r_1)} = \frac{(t_1 - t_2)}{(r_2 - r_1)/4\pi k r_1 r_2}$$

$$Q = \frac{(t_1 - t_2)}{\left[\frac{(r_2 - r_1)}{4\pi k r_1 r_2}\right]}\left[= \frac{\Delta t}{R_{th}}\right]$$

PERPINDAHAN KONDUKSI MELALUI BOLA KOMPOSIT

$$Q = h_{hf} .4\pi r_1^2 (t_{hf} - t_1) = \frac{4\pi k_A r_1 r_2 (t_1 - t_2)}{(r_2 - r_1)} = \frac{4\pi k_B r_2 r_3 (t_2 - t_3)}{(r_3 - r_2)}$$

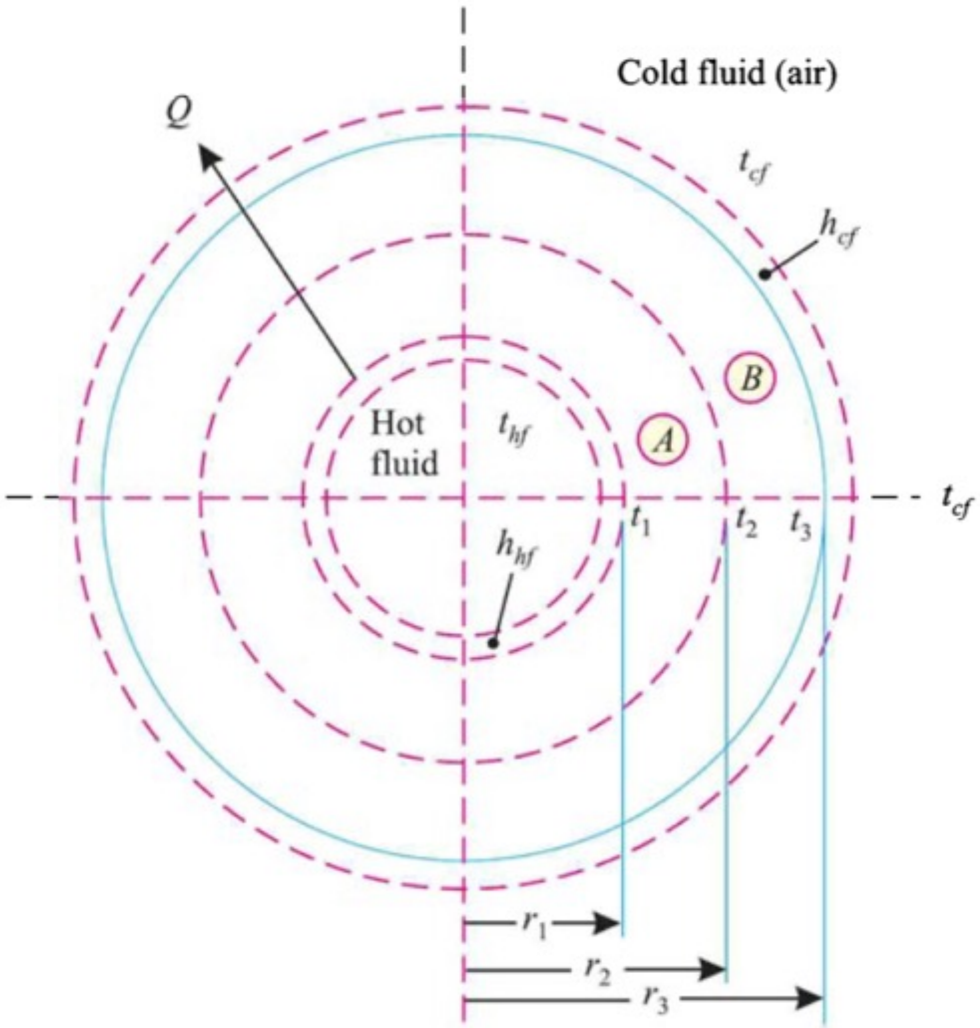
$$= h_{cf} .4\pi r_3^2 (t_3 - t_{cf})$$

$$t_{hf} - t_1 = \frac{Q}{h_{hf} .4\pi r_1^2}$$

$$t_1 - t_2 = \frac{Q (r_2 - r_1)}{4\pi k_A . r_1 r_2}$$

$$t_2 - t_3 = \frac{Q (r_3 - r_2)}{4\pi k_B . r_2 r_3}$$

$$t_3 - t_{cf} = \frac{Q}{h_{cf} .4\pi r_3^2}$$



$$\frac{Q}{4\pi}\left[\frac{1}{h_{hf}\cdot r_1^{\,2}}+\frac{(r_2-r_1)}{k_A\cdot r_1\,r_2}+\frac{(r_3-r_2)}{k_B\cdot r_2\,r_3}+\frac{1}{h_{cf}\cdot r_3^{\,2}}\right]=t_{hf}-t_{cf}$$

