PERPINDAHAN KALOR KONVEKSI PAKSA

(Forced Convection Heat Transfer)

Dosen:
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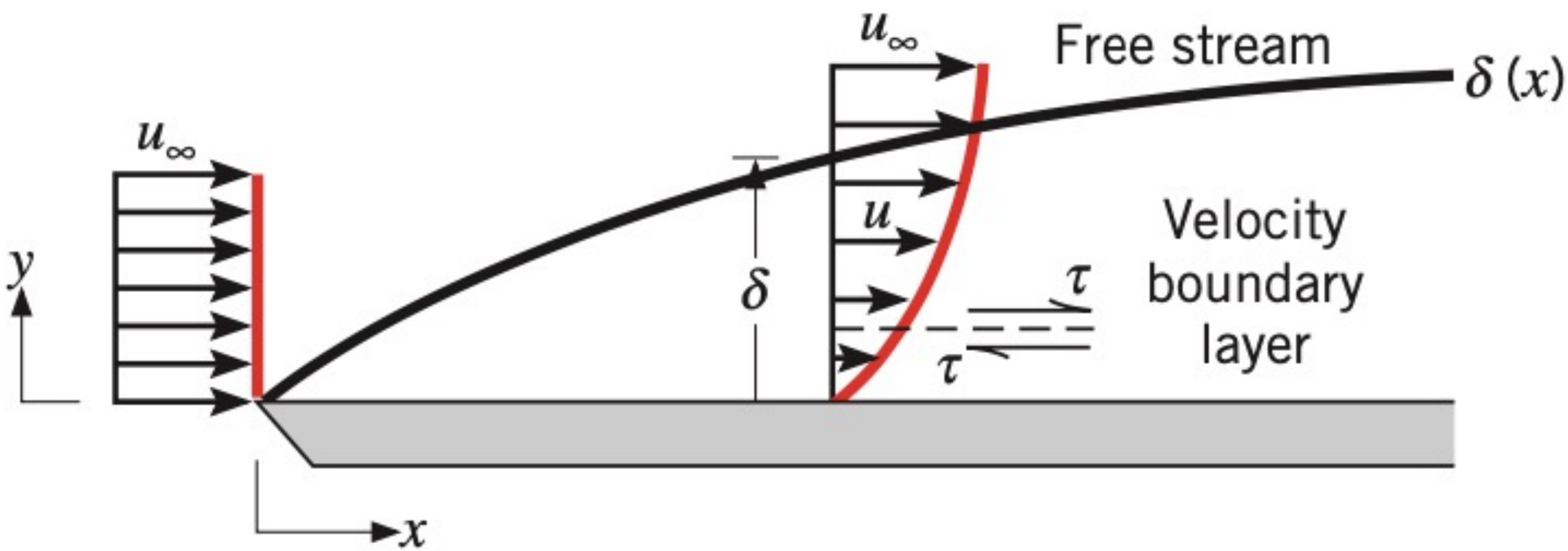
ALIRAN LAMINAR DI ATAS PLAT DATAR

a. Konsep Lapisan Batas (Boundary Layer)

- Analisa aliran fluida terbagi menjadi dua konsep dasar, yaitu :
  - aliran tanpa pengaruh gesekan oleh Leonhard Euler (1755) → persamaan Euler.
  - aliran bergesekan oleh Navier (1827), lalu oleh Stokes (1845), yaitu persamaan Navier-Stokes.
- Aliran fluida yang bergesekan akan menimbulkan lapisan batas dan akhirnya disebut dengan boundary layer (lapisan batas).
- Lapisan batas adalah suatu lapisan yang terbentuk disekitar penampang benda yang dilalui oleh fluida tertentu, karena mengalami hambatan yang disebabkan oleh beberapa faktor, seperti faktor gesekan, dan efek- efek viskos (kekentalan).
- Konsep lapisan batas oleh Ludwig Prandtl (1904).



Velocity boundary layer development on a flat plate

❖ Lapisan batas terbentuk pada saat adanya gerak relatif antara batas tersebut dengan fluida.

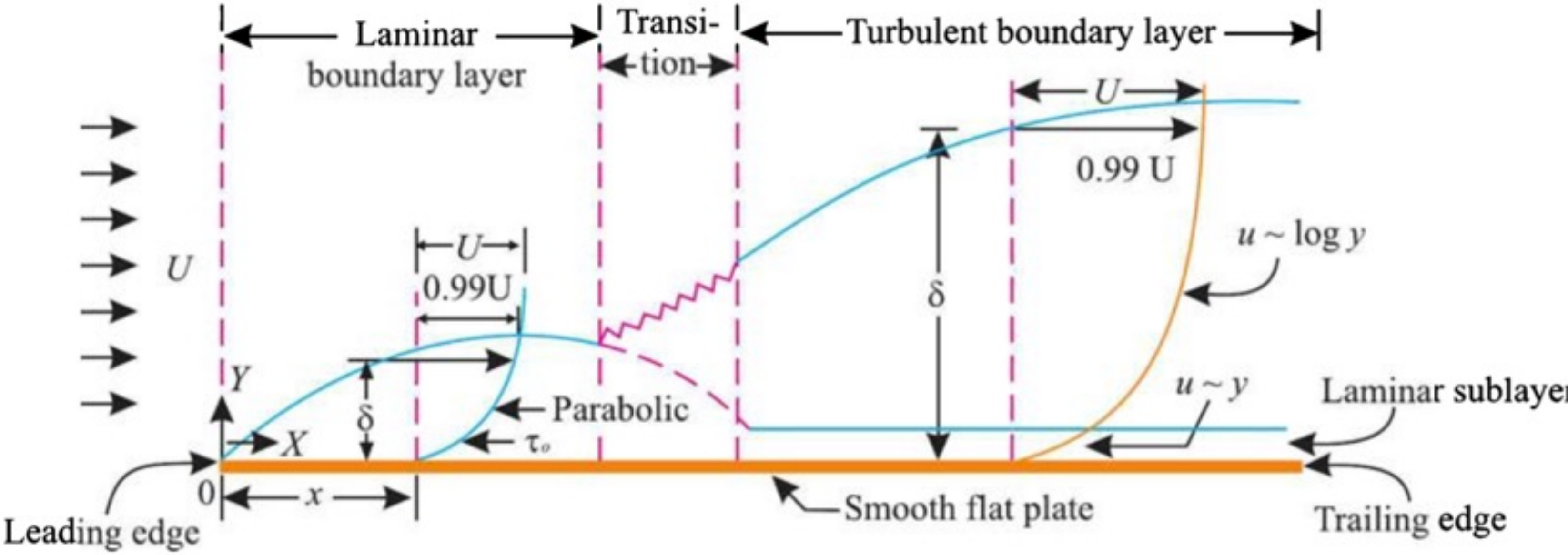
$$\tau_0 = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0},$$

τ_0 = tegangan geser
μ = viskositas dinamik
$\partial u/\partial y$ = distribusi kecepatan

Fluida menimbulkan suatu tegangan geser pada batas/*boundary* dan *boundary* menimbulkan gaya yang sama besar dan berlawanan pada fluida yang disebut **tahanan geser *(shear resistance).***

Berdasarkan teori lapisan batas, fluida yang berada disekitar benda yang bergerak di dalam fluida terdiri dari:

- suatu lapisan tipis yang berada pada batas/*boundary* yang disebut *boundary layer* dimana gesekan viskos (*viscous shear*) terjadi.
- daerah di luar lapisan batas dimana perilaku aliran sama dengan fluida ideal .



Lapisan batas pada suatu plat datar

- Ujung plat yang berhadapan dengan arah aliran disebut *leading edge.*
- Ujung plat bagian belakang disebut *trailing edge*.
- Dekat dengan *leading edge*, *boundary layer* sepenuhnya adalah laminar. Pada lapisan batas laminar distribusi kecepatan berbentuk parabol.
- **Tebal lapisan batas (δ)** meningkat sesuai jarak dari *leading edge*, x , dimana fluida diperlambat karena *viscous boundary*, dan kemudian menjadi tidak stabil dan menjadi turbulen setelah transisi.

Karakteristik lapisan batas:

(i) δ (tebal lapisan batas) membesar bila jarak dari leading x bertambah.

(ii) δ mengecil bila U bertambah.

(iii) δ bertambah bila viskositas kinematik (ν) bertambah.

(iv) $\tau_0 = \mu\left(\frac{U}{\delta}\right)$, …. τ_0 berkurang karena x meningkat. Namun bila lapisan batas menjadi turbulen, ini menunjukkan peningkatan tiba-tiba dan kemudian berkurang dengan meningkatnya x.