$$Q=\frac{4\pi(t_{hf}-t_{cf})}{\left[\frac{1}{h_{hf}\cdot r_1^2}+\frac{(r_2-r_1)}{k_A\cdot r_1\,r_2}+\frac{(r_3-r_2)}{k_B\cdot r_2\,r_3}+\frac{1}{h_{cf}\cdot r_3^{\,2}}\right]}$$

$$Q=\frac{4\pi(t_{hf}-t_{cf})}{\left[\frac{1}{h_{hf}\cdot r_1^2}+\sum_{n=1}^{n=n}\left\{\frac{r_{(n+1)}-r_n}{k_n\cdot r_n\cdot r_{(n+1)}}\right\}+\frac{1}{h_{cf}\cdot r_{(n+1)}^2}\right]}$$

Persamaan Umum:

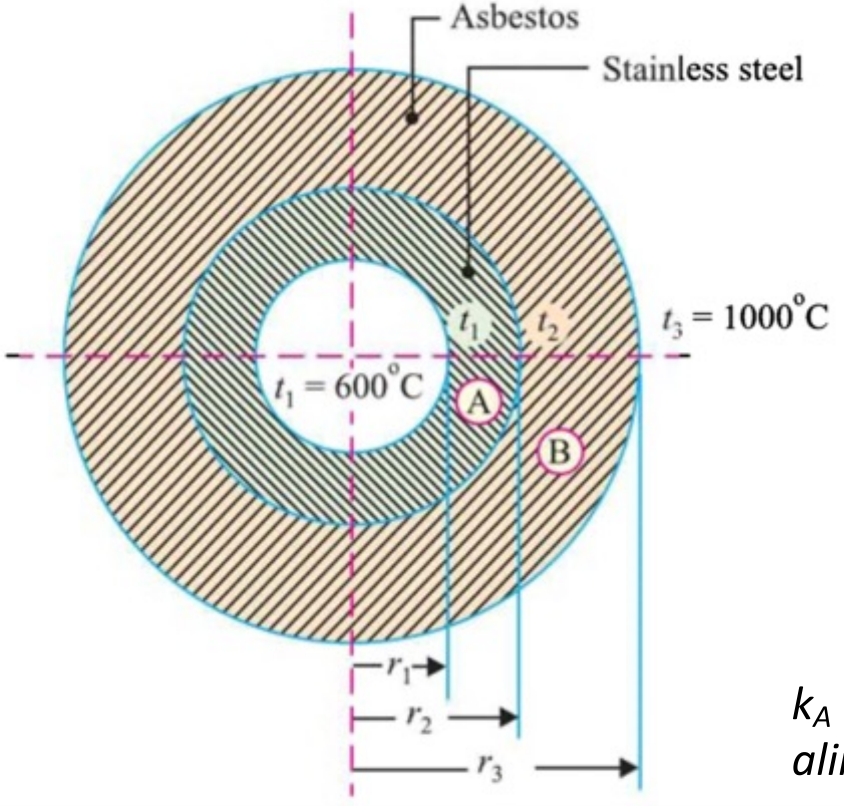
$$Q=\frac{4\pi(t_1-t_{n+1})}{\sum_{n=1}^{n=n}\left[\frac{r_{(n+1)}-r_n}{k_n\cdot r_n\cdot r_{(n+1)}}\right]}$$

CONTOH SOAL

1. Diameter dalam pipa stainless steel 20 mm dan diameter luar 40 mm dibungkus dengan insulasi asbes (k = 0,2 W/m°C) dengan ketebalan 30 mm. Jika temperatur dinding bagian dalam pipa 600 °C dan insulasi bagian luar 1000 °C, hitunglah kerugian panas *(heat loss)* per meter panjang pipa.