$\frac{Ux}{\nu} < 5 \times 10^5$ …. Lapisan batas adalah laminar (distribusi kecepatan berbentuk parabola)

$\frac{Ux}{\nu} > 5 \times 10^5$ …. Lapisan batas adalah turbulen

- **Aliran laminar dan turbulen dinyatakan dengan *Bilangan Reynolds (Re)***

$$Re = \frac{Ux}{\nu} = \frac{\rho U x}{\mu}$$

U = kecepatan aliran (m)

x = jarak dari *leading edge* (m)

ν = viskositas kinematik (m^2/s)

μ = viskositas dinamik (Ns/m^2)

- Tebal lapisan batas *(boundary layer thickness)*:

$$\eta = y\sqrt{\frac{U}{\nu x}} = \delta\sqrt{\frac{U}{\nu x}} = 5$$

$$\frac{\delta}{x} = 5\sqrt{\frac{\nu}{Ux}} = \frac{5}{\sqrt{Re_x}}$$

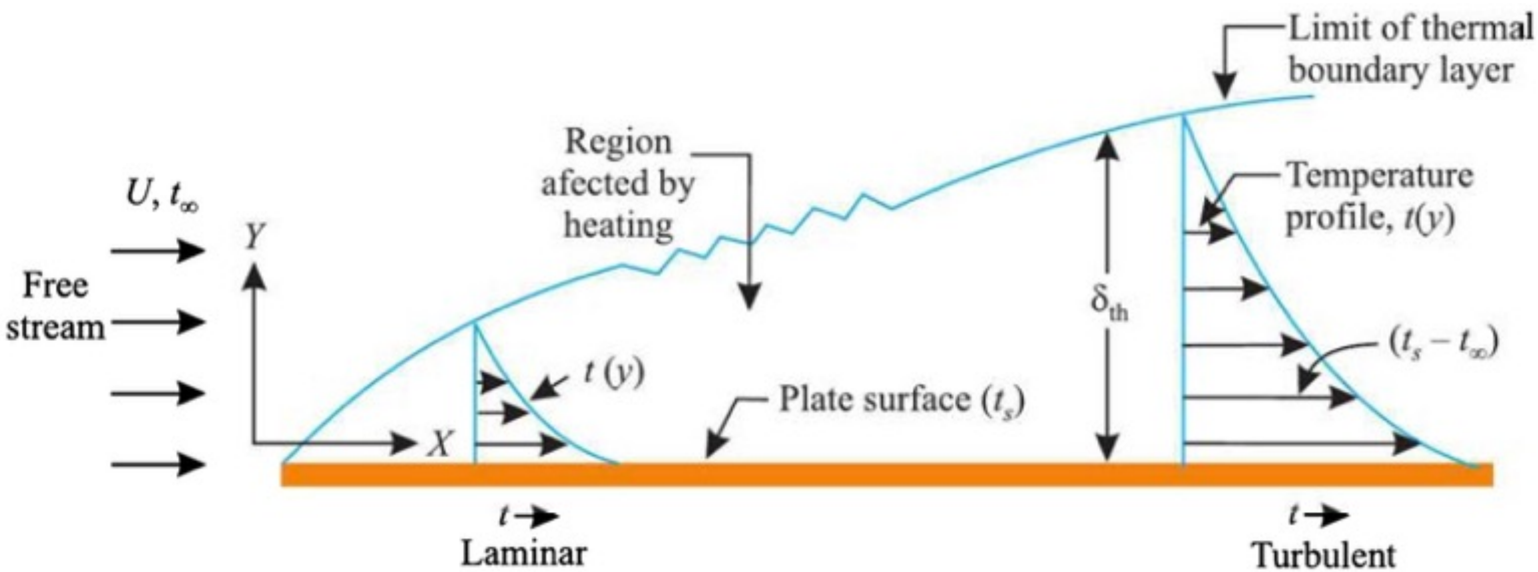
dimana, $\eta = y\sqrt{\frac{U}{\nu x}}$ menyatakan stretching factor

- Dari eksperimen yang telah dilakukan menyatakan bahwa profil kecepatan pada lokasi yang berbeda sepanjang plat secara geometri adalah sama, perbedaannya hanya oleh *stretching factor* pada arah Y.
- Hal ini menunjukkan bahwa u/U = 0,99 dapat ditunjukkan pada setiap lokasi x sebagai fungsi jarak dari dinding y/δ.

Parameter lapisan batas untuk profil kecepatan yang berbeda

S. No.	Velocity profile	Boundary conditions		δ	$\overline{C}_f$
		At $y = 0$	At $y = \delta$		
1.	$\frac{u}{U} = \frac{y}{\delta}$	$u = 0$	$u = U$	$\frac{3.46x}{\sqrt{\mathrm{Re}_x}}$	$\frac{1.155}{\sqrt{\mathrm{Re}_L}}$
2.	$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$	$u = 0$	$u = U$ $\frac{\partial u}{\partial y} = 0$	$\frac{5.48x}{\sqrt{Re_x}}$	$\frac{1.46}{\sqrt{Re_L}}$
3.	$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$	$u = 0$ $\frac{\partial^2 u}{\partial y^2} = 0$	$u = U$ $\frac{\partial u}{\partial y} = 0$	$\frac{4.64x}{\sqrt{Re_x}}$	$\frac{1.292}{\sqrt{Re_L}}$
4.	$\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$	$u = 0$	$u = U$	$\frac{4.795x}{\sqrt{Re_x}}$	$\frac{1.31}{\sqrt{Re_L}}$
5.	Blasius exact solution			$\frac{5x}{\sqrt{Re_x}}$	$\frac{1.328}{\sqrt{Re_L}}$

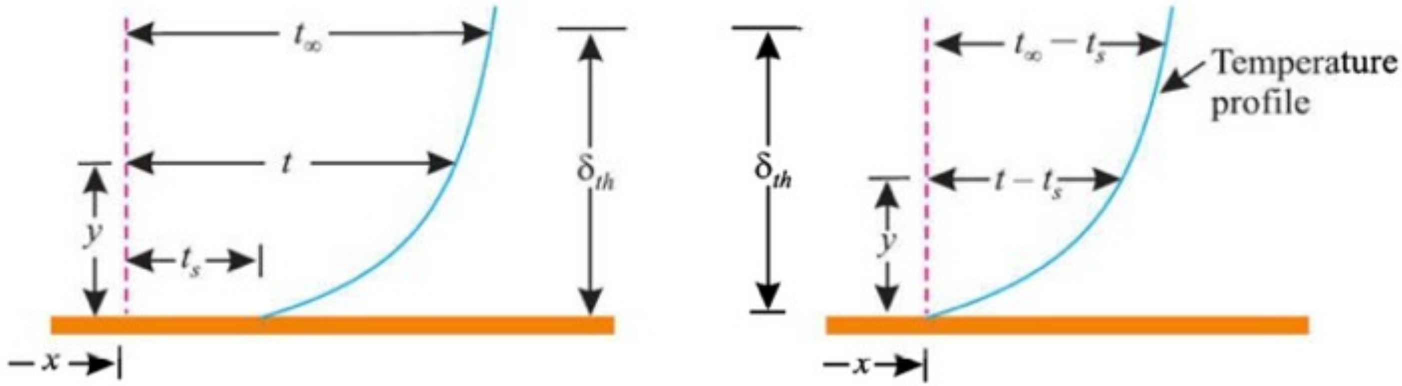
Lapisan Batas Panas *(Thermal Boundary Layer)*



Thermal boundary layer yang terbentuk pada saat aliran fluida dingin mengalir di atas plat panas.

Thermal boundary layer (δ_{th}) berada pada jarak y dari permukaan plat, dimana:

$$\frac{t_s - t}{t_s - t_\infty} = 0{,}99$$



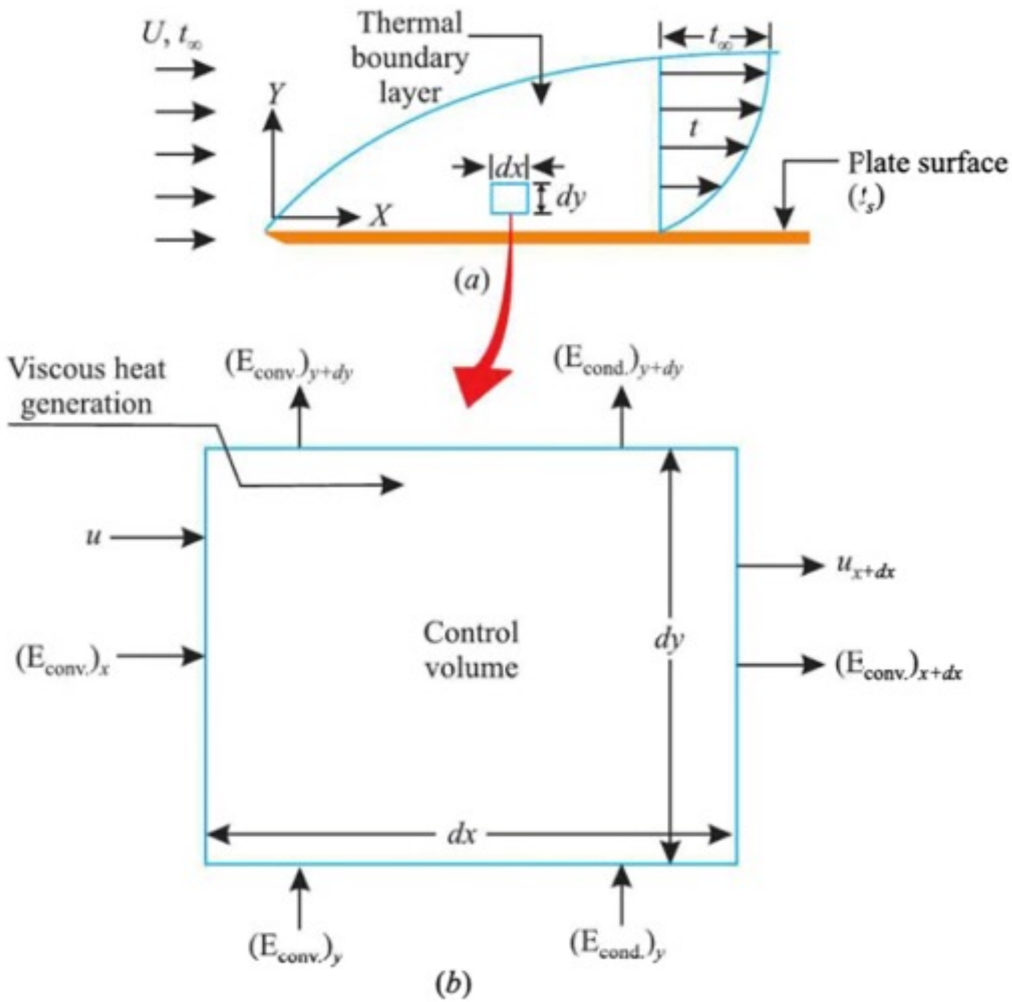
Thermal boundary layer yang terbentuk pada saat aliran fluida panas mengalir di atas plat dingin.

- Profil kecepatan lapisan batas hidrodinamik terutama tergantung pada viskositas fluida.
- Profil tempertar lapisan batas thermal tergantung pada viskositas, kecepatan aliran, panas spesifik dan konduktivitas termal fluida.
- Besaran relatif δ dan δ_{th} dipengaruhi oleh sifat-sifat termo fisik fluida.

Bilangan Prandtl: $\mathrm{Pr} = \dfrac{\mu\, Cp}{k}$

$$(i)\quad \delta_{\mathrm{th}} = \delta \ldots\ldots\ldots bila\ \ \mathrm{Pr} = 1;$$
$$(ii)\quad \delta_{\mathrm{th}} < \delta \ldots\ldots\ldots bila\ \ \mathrm{Pr} > 1;$$
$$(iii)\ \delta_{\mathrm{th}} > \delta \ldots\ldots\ldots bila\ \ \mathrm{Pr} < 1;$$

Persamaan Energi Lapisan Batas Thermal di atas Plat Datar



Energi masuk dan keluar volume atur (control volume)

- Gambar di atas menunjukkan fluida panas yang mengalir di atas plat datar dingin dan pengembangan lapisan batas termal.
- Sesuai prinsip konservasi energi untuk keadaan steady:

Konveksi energi panas pada arah X dan Y + konduksi energi panas dalam arah Y + viscous heat generation = 0

$$d(E_{conv.})_x + d(E_{conv.})_y + d(E_{cond.})_y + \text{viscous heat generation} = 0$$

$$(i) \qquad\qquad (ii) \qquad\qquad (iii) \qquad\qquad (iv)$$

(*i*) **The Energy convected in *X*-direction :**

$$(E_{conv.})_x = \text{Mass} \times \text{specific heat} \times \text{temperature}$$

$$= [\rho u(dy \times 1)]\, c_p t = (\rho u\, dy)\, c_p t$$

$$(E_{conv.})_{x+dx} = \left[\rho\left(u + \frac{\partial u}{\partial x} dx\right) dy\right] c_p \left(t + \frac{\partial t}{\partial x}\right) dx$$

$$= \rho c_p\, dy \left[ut + u + \frac{\partial u}{\partial x} dx + t \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial x} \frac{\partial t}{\partial x} (dx)^2\right]$$

$$= \rho c_p\, dy \left[ut + u \frac{\partial t}{\partial x} dx + t \frac{\partial u}{\partial x} dx\right]$$

... neglecting the product of small quantities

$\therefore$ Net energy convected in X-direction,

$$d(E_{conv.})_x = (E_{conv.})_x - (E_{conv.})_{x+dx}$$

$$= (\rho u\, dy)\, c_p t - \left[\rho c_p dy \left\{ ut + u\frac{\partial t}{\partial x}dx + t\frac{\partial u}{\partial x}dx \right\}\right]$$

or, $$d(E_{conv.})_x = -\rho c_p \left[u\frac{\partial t}{\partial x} + t\frac{\partial u}{\partial x} \right] dx\, dy$$

(*ii*) The energy convected in Y-direction :

The net energy convected in Y-direction,

$$d(E_{conv.})_y = (E_{conv.})_y - (E_{conv.})_{y+dy}$$

$$= (\rho \upsilon\, dx)\, c_p t - \left[\rho\left(\upsilon + \frac{\partial \upsilon}{\partial y}dy\right)dx\right] c_p \left(t + \frac{\partial t}{\partial y}dy\right)$$

or, $$d(E_{conv.})_y = -\rho c_p \left[\upsilon\frac{\partial t}{\partial y} + t\frac{\partial \upsilon}{\partial y}\right] dx\, dy$$

... neglecting the product of small quantities.

(*iii*) The heat conduction in the *Y*-direction :

$$d(E_{cond.})_y = (E_{cond.})_y - (E_{cond.})_{y+dy}$$

$$= -k\,(dx \times 1)\frac{\partial t}{\partial y} - \left[-k\,(dx \times 1)\left\{\frac{\partial t}{\partial y} + \frac{\partial}{\partial y}\left(\frac{\partial t}{\partial y}\right)\right\} dy \right]$$

or, $$d(E_{cond.})_y = k\frac{\partial^2 t}{\partial y^2}\, dx\, dy$$

(*iv*) **Viscous heat generation :**

Owing to relative motion of fluid in the boundary layer (fluid on the top face of the control volume moves faster than fluid on bottom face), there will be viscous effects which will cause generation of heat.

Viscous heat generation = Viscous force (average) × distance travelled by the viscous force (this is determined by the relative velocity of fluid flow at the upper and lower faces of the element).

= [Shear stress (τ) × area upon which it acts)] × distance travelled

$$= \left[\mu \frac{\partial u}{\partial y}(dx \times 1)\right] \times \left(\frac{\partial u}{\partial y} dy\right)$$

or, Viscous heat generation

$$= \mu \left(\frac{\partial u}{\partial y}\right)^2 dx\ dy \qquad \text{...(7.45)}$$

Pohlhausen menyatakan persamaan yang menghubungkan lapisan batas termal dan lapisan batas hidrodinamik:

$$\delta_{th} = \frac{\delta}{(Pr)^{1/3}}$$

Koefisien perpindahan panas lokal:

$$\frac{Q}{A} = h_x (t_s - t_\infty) = -k\left(\frac{\partial t}{\partial y}\right)_{y=0}$$

$$\left(\frac{\partial t}{\partial y}\right)_{y=0} = -(t_s - t_\infty)\sqrt{\frac{U}{\nu x}} \times \left(\frac{\partial \theta}{\partial y}\right)_{\eta=0}$$

$$= -(t_s - t_\infty)\sqrt{\frac{U}{\nu x}} \times 0.332(Pr)^{1/3}$$

$$= -\frac{0.332}{x}(t_s - t_\infty)\sqrt{\frac{U}{\nu}}(Pr)^{1/3}$$

$$= -\frac{0.332}{x}(t_s - t_\infty)(Re_x)^{1/2}(Pr)^{1/3}$$

$$\frac{Q}{A} = h_x (t_s - t_\infty) = 0.332\frac{k}{x}(t_s - t_\infty)(Re_x)^{1/2}(Pr)^{1/3}$$

$$h_x = 0.332\,\frac{k}{x}(Re_x)^{1/2}(Pr)^{1/3}$$

$$Nu_x = \frac{h_x x}{k} = 0.332\,(Re_x)^{1/2}(Pr)^{1/3}$$

h_x = *koefisien perpindahan panas konveksi lokal*

Nu_x = *Bilangan Nusselt local (….pada jarak x dari ujung masuk pelat atau leading edge)*

Koefisien perpindahan panas konveksi rata-rata:

$$\bar{h} = \frac{1}{L}\int_0^L h_x \, . dx = \frac{1}{L}\int_0^L 0.332\frac{k}{x}(Re_x)^{1/2}(Pr)^{1/3}\,dx$$

$$= \frac{1}{L}\int_0^L 0.332\,k\,(Pr)^{1/3}\sqrt{\left(\frac{U}{v}\right)}\,x^{-1/2}\,dx$$

$$\bar{h} = 0.664\left(\frac{k}{L}\right)(Re_L)^{1/2}(Pr)^{1/3}$$

$atau \; \dots . \quad \bar{h} = 2h_x$

$\overline{Nu} = Nilai\ rata - rata\ bilangan\ Nusselt$

$$= \frac{\bar{h}L}{k} = 0.664\,(Re_L)^{1/2}\,(Pr)^{1/3}$$

CONTOH SOAL
PERPINDAHAN KALOR KONVEKSI PAKSA
(Forced Convection)

Contoh No. 1:

Udara 20 ^oC dan tekanan 1 bar mengalir di atas plat datar dengan kecepatan 3 m/s. Jika lebar plat 280 mm dan temperatur plat 56 ^oC, hitunglah pada x = 280 mm, dimana sifat-sifat udara pada *bulk mean temperature* (20 + 56) / 2 = 38 ^oC (dari tabel):

$\rho = 1.1374\ kg/m^3;\ k = 0.02732\ W/m°C;\ c_p = 1.005\ kJ/kgK;\ \nu = 16.768 \times 10^{-6}\ m^2/s;\ Pr = 0.7.$

i. Tebal lapisan batas *(Boundari layer thickness)*
ii. Koefisien gesek local *(Local friction coefficient)*
iii. Koefisien gesek rata-rata *(Average friction coefficient)*
iv. Tegangan geser karena gesekan *(Shearing stress due to friction)*
v. Tebal lapisan batas termal *(Thickness of thermal boundary layer)*
vi. Koefisien perpindahan panas konveksi local *(Local convective heat transfer coefficient)*
vii. Koefisien perpindahan panas konveksi rata-rata *(Average convective heat transfer coefficient)*
viii. Laju perpindahan panas konveksi *(Rate of heat transfer by convection)*
ix. Gaya hambat total pada plat *(Total drag force on the plat)*
x. Laju massa aliran total melalui batas *(Total mass flow rate through the boundary)*

Solution. *Given* : $U = 3\text{m/s}$, $x = 280 \text{ mm} = 0.28 \text{ m}$, $\rho = 1.1374 \text{ kg/m}^3$, $k = 0.02732 \text{ W/m°C}$, $c_p = 1.005 \text{ kJ/kgK}$, $\nu = 16.768 \times 10^{-6} \text{ m}^2\text{/s}$.

Let us first ascertain the type of the flow, whether laminar or turbulent.

$$Re_x = \frac{Ux}{\nu} = \frac{3 \times 0.28}{16.768 \times 10^{-6}} = 5.0 \times 10^4$$

Since $Re_x < 5 \times 10^5$, hence flow in *laminar*.