Penyelesaian:

$$r_1 = \frac{20}{2} = 10\,\text{mm}$$
$$= 0.01\text{ m}$$
$$r_2 = \frac{40}{2} = 20\,\text{mm}$$
$$= 0.02\,\text{m}$$
$$r_3 = 20 + 30 = 50\,\text{mm}$$
$$= 0.05\text{m}$$
$$t_1 = 600^\circ\text{C}$$
$$t_3 = 1000^\circ\text{C}$$
$$k_B = 0.2\,\text{W/m}^\circ\text{C}$$



Heat transfer per metre of a length, Q/L:

$$Q = \frac{2\pi L (t_1 - t_3)}{\dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{\ln(r_3/r_2)}{k_B}}$$

$$\frac{Q}{L} = \frac{2\pi(t_1 - t_3)}{\dfrac{\ln(r_3/r_2)}{k_B}} = \frac{2\pi(600 - 1000)}{\dfrac{\ln(0.05/0.02)}{0.2}}$$

$$= \mathbf{-548.57\ W/m\ \ (Ans.)}$$

k_A tidak diketahui jadi diabaikan. Hasil (-) menunjukkan aliran panas dari luar ke dalam pipa.

2. Suatu pipa baja dengan diameter luar 50 mm di bungkus dengan insulasi asbes (k = 0,166 W/mK) yang tebalnya 6,4 mm diikuti dengan insulasi fiber-glass (k = 0,0485 W/mK) yang tebalnya 25 mm. Temperatur dinding pipa 393 K dan temperatur luar insulasi 311 K. Hitunglah temperatur interface antara asbes dan fiber-glass.

Solution.