(*i*) Boundary layer thickness at $x = 0.28$ m, δ :

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

or,

$$\delta = \frac{5 \times 0.28}{\sqrt{5 \times 10^4}} = 0.00626 \text{ m or } \mathbf{6.26 \text{ mm (Ans.)}}$$

(*ii*) Local friction coefficient, C_{fx} :

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}}$$

or,

$$C_{fx} = \frac{0.664}{\sqrt{5 \times 10^4}} = \mathbf{0.002969 \text{ (Ans.)}}$$

(*iii*) **Average friction coefficient, C_f:**

$$\overline{C}_f = \frac{1.328}{\sqrt{Re_L}}$$

or,

$$\overline{C}_f = \frac{1.328}{\sqrt{5 \times 10^4}} = \mathbf{0.005939\ (Ans.)}$$

(*iv*) **Shearing stress due to friction, τ_0 :**

$$\tau_0 = C_{fx} \times \frac{\rho U^2}{2}$$

$$= 0.002969 \times \frac{1.1374 \times 3^2}{2} = \mathbf{0.01519\ N/m^2\ (Ans.)}$$

(*v*) **Thickness of thermal boundary layer, δ_{th} :**

$$\delta_{th} = \frac{\delta}{(Pr)^{1/3}}$$

$$= \frac{0.00626}{(0.7)^{1/3}} = 0.00705 \text{ , m or } \mathbf{7.05\ mm\ (Ans.)}$$

(*vi*) **Local convective heat transfer coefficient, h_x :**

$$h_x = 0.332 \frac{k}{x} (Re_x)^{1/2} (Pr)^{1/3}$$

$$= 0.332 \times \frac{0.02732}{0.28} \times (5 \times 10^4)^{1/2} \times (0.7)^{1/3}$$

$$= \mathbf{6.43\ W/m^2 {}^\circ C\ (Ans.)}$$

(*vii*) Average convective heat transfer coefficient, $\bar{h}$:

$$\bar{h} = 0.664\left(\frac{k}{L}\right)(Re_L)^{1/2}\ (Pr)^{1/3}$$

$$= 0.664\left(\frac{0.02732}{0.28}\right)(5 \times 10^4)^{1/2}\ (0.7)^{1/3} = \mathbf{12.86\ W/m^{2}°C\ (Ans.)}$$

$$\ldots (\bar{h} = 2h_x)$$

(*viii*) Rate of heat transfer by convection, Q_{conv}.:

$$Q_{conv.} = \bar{h}\, A_s\,(t_s - t_\infty)$$

$$= 12.85 \times (0.28 \times 0.28)\ (56 - 20) = \mathbf{36.29\ W\ (Ans.)}$$

(*ix*) Total drag force on the plate, F_D :

$$F_D = \tau_0 \times \text{ area of plate on one side upto 0.28 m}$$

$$= 0.01519 \times 0.28 \times 0.28 = \mathbf{0.00119\ N\ (Ans.)}$$

(*x*) Total mass flow rate through the boundary, m :

$$m = \frac{5}{8}\rho U\,(\delta_2 - \delta_1)$$

(where $\delta_1 = 0$ at $x = 0$ and $\delta_2 = \delta$ at $x = 0.28$ m)

$$= \frac{5}{8} \times 1.1374 \times 3\ (0.00626 - 0) = \mathbf{0.01335\ kg/s\ (Ans.)}$$

Contoh No. 2:

Udara 200 °C dan tekanan atmosfir mengalir di atas plat datar dengan kecepatan 5 m/s. Lebar plat 15 mm dan temperatur plat 120 °C, hitunglah lapisan batas hidrodinamik dan thermal, koefisien perpindahan panas local pada jarak 0,5 m dari ujung depan (leading edge) plat. Asumsi bahwa aliran hanya pada satu sisi plat.

$\rho = 0.815\ kg/m^3;\ \mu = 24.5 \times 10^{-6}\ Ns/m^2;\ Pr = 0.7,\ k = 0.0364\ W/m\ K.$

Penyelesaian:

U = 5 m/s; x = 0,5 m;

$$Re_x = \frac{Ux}{\nu} = \frac{5 \times 0.5}{\mu/\rho} = \frac{5 \times 0.5}{(24.5 \times 10^{-6}/0.815)} = 83163$$

Since $Re_x < 5 \times 10^5$, hence flow is *laminar*.

Boundary layer thickness at $x = 0.5$ m, δ:

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 0.5}{\sqrt{83163}} = 8.669 \times 10^{-3}\ \text{m}$$

or **8.669 mm (Ans.)**

Thickness of thermal boundary layer, at $x = 0.5$ m, δ_{th}

$$\delta_{th} = \frac{\delta}{(Pr)^{1/3}} = \frac{8.669}{(0.7)^{1/3}} = \mathbf{9.763\ mm}\ \textbf{(Ans.)}$$

Local heat transfer coefficient, h_x:

$$h_x = 0.332 \times \frac{k}{x}(Re_x)^{1/2}(Pr)^{1/3}$$

$$= 0.332 \times \frac{0.0364}{0.5} \times (83163)^{1/2} \times (0.7)^{1/3} = \mathbf{6.189\ W/m^2K}$$

Contoh No. 3:

Udara pada tekanan atmosfir dan 40 ^oC mengalir di atas plat datar yang panjang 2 m dengan kecepatan 5 m/s. Temperatur plat 120 ^oC, hitunglah koefisien perpindahan panas rata-rata pada panjang plat 2 m dan laju perpindahan panas antara plat dengan udara jika lebar plat 1 m.

Udara pada 1 atm dan 80 ^oC, $v = 2{,}107 \times 10^{-5}\ m^2/s$, k = 0,03025 W/mK, Pr = 0,6965

Penyelesaian:

Average heat transfer coefficient, $\bar{h}$:

$$Re = \frac{UL}{\nu} = \frac{5 \times 2}{2.107 \times 10^{-5}} = 4.746 \times 10^5$$

Assuming $Re_{cr} = 5 \times 10^5$, the flow is *laminar*.

Using *exact solution*, the average Nusselt number is given by

$$\overline{Nu} = 0.664\ (Re_L)^{1/2}\ (Pr)^{1/3}$$

or,

$$\frac{\bar{h}L}{k} = 0.664\ (4.746 \times 10^5)^{1/2}\ (0.6965)^{1/3} = 405.48$$

$\therefore$

$$\bar{h} = \frac{k}{L} \times 405.48 = \frac{0.03025}{2} \times 4054.48 = \mathbf{6.133\ W/m^3K}$$

Rate of heat transfer, Q :

$$Q = \bar{h}\ A_s\ (t_s - t_\infty)$$

$$= 6.133 \times (2 \times 1)\ (120 - 40) = \mathbf{981.28\ W} \quad \textbf{(Ans.)}$$

Contoh No. 4:

Udara pada 20 °C mengalir di atas plat datar dimana lebar 200 mm dan panjang 500 mm. Temperatur plat 100 °C. Hitunglah heat loss per hour dari plat jika udara mengalir parallel panjang plat sampai 500 mm dengan kecepatan 2 m/s. Juga hitung bila mengalir sesuai lebar 200 mm.

Sifat udara pada (100 + 20)/2 = 60 °C:, $v = 18,97 \times 10^{-6}$ m²/s k = 0,025 W/m°C, Pr = 0,7

Penyelesaian: *Given* : $U = 2$ m/s, $v = 18.97 \times 10^{-6}$ m²/s, $k = 0.025$ W/m°C and $Pr = 0.7$.

Heat loss per hour from the plate, Q :

Case I. *When the flow is parallel to 500 mm side :*

$$\overline{Nu} = \frac{\bar{h} L}{k} = 0.664\,(Re_L)^{1/2}\,(Pr)^{1/3}$$

where,

$$Re_L = \frac{UL}{v} = \frac{2 \times 0.5}{18.19 \times 10^{-6}} = 5.27 \times 10^4$$

$$\bar{h} = \frac{k}{L} \times 0.664\,(Re_L)^{1/2}\,(Pr)^{1/3}$$

$$= \frac{0.025}{0.5} \times 0.664\,(5.27 \times 10^4)^{1/2}\,(0.7)^{1/3} = 6.767\ \text{W/m}^2{}^\circ C$$

$$Q = \bar{h} A_s (t_s - t_\infty) = 6.767 \times (0.5 \times 0.2)\,(100 - 20) = \mathbf{54.14\ W}$$

Case II. *When the flow is parallel to 200 mm side* :

$$Re_L = \frac{2 \times 0.2}{18.97 \times 10^{-6}} = 2.11 \times 10^4$$

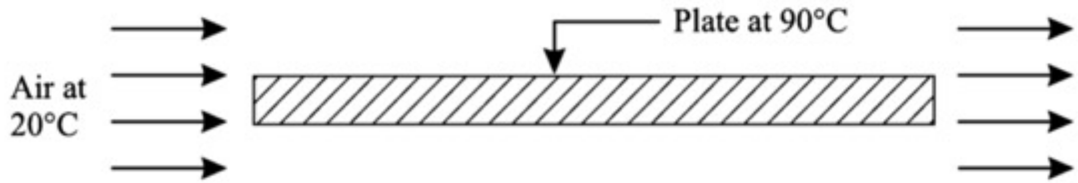
$$\bar{h} = \frac{0.025}{0.2} \times 0.664 \times (2.11 \times 10^4)^{1/2}\,(0.7)^{1/3} = \mathbf{10.7\ W/m^2\,{}^\circ C}$$

$$Q = \bar{h} \times A_s (t_s - t_\infty) = 10.7 \times (0.2 \times 0.5) \times (100 - 20) = \mathbf{85.6\ W}$$

Contoh No. 5:

Dalam proses pembuatan kaca berbentuk *square* yang luasnya 1 m^2 dan tebal 3 mm dipanaskan merata sampai 90 ^{o}C yang didinginkan oleh udara 20 ^{o}C yang mengalir paralel pada kedua permukaan kaca dengan kecepatan 2 m/s. Hitunglah nilai awal pendinginan kaca. Untuk kaca: ρ = 2500 kg/m^3 dan c_p = 0,67 kJ/kgK.
Sifat udara: ρ = 1,076 kg/m^3 ; c_p = 1008 J/kgK; k = 0,0286 W/m^{o}C; μ = 19,8 x 10^{-6} N.s/m^2.

Penyelesaian: *Given* : A = 1 m^2 ; t_s = 90°C; t_∞ = 20°C, U = 2 m/s.



$$\overline{Nu} = \frac{\bar{h}\,L}{k} = 0.664(Re_L)^{1/2}(Pr)^{1/3}$$

(valid for $Pr > 0.5$)

$$Re_L = \frac{\rho U L}{\mu} = \frac{1.076 \times 2 \times 1}{19.8 \times 10^{-6}} = 1.087 \times 10^5$$

$$Pr = \frac{\mu c_p}{k} = \frac{19.8 \times 10^{-6} \times 1008}{0.0286} = .0698$$

$$\frac{\bar{h}\,L}{k} = 0.664 \times (1.087 \times 10^5)^{1/2} \times (0.698)^{1/3} = 194.19$$

$$\bar{h} = \frac{k}{L} \times 194.19 = \frac{0.0286}{1} \times 194.19 = 5.55 \text{ W/m}^2\,°\text{C}$$

Aliran panas pada kedua permukaan plat:

$$Q = 2\bar{h}\,A(t_s - t_\infty) = 2 \times 5.55 \times 1 \times (90 - 20) = 777 \text{ W}$$

Heat lost : $Q = m\,c_p\,\Delta t = 777$

$$m = (\text{Area} \times \text{thickness}) \times \rho$$

$$= 1 \times \frac{3}{1000} \times 2500 = 7.5 \text{ kg}$$

$$777 = 7.5 \times (0.67 \times 10^3) \times \Delta t$$

$$\Delta t = \frac{777}{7.5 \times (0.67 \times 10^3)} = \mathbf{0.155°C/s}$$

Contoh No. 5:
Suatu plat datar, lebar 1 m dan panjang 1,5 m mempunyai temperatur 90 °C berada di udara yang mengalir bebas dengan temperatur 10 °C. Hitunglah kecepatan dimana udara harus mengalir di atas plat sepanjang 1,5 m sehingga disipasi energi 3,75 kW.
Sifat udara pada 50 °C: ρ = 1,09 kg/m³ ; c_p = 1,007 kJ/kg°C; k = 0,028 W/m°C; μ = 2,03 x 10^{-5} N.s/m², Pr = 0,7

Penyelesaian: *Given* : $L = 1.5$ m, $B = 1$m, $t_s = 90°C$, $t_\infty = 10°C$, $Q = 3.75$ kW

Free stream velocity, *U* :

The heat flow from the plate to air is given by

$$Q = \bar{h}\, A_s (t_s - t_\infty)$$

where,

$$\bar{h} = \frac{k}{L} \times 0.664\, (Re_L)^{1/2}\, (Pr)^{1/3} \qquad \text{...[}$$

$$= \frac{0.028}{1.5} \times 0.664 \left(\frac{\rho L U}{\mu} \right)^{1/2} (0.7)^{1/3}$$

$$= \frac{0.028}{1.5} \times 0.664 \left(\frac{1.09 \times 1.5 \times U}{2.03 \times 10^{-5}} \right)^{1/2} (0.7)^{1/3} = 3.123\sqrt{U}$$

$\therefore$

$$3.75 \times 6000 = 3.123\ \sqrt{U}\ \times (1.5 \times 1)\,(90 - 10)$$

or,

$$\sqrt{U} = \frac{3.75 \times 1000}{3.123 \times (1.5 \times 1)\,(80)} = 10$$

or,

$$\mathbf{U = 100\ m/s} \quad \textbf{(Ans.)}$$

Contoh No. 6:

Udara pada 20 °C dan tekanan atmosfir mengalir di atas plat datar pada kecepatan 1,8 m/s. Jika Panjang plat 2,2 m dan temperatur 100 °C, hitunglah laju perpindahan panas per satuan lebar menggunakan (i) metode exact dan (ii) metode aproximasi.

Sifat udara pada (100 + 20)/2 = 60 °C: ρ = 1,06 kg/m³ ; c_p = 1,005 kJ/kg°C; k = 0,02894 W/m°C; ν = 18,97 x 10^{-6} m²/s, Pr = 0,696

Penyelesaian: *Given* : $t_\infty = 20°C$, $t_s = 100°C$, $U = 1.8$ m/s, $L = 2.2$ m, $B = 1$ m

Heat transfer rate per unit width :

Reynolds number, $$Re_L = \frac{UL}{\nu} = \frac{1.8 \times 2.2}{18.97 \times 10^{-6}} = 2.087 \times 10^5$$

Since Reynolds number is less than 5×10^5 hence flow is *laminar*.

(*i*) **Using exact solution :**

The average Nusselt number is given by

$$\overline{Nu} = 0.664\,(Re_L)^{1/2}\,(Pr)^{1/3}$$

or, $$\frac{\bar{h}L}{k} = 0.664\,(2.087 \times 10^5)^{1/2}\,(0.696)^{1/3} = 268.82$$

or, $$\bar{h} = \frac{268.82\,k}{L} = \frac{268.82 \times 0.02894}{2.2} = 3.536 \text{ W/m}^2\text{°C}$$

∴ Heat transfer rate from the plate,

$$Q = \bar{h}\,A_s\,(t_s - t_\infty) = 3.536 \times (2.2 \times 1)\,(100 - 20) = \mathbf{622.34\ W}$$

(*ii*) **Using approximate solution :**

$$\overline{Nu} = \frac{\bar{h}L}{k} = 0.646\,(Re_L)^{1/2}\,(Pr)^{1/3}$$

or, $$\frac{\bar{h}\,L}{k} = 0.646\,(2.087 \times 10^5)^{1/2}\,(0.696)^{1/3} = 261.53$$

or, $$\bar{h} = \frac{261.53\,k}{L} = \frac{261.53 \times 0.02894}{2.2} = 3.44\ \text{W/m}^2\,°C$$

∴ Heat transfer rate from the plate,

$$Q = \bar{h}\,A_s\,(t_s - t_\infty) = 3.44 \times (2.2 \times 1) \times (100 - 20) = \mathbf{605.44\ W} \quad \textbf{(Ans.)}$$

Contoh No. 7:

Udara pada 30 °C mengalir dengan kecepatan 2,8 m/s di atas plat datar yang berukuran: panjang 1000 mm x lebar 600 mm x tebal 25 mm. Temperatur permukaan atas plat 90 °C. Jika konduktivitas termal material plat 25 W/m°C, hitunglah: (i) Kerugian panas oleh plat, (ii) temperatur bagian bawah plat untuk keadaan steady.

Sifat udara pada (90 + 30)/2 = 60 °C: ρ = 1,06 kg/m^3 ; c_p = 1,005 kJ/kg°C; k = 0,02894 W/m°C; ν = 18,97 x 10^{-6} m^2/s, Pr = 0,696

Solution. *Given* : $t_\infty = 30°C$, $t_s = 90°C$, U = 2.8 m/s, k_{plate} = 25 W/m°C, L = 1000 mm = 1m, B = 600 mm = 0.6 m, δ = 25 mm = 0.025m.

(*i*) **Heat lost by the plate :**

Reynolds number at the trailing edge,

$$Re_L = \frac{UL}{\nu} = \frac{2.8 \times 1.0}{18.97 \times 10^{-6}} = 1.476 \times 10^5$$

Since Reynolds number is less than 5 × 10^5, hence flow is *laminar* throughout the length,

$$\overline{Nu} = 0.664\,(Re_L)^{1/2}\,(Pr)^{1/3} = \frac{\bar{h}L}{k} \qquad \text{...[Eqn. (7.68)]}$$

or, $\bar{h}$ (average heat transfer coefficient) $= \dfrac{\overline{Nu} \times k}{L} = \dfrac{0.664\,(Re_L)^{1/2}\,(Pr)^{1/3} \times k}{L}$

or,

$$\bar{h} = \frac{0.664\,(1.476 \times 10^5)^{1/2}\,(0.696)^{1/3} \times 0.02894}{1.0}$$

$$= 6.542 \text{ W/m}^2\,°\text{C}$$

(*ii*) Bottom temperature of the plate, t_b:

Heat lost by the plate Q (calculated above) must be conducted through the plate, hence exchange from top to bottom surface is

$$Q = -\frac{kA(t_s - t_\infty)}{\delta} = 235.5$$

or,

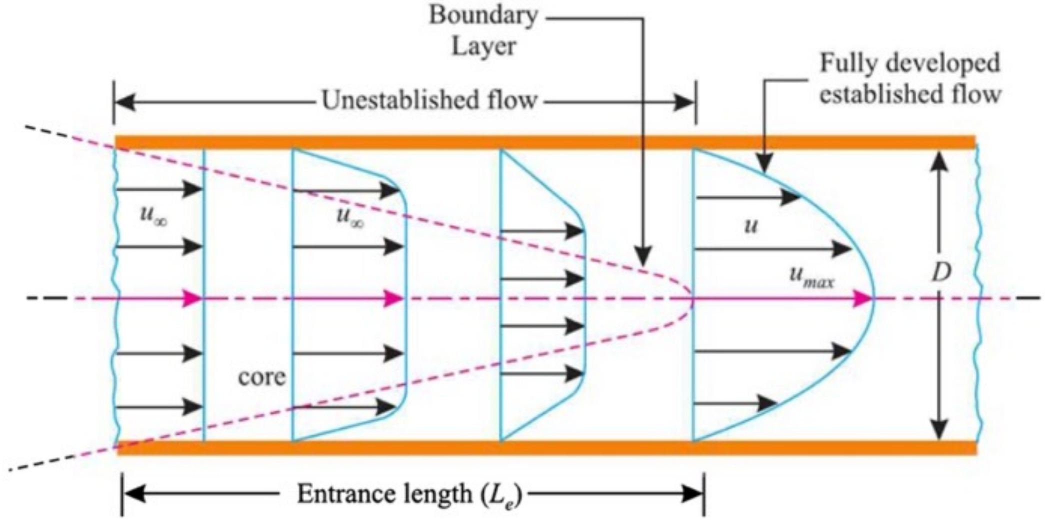
$$= \frac{25 \times (1.0 \times 0{\cdot}6)\,(90 - t_b)}{0.025} = 235.5$$

or,

$$t_b = 90 + \frac{0.025 \times 235.5}{25\,(1.0 \times 0.6)} = \mathbf{90.39°C} \qquad \textbf{(Ans.)}$$