Given : $r_1 = \frac{50}{2} = 25\,\text{mm} = 0.025\,\text{m};$

$$r_2 = r_1 + 6.4 = 25 + 6.4 = 31.4 \text{ mm or } 0.0314 \text{ m};$$

$$r_3 = r_2 + 25 = 31.4 + 25 = 56.4 \text{ mm} = 0.0564 \text{ m};$$

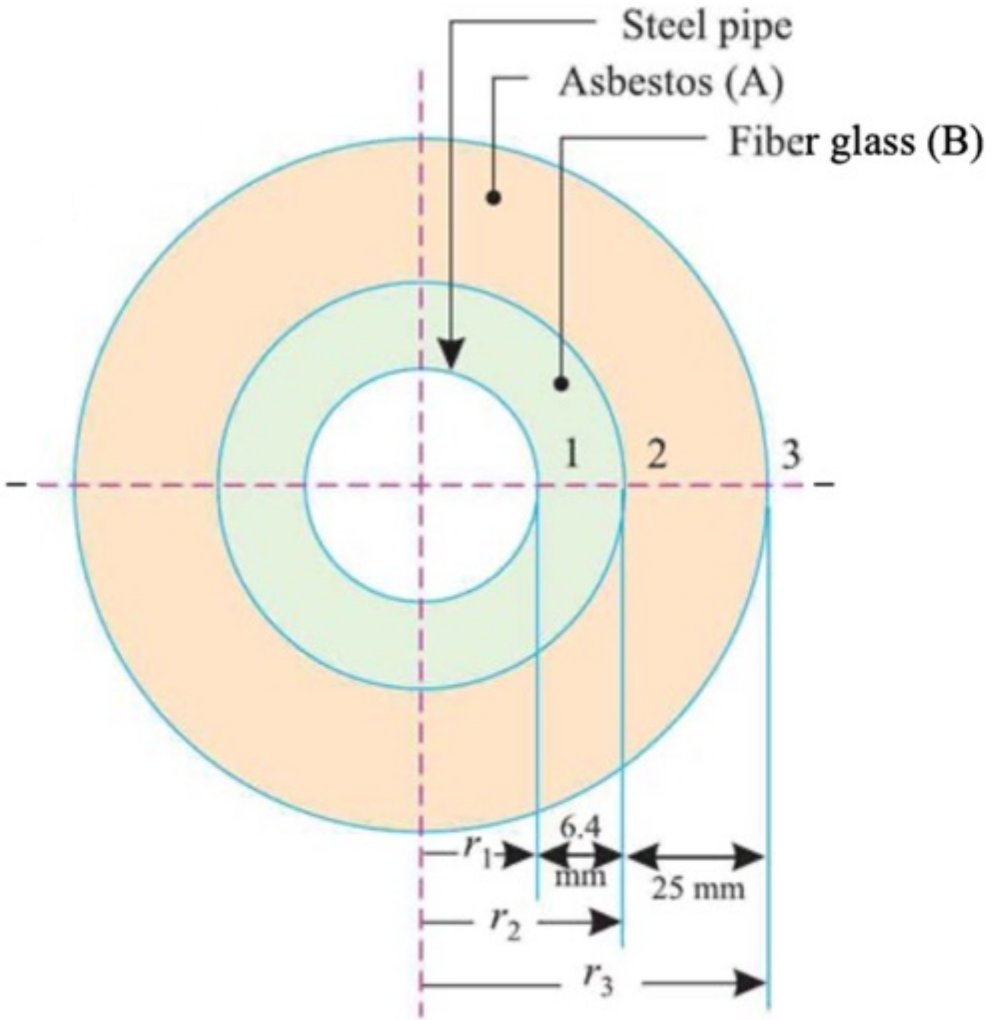
$T_1 = 393$ K; $T_3 = 311$ K

$k_A = 0.166$ W/mK;

$k_B = 0.0485$ W/mK.

Interface temperature between the asbestos and fiber-glass, t_2 :

We know that,
$$Q = \frac{2\pi L(T_1 - T_3)}{\frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B}}$$



$$\frac{Q}{L} = \frac{2\pi(T_1 - T_3)}{\dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{\ln(r_3/r_2)}{k_B}}$$

$$= \frac{2\pi(393 - 311)}{\dfrac{\ln(0.0314/0.025)}{0.166} + \dfrac{\ln(0.0564/0.0314)}{0.0485}}$$

$$= \frac{515.22}{1.373 + 12.075} = 38.31\,\text{W/m}$$

Also, $$\frac{Q}{L} = \frac{2\pi(T_1 - T_2)}{\dfrac{\ln(r_2/r_1)}{k_A}}$$

or, $$38.31 = \frac{2\pi(393 - T_2)}{\left[\dfrac{\ln(0.0314/0.025)}{0.166}\right]}$$

$$38.31 = \frac{2\pi(393 - T_2)}{1.373}$$

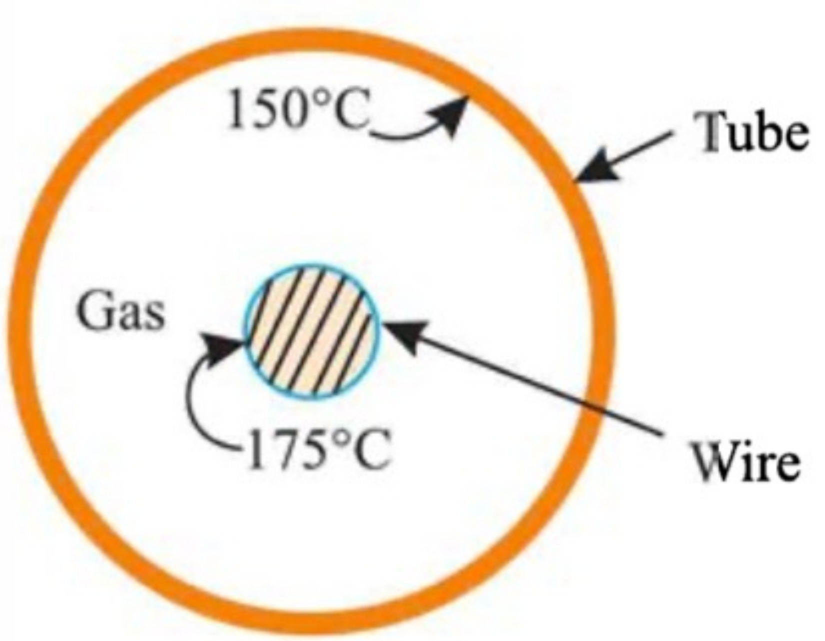
$\therefore$ $$T_2 = 393 - \frac{38.31 \times 1.373}{2\pi} = 384.6\,\text{K}$$

or, $t_2 = 384.6 - 273 =$ **111.6° C** **(Ans.)**

3. Diameter dalam suatu pipa gas 2 mm dan Panjang 25 cm. Gas dipanaskan dengan kawat listrik berdiameter 50 mikron (0,05 mm) yang ditempatkan sepanjang sumbu pipa. Arus listrik 0,5 Ampere dan voltage drop sepanjang elemen pemanas 4 Volt. Jika temperatur kawat 175 ^oC dan temperatur dinding pipa bagian dalam 150 ^oC, hitunglah konduktivitas termal pipa gas.