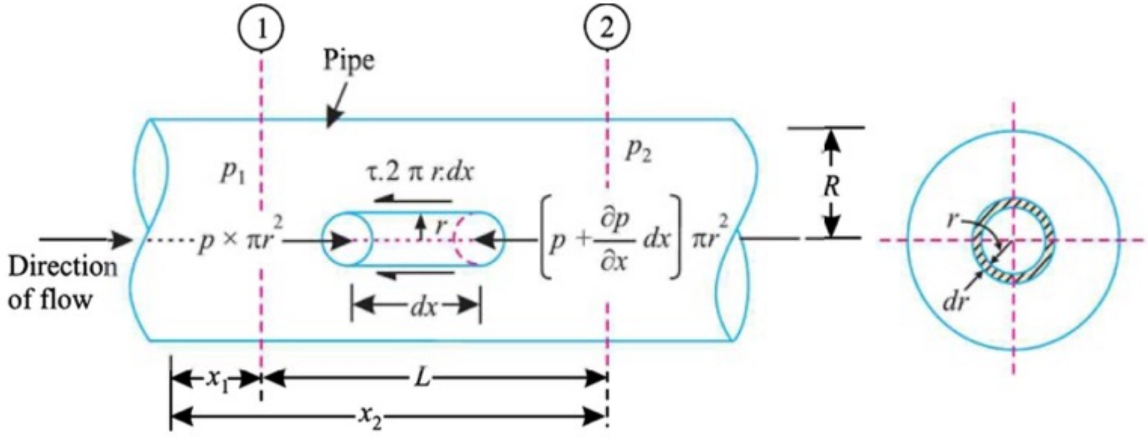
ALIRAN LAMINAR PADA PIPA

- Perkembangan lapisan batas *(boundary layer)* aliran yang mengalir dalam pipa adalah sama seperti pada bidang datar.
- Fluida yang mengalir masuk pipa dengan kecepatan yang sama mengalami perlambatan dekat dinding pipa dan perkembangan lapisan batas digambarkan dengan garis putus-putus pada gambar di bawah.
- Lapisan batas yang terbentuk akan bertemu pada garis sumbu pipa dan keseluruhan aliran mempunyai karakteristik dari lapisan batas.
- Ketika tebal lapisan batas sama dengan radius pipa dan tidak ada perubahan dalam distribusi kecepatan, maka distibusi kecepatan disebut *fully developed velocity profile* atau profil kecepatan yang berkembang penuh.



DISTRIBUSI KECEPATAN

- Gambar di bawah menunjukkan pipa bundar horizontal dengan radius R, dimana mengalir aliran laminar.
- Tegangan geser *(shear stress)* yang terjadi: $F = \tau$ x 2πr x dx
- p = intensitas tekanan pada bagian kiri dan intensitas tekanan pada bagian kanan $\left(p + \frac{\partial p}{\partial x}.dx\right)$
- Gaya yang bekerja pada elemen fluida:
    1. Gaya geser, τ x 2πr x dx pada permukaan elem fluida.
    2. Gaya tekanan, $p\ x\ \pi$r^2 pada ujung sisi kiri.
    3. Gaya tekanan, $\left(p + \frac{\partial p}{\partial x}.dx\right)$ πr^2 pada sisi kanan.



- Untuk aliran steady, net force pada silinder harus nol.

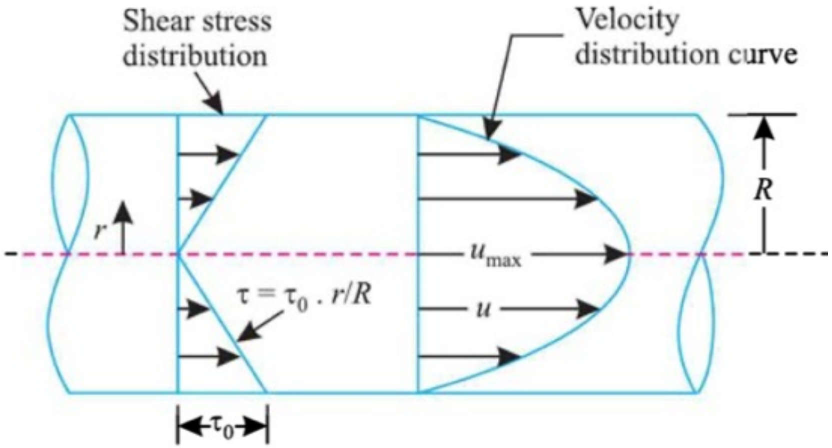
$$\left[p \times \pi r^2 - \left(p + \frac{\partial p}{\partial x} .dx \right) \pi r^2 \right] - \tau \times 2\pi r \times dx = 0$$

$$-\frac{\partial p}{\partial x} . dx \times \pi r^2 - \tau \times 2\pi r \times dx = 0$$

$$\tau = -\frac{\partial p}{\partial x} . \frac{r}{2}$$

- Persamaan di atas menunjukkan bahwa aliran akan terjadi hanya jika ada perbedaan tekanan dalam arah aliran dan tegangan geser secara linear sepanjang penampang diman nilainya nol pada sumbu pipa (r = 0) dan maksimum pada dinding pipa yang dinyatakan oleh:

$$\tau_0 = -\frac{\partial p}{\partial x} \left(\frac{R}{2} \right)$$



- Dari Hukum Newton viskositas:

$$\tau = \mu . \frac{du}{dy}$$

- Perbedaan tekanan antara dua sisi 1 dan 2 (lihat gambar) pada jarak x_1 dan x_2:

$$-\int_{p_1}^{p_2} \partial p = \frac{8\mu\,\bar{u}}{R^2} \int_{x_1}^{x_2} \partial x$$

$$(p_1 - p_2) = \frac{8\,\mu\,\bar{u}}{R^2}(x_2 - x_1) = \frac{8\mu\bar{u}\,L}{R^2} = \frac{32\,\mu\,\bar{u}\,L}{D^2} = \frac{128\,\mu\,Q\,L}{\pi D^4}$$

$$\frac{p_1 - p_2}{w}\,(= h_L) = \frac{128\,\mu\,Q\,L}{w\pi D^4}$$

- Distribusi temperatur:

$$t = \frac{1}{\alpha}\frac{\partial t}{\partial x}u_{max}\left(\frac{r^2}{4} - \frac{r^4}{16R^2}\right) + \left[t_s - \frac{1}{\alpha}\frac{\partial t}{\partial r}u_{max}\frac{3R^2}{16}\right]$$

$$t_s - t = \frac{u_{max}}{\alpha}.\frac{\partial t}{\partial x}\left[\frac{3R^2}{16} - \frac{r^2}{4} + \frac{r^4}{16R^2}\right]$$

- Koefisien perpindahan panas, h:

$$h = \frac{k \times \frac{u_{max}R}{4\alpha} \cdot \frac{\partial t}{\partial x}}{\frac{11}{96}\frac{u_{max}}{\alpha} \cdot R^2 \cdot \frac{\partial t}{\partial x}} = \frac{24k}{11R} = \frac{48k}{11D}$$

- Bilangan Nusselt: $Nu = \dfrac{hD}{k} = 3.65$

Contoh 8:

Minyak pelumas pada temperatur 60 °C mengalir masuk pipa yang berdiameter 1 cm dan kecepatan aliran 3 m/s. Temperatur permukaan pipa 40 °C. Diasumsikan minyak mempunyai sifat rata-rata sebagai berikut, hitunglah panjang pipa yang diperlukan untuk mendinginkan minyak sampai 45 °C.

$\rho = 865\ kg/m^3;\ k = 0.14\ W/m\ K;\ c_p = 1.78\ kJ/kg°C.$

Asumsi bahwa aliran laminar (fully developed): $\overline{Nu} = 3{,}657$

Penyelesaian:

Length required, L :

$$Q = m\,c_p\,(t_i - t_0)$$

$$= (\rho A_f U)\,c_p\,(t_i - t_0)$$

(where U = average velocity, A_f = flow area)

$$= \left(\rho \frac{\pi}{4} D^2 U\right) c_p\,(t_i - t_0)$$

$$= (865 \times \frac{\pi}{4} \times 0.01^2 \times 3) \times 1.78 \times 10^3\,(60 - 45) = 5441.7\ \text{W}$$

Also, $Q = \bar{h}\,A\theta_m$

where, A = heat transfer area $= \pi DL$, and

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} = \frac{(60-40)-(45-40)}{ln\left[\frac{(60-40)}{(45-40)}\right]} = \frac{15}{1.386} = 10.82°C$$

$$\overline{Nu} = \frac{\bar{h}\,D}{k} = 3.657 \qquad \text{...(Given)}$$

$$\bar{h} = \frac{3.657\,k}{D} = \frac{3.657 \times 0.140}{0.01} = 51.2\ \text{W/m}^2\text{K}$$

Now, $Q = 5441.7 = 51.2 \times \pi DL \times 10.82$

$$\therefore \quad L = \frac{5441.7}{51.2 \times \pi \times 0.01 \times 10.82} = \mathbf{312.7\ m} \quad \textbf{(Ans.)}$$

Contoh 9:

0,5 kg air mengalir melalui pipa yang berdiameter 20 mm, yang dipanaskan dari 20 °C sampai 50 °C, Pemanasan dilakukan dengan uap kondensasi pada permukaan pipa dimana temperature permukaan pipa dipertahankan 85 °C. Hitunglah panjang pipa yang diperlukan untuk aliran berkembang penuh.