Solution. *Given* :

Inside radius of the tube,	r_t	= 2mm
Length of the tube,	L	= 25 cm = 0.25 m
Radius of the electric wire,	r_w	= 0.025 mm
Inside tube temperature,	t_t	= 150°C
Wire temperature,	t_w	= 175°C
Current through the element		= 0.5 A
Voltage across the element		= 4 V



Thermal conductivity of the gas, k :

Heat transferred through a cylinder,

$$Q = \frac{2\pi k L (t_w - t_t)}{\ln (r_t / r_w)}$$

$$= \frac{2\pi k \times 0.25 (175 - 150)}{\ln (1/0.025)} = 10.645 \text{ kW} \quad ...(i)$$

Also, $$Q = VI = 4 \times 0.5 = 2.0 \text{ W} \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$10.645\, k = 2.0$$

or, k = **0.188 W/m°C. (Ans.)**

4. Diameter suatu pipa uap 240 mm dan panjang 210 m dilapisi dengan high temperature insulation (k = 0,092 W/m°C) yang tebalnya 50 mm dan low temperatur insulation (k = 0,062 W/m°C). Temperatur permukaan dalam dan luar masing-masing 390 °C dan 40 °C.

   Hitunglah:

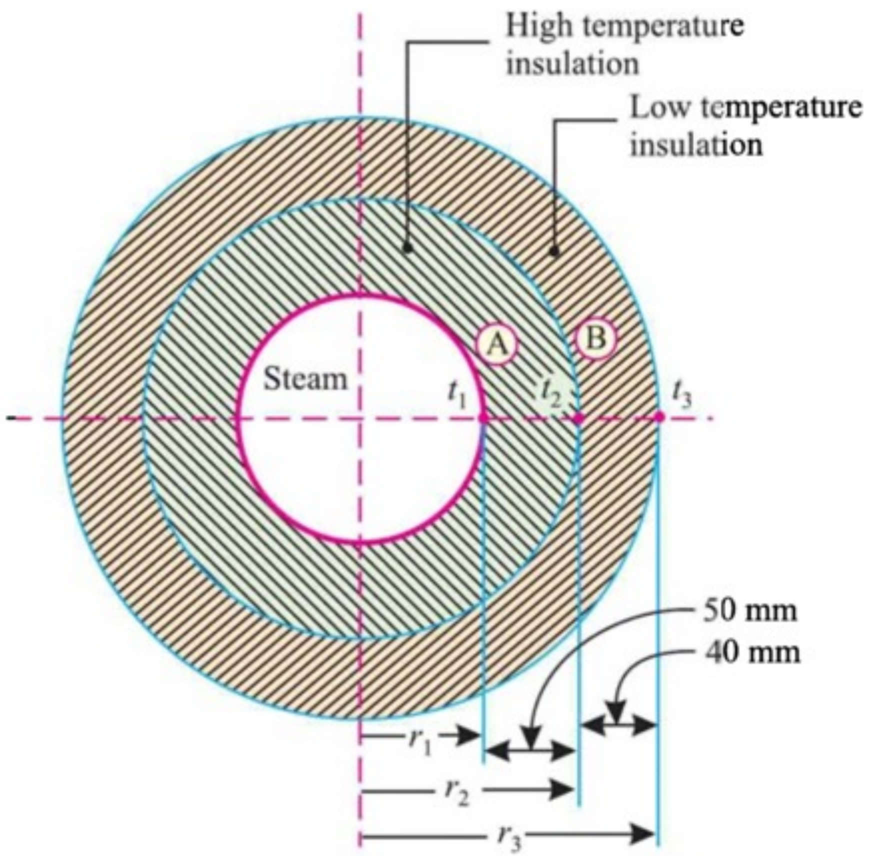
   (i) Total heat loss per jam.

   (ii) Heat loss per m^2 permukaan pipa.

   (iii) Total heat loss per m^2 permukaan luar.

   (iv) Temperatur antara dua lapisan insulasi.

   Abaikan konduksi panas melalui material pipa.



Penyelesaian:

Given :

$$r_1 = \frac{240}{2} = 120\,\text{mm} = 0.12\,\text{m}$$

$$r_2 = 120 + 50 = 170\,\text{mm} = 0.17\,\text{m}$$

$$r_3 = 120 + 50 + 40 = 210\,\text{mm} = 0.21\,\text{m}$$

$$k_A = 0.092\,\text{W/m°C};\ k_B = 0.062\,\text{W/m°C}$$

$$t_1 = 390\text{°C};\ t_3 = 40\text{°C}$$

Length of steam main, $L = 210$ m.

(*i*) **Total heat loss per hour :**

$$Q = \frac{2\pi L(t_1 - t_3)}{\left[\dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{\ln(r_3/r_2)}{k_B}\right]} \qquad \text{[Eqn. (2.71)]}$$

$$= \frac{2\pi \times 210(390 - 40)}{\left[\dfrac{\ln(0.17/0.12)}{0.092} + \dfrac{\ln(0.21/0.17)}{0.062}\right]}$$

$$= \frac{461814}{(3.786 + 3.408)} = 64194.3 \text{ W}$$

$$= \frac{64194.3 \times 3600}{1000} = 231099.5 \text{ kJ/h}$$

i.e., The total heat loss per hour = **231099.5 kJ/h** **(Ans.)**

(*ii*) **Total heat loss per m² of the pipe surface :**

Total heat loss per m^2 of the surface

$$= \frac{231099.5}{2\pi r_1 . L} = \frac{231099.5}{2\pi \times 0.12 \times 210} = \mathbf{1459.55 \text{ kJ/h}} \quad \textbf{(Ans.)}$$

(*iii*) Total heat loss per m² of the outer surface :

Total heat loss per m² of the outer surface

$$= \frac{231099.5}{2\pi r_3.L} = \frac{231099.5}{2\pi \times 0.21 \times 210} = \mathbf{834.03\ kJ/h} \quad \textbf{(Ans.)}$$

(*iv*) The temperature between two layers, t_2 :

$$Q = \frac{2\pi L(t_1 - t_2)}{\left[\dfrac{\ln(r_2/r_1)}{k_A}\right]}$$

$$64194.3 = \frac{2\pi \times 210(390 - t_2)}{\left[\dfrac{\ln(0.17/0.12)}{0.092}\right]} = 348.5(390 - t_2)$$

$$\therefore \quad t_2 = 390 - \frac{64194.3}{348.5} = \mathbf{205.8°\,C} \quad \textbf{(Ans.)}$$

No. 5. . *Hot air at a temperature of 65°C is flowing through a steel pipe of 120 mm diameter. The pipe is covered with two layers of different insulating materials of thickness 60 mm and 40 mm, and their corresponding thermal conductivities are 0.24 and 0.4 W/m°C. The inside and outside heat transfer coefficients are 60 W/m°C and 12 W/m°C respectively. The atmosphere is at 20°C. Find the rate of heat loss from 60 m length of pipe.*

Solution. Refer to Fig. 2.61.

Given :