Sifat termofisik air pada 60 °C:

$$\rho = 983.2\ kg/m^3,\ c_p = 4.178\ kJ/kgK,\ k = 0.659\ W/m°C,\ \nu = 0.478 \times 10^{-6}\ m^2/s$$

Penyelesaian: Diketahui: $m = 0,5\ kg/min,\ D = 20\ mm = 0,02\ m,\ t_i = 20\ °C,\ t_o = 50\ °C.$

Panjang pipa yang diperlukan agar aliran berkembang penuh:

$$m = \rho A\bar{u} = 983.2 \times \frac{\pi}{4} \times (0.02)^2 \times \bar{u} = \frac{0.5}{60}\ (\text{kg/s})$$

$$\bar{u} = \frac{0.5}{60} \times \frac{4}{\pi} \times \frac{1}{983.2 \times (0.02)^2} = 0.0269\ \text{m/s}$$

$$Re = \frac{D.\bar{u}}{\nu} = \frac{0.02 \times 0.0269}{0.478 \times 10^{-6}} = 1125.5$$

(Re < 2000, aliran laminar)

$$Nu = \frac{hD}{k} = 3.65$$

$$h = \frac{3.65\,k}{D} = \frac{3.65 \times 0.659}{0.02} = 120.26\ \text{W/m}^2\ °\text{C}$$

$$Q = A_s h\,(t_s - t_\infty) = m\,c_p\,(t_o - t_i)$$

$$t_\infty = \frac{20 + 50}{2} = 35°\text{C} = t_b$$

$$(\pi \times 0.02 \times L) \times 120.26 \times (85 - 35) = \frac{0.5}{60} \times (4.178 \times 10^3)(50 - 20)$$

$$377.8\,L = 1044.5$$

$$L = \frac{1044.5}{377.5} = \mathbf{2.76\,m\ (Ans.)}$$

ALIRAN TURBULEN PADA PLAT DATAR

Contoh 10:

Suatu plat datar panjang 5 m dan lebar 0,75 m ditempatkan paralel dengan arah aliran yang mengalir dengan kecepatan 5 m/s. Jika koefisien hambat rata-rata *(average drag coefficient)* untuk aliran turbulen melalui plat datar dinyatakan sebagai: $\overline{C}_f = \dfrac{0.455}{(\log_{10} Re_L)^{2.58}}$

Hitunglah gaya hambat *(drag force)* pada kedua sisi plat. Diambil v = 0,011 x 10^{-4} m^2/s.

$$Re_L = \frac{UL}{v} = \frac{5 \times 5}{0.011 \times 10^{-4}} = 22.73 \times 10^6$$

Re > 5 x 10^5 maka aliran turbulen

$$\overline{C}_f = \frac{0.455}{[\log_{10} Re_L]^{2.58}}$$

$$= \frac{0.455}{(\log_{10} 22.73 \times 10^6)^{2.58}} = 2.642 \times 10^{-3}$$

$$F_D = 2 \times \overline{C}_f \times \left(\frac{1}{2} \rho A U^2 \right)$$

$$= 2 \times 2.642 \times 10^{-3} \times \left[\frac{1}{2} \times 1000 \times (5 \times 0.75) \times 5^2 \right] = \mathbf{247.68\ N}$$

Contoh 11:

Udara 20 °C mengalir dengan kecepatan 10 m/s paralel terhadap dinding yang lebarnya 5 m dan tinggi 3 m. Hitunglah laju perpindahan panas yang terjadi jika temperatur dinding 40 °C. Sifat-sifat udara pada temperatur film rata-rata: k = 0,0263 W/mK. ν = 125,89 x 10^{-6} m^2/s dan Pr = 0,707. Jika lapisan batas adalah turbulen, berapakan prosentase error laju perpindahan panas?

Korelasi berikut dapat digunakan:

$$\overline{Nu} = 0.664\ Re_L^{0.5}\ Pr^{1/3}$$

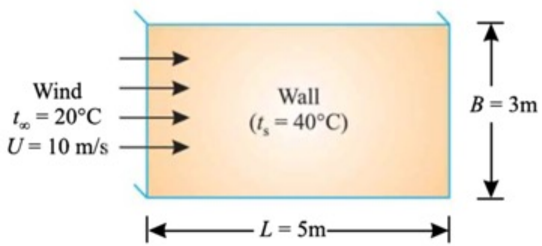
$$\overline{Nu} = 0.0375\ Re_L^{0.8}\ Pr^{1/3}$$

$$\overline{Nu} = 0.0375\ [Re_L^{0.8} - 23200]\ Pr^{1/3}$$

Penyelesaian:

$t_\infty = 20°C;\ U = 10\ m/s;\ L = 5\ m;\ B = 3\ m;\ t_s = 40°C;\ (Re)_{cr} = 5 \times 10^5$

$k = 0.0263\ W/m\ K;\ \nu = 15.89 \times 10^{-6}\ m^2/s;\ Pr = 0.707.$



Percentage error in computation of heat transfer rate :

$$Re_L = \frac{UL}{\nu} = \frac{10 \times 5}{15.89 \times 10^{-6}} = 3.1466 \times 10^6$$

i.e., $> (Re)_{cr}$

$$\overline{Nu} = \frac{\bar{h}L}{k} = 0.0375\left[Re_L^{0.8} - 23200\right] Pr^{1/3}$$

...(*Combination of laminar and turbulent flow*)

$$\bar{h} = \frac{k}{L} \times 0.0375\left[Re_L^{0.8} - 23200\right] Pr^{1/3}$$

$$= \frac{0.0263}{5} \times 0.0375\ [(3.1466 \times 10^6)^{0.8} - 23200] \times (0.707)^{1/3}$$

$$= 0.00019725\ (157860.4 - 23200) \times 0.8908 = 23.66\ W/m^2\ °C$$

$$Q = \bar{h}A\ (t_s - t_\infty) = 23.66 \times (5 \times 3) \times (40 - 20) = 7098\ W$$

If *entire boundary is assumed turbulent,*

$$\overline{Nu} = \frac{\bar{h}L}{k} = 0.0375\ Re_L^{0.8}\ Pr^{1/3}$$

$$\therefore \qquad \bar{h} = \frac{k}{L} \times 0.0375\ Re_L^{0.8}\ Pr^{1/3}$$

$$= \frac{0.0263}{5} \times 0.0375 \times (3.1466 \times 10^6)^{0.8} \times (0.707)^{1/3}$$

$$= 0.00019725 \times 157860.4 \times 0.8908 = 27.74\ \text{W/m}^2\,{}^\circ\text{C}$$

$$Q = 27.74 \times (5 \times 3) \times 20 = 8322\ \text{W}$$

$\therefore$ **Percentage error** $= \dfrac{8322 - 7098}{7098} \times 100$ **= 17.24%** **(Ans.)**

Contoh 12:

Suatu crankcase dari suatu mesin pembakaran dalam *(internal combustion engine)* pada suatu kendaraan yang diasumsikan berupa plat datar berukuran 80 cm x 20 cm. Kendaraan tersebut bergerak dengan kecepatan 90 km/jam dan *crankcase* didinginkan oleh aliran udara mengalir melewatinya pada kecepatan yang sama. Hitunglah kerugian panas *(heat loss)* dari permukaan *crank* yang bertemperatur 85 °C terhadap udara sekitar yang bertemperatur 15 °C. *Boundary layer* berupa turbulen.

Solution. *Given :* $U = 90 \text{ km/h} = \dfrac{90 \times 1000}{3600} = 25 \text{ m/s}$; $t_s = 85°\text{C}$; $t_\infty = 15°\text{C}$; $L = 80 \text{ cm} = 0.8 \text{ m}$; $B = 20 \text{ cm} = 0.2\text{m}$.

The properties of air at $t_f = \dfrac{85 + 15}{2} = 50°\text{C}$ are :

$k = 0.02824 \text{ W/m°C}$, $\nu = 17.95 \times 10^{-6} \text{ m}^2\text{/s}$, $Pr = 0.698$ ... (From tables)

Heat loss from the crankcase, *Q* :

The Reynolds number, $Re_L = \dfrac{UL}{\nu} = \dfrac{25 \times 0.8}{17.95 \times 10^{-6}} = 1.114 \times 10^6$

Since $Re_L > 5 \times 10^5$, the nature of flow is ***turbulent***.

For turbulent boundary layer,

$$\overline{Nu} = \frac{\bar{h}L}{k} = 0.036\,(Re_L)^{0.8}\,(Pr)^{0.333} = 0.036\,(1.114 \times 10^6)^{0.8}\,(0.698)^{0.333} = 2196.92$$

or, $$\bar{h} = \frac{k}{L} \times 2196.92 = \frac{0.02824}{0.8} \times 2196.92 = 77.55 \text{ W/m}^2\text{°C}$$

$$Q = \bar{h}A\,(t_s - t_\infty) = 77.55 \times (0.8 \times 0.2)\,(85 - 15) = \mathbf{868.56\ W}$$

Contoh 13:

Udara pada 20 °C dan 1,013 bar mengalir di atas suatu plat datar pada kecepatan 40 m/s. Panjang plat 1 m, lebar 1 m dan temperaturnya 60 °C. Hitunglah laju perpindahan panas dari plat. Gunakan korelasi sebagai berikut:

$$Nu_L = (Pr)^{0.33}\,[0.037\,(Re_L)^{0.8} - 850]$$

Solution. *Given :* $t_\infty = 20°C$; $U = 40$ m/s; $L = 1$m; $B = 1$ m; $t_s = 60°C$,

Properties of air at (60 + 20)/2 = 40°C, from the tables:

$\rho = 1.128$ kg/m³; $c_p = 1.005$ kJ/kg°C; $k = 0.0275$ W/m°C; $\nu = 16.96 \times 10^{-6}$ m²/s; $Pr = 0.699$.