$$r_1 = \frac{120}{2} = 60\text{ mm} = 0.06\text{ m}$$

$$r_2 = 60 + 60 = 120\text{ mm} = 0.12\text{ m}$$

$$r_3 = 60 + 60 + 40 = 160\text{ mm} = 0.16\text{ m}$$

$$k_A = 0.24\text{ W/m°C}; \qquad k_B = 0.4\text{ W/m°C}$$

$$h_{hf} = 60\text{ W/m}^2\text{°C}; \qquad h_{cf} = 12\text{ W/m}^2\text{°C}$$

$$t_{hf} = 65\text{°C}; \; t_{cf} = 20\text{°C}$$

Length of pipe, $L = 60$ m

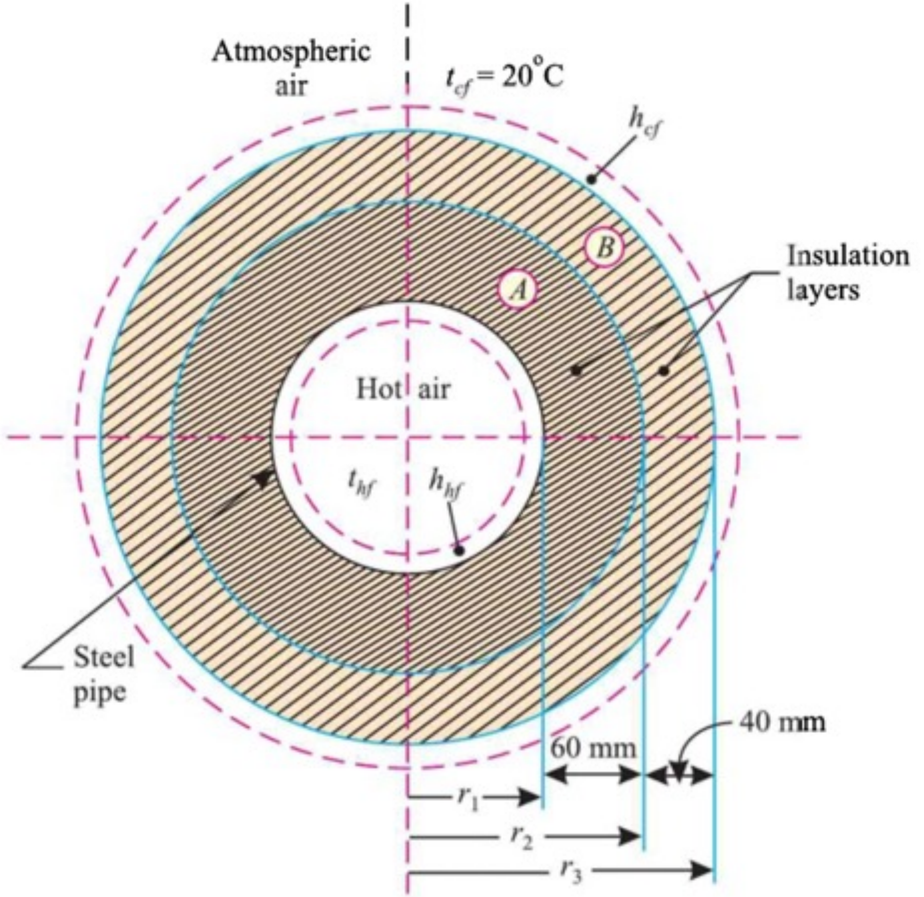


Fig. 2.61.

Rate of heat loss, Q :

Rate of heat loss is given by

$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} \cdot r_1} + \frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B} + \frac{1}{h_{cf} \cdot r_3}\right]} \qquad \text{[Eqn. (2.69)]}$$

$$= \frac{2\pi \times 60(65 - 20)}{\left[\frac{1}{60 \times 0.06} + \frac{\ln(0.12/0.06)}{0.24} + \frac{\ln(0.16/0.12)}{0.4} + \frac{1}{12 \times 0.16}\right]}$$

$$= \frac{16964.6}{0.2777 + 2.8881 + 0.7192 + 0.5208} = 3850.5 \text{ W}$$

i.e., Rate of heat loss = **3850.5 W** **(Ans.)**

No. 6. l. *A 150 mm steam pipe has inside diameter of 120 mm and outside diameter of 160 mm. It is insulated at the outside with asbestos. The steam temperature is 150°C and the air temperature is 20°C. h (steam side) = 100 W/m²°C, h (air side) = 30 W/m²°C, k (asbestos) = 0.8 W/m°C and k (steel) = 42 W/m°C. How thick should the asbestos be provided in order to limit the heat loses to 2.1 kW/m² ?* **(AMIE Winter, 2002)**

Solution. Refer to Fig. 2.65.

Given :

$$r_1 = \frac{120}{2} = 60\,\text{mm} = 0.06\,\text{m}$$

$$r_2 = \frac{160}{2} = 80\,\text{mm} = 0.08\,\text{m}$$

$k_A = 42$ W/m°C; $\quad k_B = 0.8$ W/m°C

$t_{hf} = 150$°C; $\quad t_{cf} = 20$°C

$h_{hf} = 100$ W/m²°C; $\quad h_{cf} = 30$ W/m²°C

Heat loss = 2.1 kW/m²

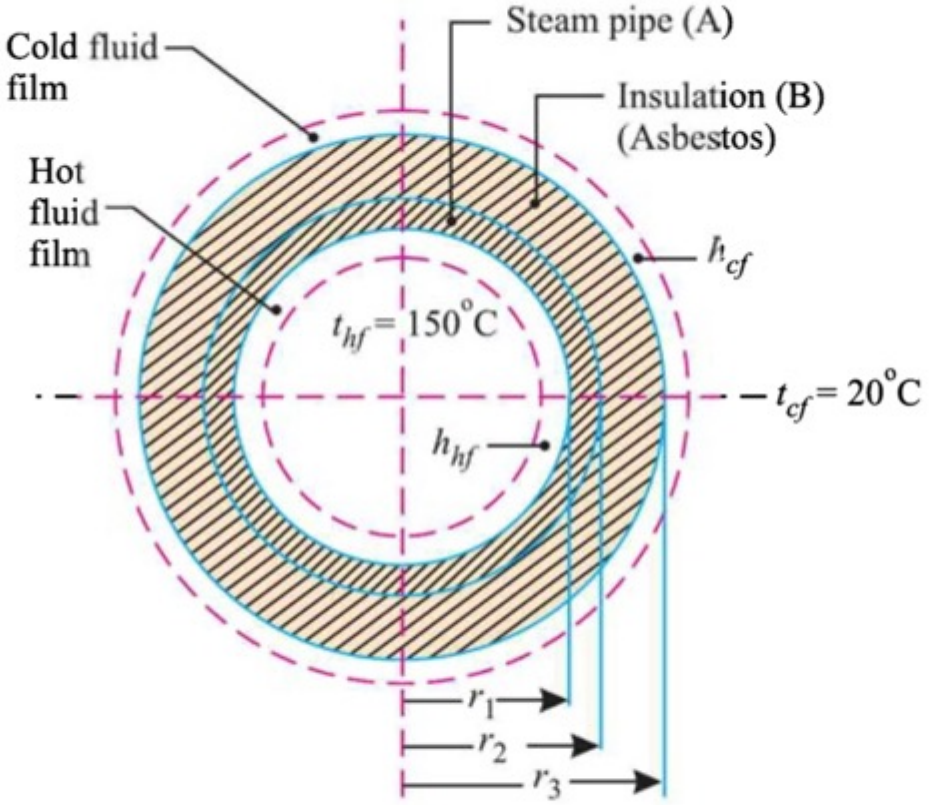


Fig. 2.65.

Thickness of insulation (asbestos), $(r_3 - r_2)$:

Area for heat transfer $= 2\pi\, r\, L$ (where L = length of the pipe)

$$\therefore \quad \text{Heat loss} = 2.1 \times 2\pi\, rL \text{ kW}$$

$$= 2.1 \times 2\pi \times 0.075 \times L = 0.989\, L \text{ kW}$$

$$= 0.989 \text{ L} \times 10^3 \text{ watts}$$

(where r, mean radius $= \dfrac{150}{2} = 75$ mm or 0.075 m ... given]

Heat transfer rate in such a case is given by

$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf}\,.\,r_1} + \dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{\ln(r_3/r_2)}{k_B} + \dfrac{1}{h_{cf}\,.\,r_3}\right]} \qquad \text{[Eqn. 2.69]}$$

$$0.989\ L \times 10^3 = \frac{2\pi L(150 - 20)}{\left[\dfrac{1}{100 \times 0.06} + \dfrac{\ln(0.08/0.06)}{42} + \dfrac{\ln(r_3/0.08)}{0.8} + \dfrac{1}{30 \times r_3}\right]}$$

$$0.989 \times 10^3 = \frac{816.81}{\left[0.16666 + 0.00685 + \dfrac{\ln(r_3/0.08)}{0.8} + \dfrac{1}{30\, r_3}\right]}$$

or, $$\frac{\ln(r_3/0.08)}{0.8} + \frac{1}{30\,r_3} = \frac{816.81}{0.989 \times 10^3} - (0.16666 + 0.00685) = 0.6524$$

or, $$1.25 \ln\,(r_3/0.08) + \frac{1}{30\,r_3} - 0.6524 = 0$$

Solving by hit and trial, we get

$$r_3 \simeq 0.105\ \text{m or } 105\ \text{mm}$$

∴ Thickness of insulation = $r_3 - r_2$ = 105 – 80 = **25 mm (Ans.)**