Heat transfer from the plate, Q :

Reynolds number,
$$Re_L = \frac{UL}{\nu} = \frac{40 \times 1}{16.96 \times 10^{-6}} = 2.36 \times 10^6$$

∴
$$Nu_L = \frac{\bar{h}L}{k} = (0.699)^{0.33}\,[0.037\,(2.36 \times 10^6)^{0.8} - 850] = 3365.6$$

or,
$$\bar{h} = \frac{0.0275 \times 3365.6}{1} = 92.55 \text{ W/m}^2\,°\text{C}$$

∴
$$Q = \bar{h}\,A_s\,(t_s - t_\infty) = 92.55 \times (1 \times 1)\,(60 - 20)$$
$$= 3702 \text{ W} \quad \text{or,} \quad \mathbf{3.702\ kW} \qquad \textbf{(Ans.)}$$

Contoh 14:

Pada suatu sistem turbin gas, gas panas 950 °C mengalir dengan kecepatan 70 m/s di atas permukaan combustion chamber dimana temperatur 280 °C. Hitunglah kerugian panas *(heat loss)* dari gas ke *combustion chamber* dimana diidealisasikan berupa plat datar yang berukuran 120 cm x 80 cm. Aliran paralel sampai sisi 120 cm dan bilangan Reynolds transisi 5 x 10^5.

Sifat-sifat gas: $\rho = 0.494\ kg/m^3,\ k = 0.075\ W/m°C,\ \nu = 95 \times 10^{-6}\ m^2/s,\ Pr = 0.625.$

Solution. *Given:* $L = 120\ \text{cm} = 1.20\ \text{m},\ B = 80\ \text{cm} = 0.8\ \text{m},\ Re_c = 5 \times 10^5,\ U = 70\ \text{m/s}.$

Heat loss from the gases to the combustion chamber :

The average Nusselt number for a flow over a flat plate when both laminar and turbulent boundary layers are present is given by

$$\overline{Nu} = \left[0.664\,(Re_L)^{0.5} + 0.036\left\{(Re_L)^{0.8} - (Re_c)^{0.8}\right\}\right](Pr)^{0.333}$$

If transition occurs at $Re_c = 5 \times 10^5$, then the above expression reduces to

$$\overline{Nu} = [0.036\,(Re_L)^{0.8} - 836]\,(Pr)^{0.333}$$

The Reynolds number at the end of the plate is

$$Re_L = \frac{UL}{\nu} = \frac{70 \times 1.2}{95 \times 10^{-6}} = 8.84 \times 10^5$$

This shows that both laminar and turbulent boundary layers are present and,

Hence, $\overline{Nu} = [0.036\,(8.84 \times 10^5)^{0.8} - 836]\,(0.625)^{0.333} = 1045$

or, $\dfrac{\bar{h}\,L}{k} = 1045$ or $\bar{h} = \dfrac{k}{L} \times 1045$

or, $\bar{h} = \dfrac{0.075}{1.2} \times 1045 = 65.31\ \text{W/m}^2\,°\text{C}$

Heat loss, $Q = \bar{h} A\,\Delta t$

$= 65.31 \times (1.2 \times 0.8) \times (950 - 280) = 42007.4\ \text{W} \simeq$ **42 kW** **(Ans.)**

Contoh 15:

Udara yang bertemperatur 20 °C dan 1,013 bar mengalir dengan kecepatan 35 km/jam di atas kontener persegi dimana bagian permukaan atas panjang 750 mm dalam arah aliran dan lebar 1 m. Hitunglah perpindahan panas dari permukaan atas yang bertemperatur 60 °C. Gunakan sifat-sifat udara pada temperature 40 °C:

$\mu = 1.906 \times 10^{-5}$ *kg/ms*, $c_p = 1.007$ *kJ/kg°C and* $k = 0.0272$ *W/m°C,*

and the following co-relations for finding average heat transfer coefficient :

$$\overline{Nu} = 0.664\,(Re_L)^{0.5}\,(Pr)^{0.33} \text{ if } Re_L \le 5 \times 10^5$$

$$\overline{Nu} = [0.037\,(Re_L)^{0.5} - 850]\,(Pr)^{0.33} \text{ if } Re_L > 5 \times 10^5$$

Solution. *Given :* $p = 1.013$ bar, $L = 750$ mm $= 0.75$ m, $B = 1$ m, $U = 35$ m/s, Mean film temp.

$t_s = 60°C,\ t_\infty = 20°C,\ t_f = \left(\frac{60+20}{2}\right) = 40°C$

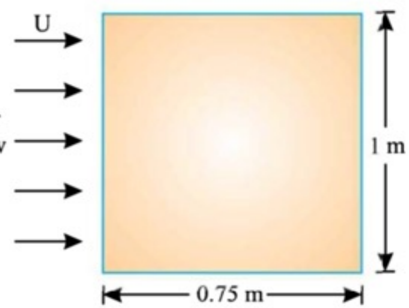


Fig. 7.21.

Heat transfer from the top surface, Q :

$$Pr = \frac{\mu c_p}{k} = \frac{1.906 \times 10^{-5} \times (1.007 \times 10^3)}{0.0272} = 0.706$$

Using the gas equation for air, we have

$$p = \rho RT \quad \text{or,} \quad \rho = \frac{p}{RT}$$

or, $$\rho = \frac{1.013 \times 10^5}{287 \times (20 + 273)} = 1.2 \text{ kg/m}^3$$

$$Re_L = \frac{\rho UL}{\mu} = \frac{1.2 \times 35 \times 0.75}{1.906 \times 10^{-5}} = 16.53 \times 10^5$$

Since $Re > 5 \times 10^5$, we shall use eqn. (*ii*) for finding average heat transfer coefficient.

$$\therefore \quad \overline{Nu} = \frac{\bar{h}L}{k} = [0.037\,(Re_L)^{0.8} - 850]\,(Pr)^{0.33}$$

$$\bar{h} = \frac{k}{L} = [0.037\,(Re_L)^{0.8} - 850]\,(Pr)^{0.33}$$

$$= \frac{0.0272}{0.75}[0.037\,(16.53 \times 10^5)^{0.8} - 850] \times (0.706)^{0.33}$$

$$= 0.03627\,(3489.96 - 850) \times 0.8915 = 85.36 \text{ W/m}^2\text{°C}$$

$$Q = \bar{h}A_s\,(t_s - t_\infty)$$

$$= 85.36 \times (0.75 \times 1)\,(60 - 20) = \mathbf{2560.8\ W} \quad \textbf{(Ans.)}$$

Contoh 16:

Suatu plat lebar 1 m dan panjang 1,5 m dengan temperatur dipertahankan pada 90 °C dimana temperatur aliran bebas 10 °C. Hitunglah kecepatan dimana udara haru mengalir di atas plat tersebut sehingga besarnya disipasi energi dari plat 3,75 kW.

Use : $\bar{N}u = \frac{\bar{h}L}{k} = 0.664\ (Re_L)^{1/2}\ (Pr)^{1/3}$ ... *for laminar flow*

$\bar{N}u = \frac{\bar{h}L}{k} = [0.036\ (Re_L)^{0.8} - 836]\ (Pr)^{1/3}$ ... *for turbulent flow*

Take the following air properties at 50°C :

$\rho = 1.0877\ kg/m^3$, $k = 0.02813\ W/m°C$, $c_p = 1007.3\ J/kg°C$, $\mu = 2.029 \times 10^{-5}\ kg/ms$ and $Pr = 0.703$.

(P.U)

Solution. *Given :* $L = 1.5$ m, $B = 1$ m, $t_s = 90°C$, $t_\infty = 10°C$, $Q = 3.75$ kW,

Mean film temperature, $t_f = (90 + 10)/2 = 50°C$

Velocity of flow, *U* :

$$Q = \bar{h}A_s\ (t_s - t_\infty) = \bar{h}\ (L \times B)\ (t_s - t_\infty)$$

$$3.75 \times 10^3 = \bar{h} \times (1.5 \times 1) \times (90 - 10)$$

or,

$$\bar{h} = \frac{3.75 \times 10^3}{(1.5 \times 1) \times (90 - 10)} = 31.25\ W/m^2\,°C$$

The $\bar{h}$ value indicates that the flow must be ***turbulent***. Considering the flow to be parallel to the length of the plate, we have

$$\bar{N}u = \frac{\bar{h}L}{k} = [0.036\,(Re_L)^{0.8} - 836]\,(Pr)^{1/3}$$

or, $$\frac{31.25 \times 1.5}{0.02813} = [0.036\,(Re_L)^{0.8} - 836]\,(0.703)^{0.333}$$

or, $$0.036\,(Re_L)^{0.8} = \frac{31.25 \times 1.5}{0.02813} \times \frac{1}{(0.703)^{0.333}} + 836 = 2709.8$$

or, $$Re_L = \left(\frac{2709.8}{0.036}\right)^{1/0.8} = 1246790$$

$$Re_L = \frac{\rho U L}{\mu} = 1246790$$

or, $$U = \frac{1246790 \times \mu}{\rho L} = \frac{1246790 \times 2.029 \times 10^{-5}}{1.0877 \times 1.5}$$

= 15.5 m/s (Ans.)

Contoh 17:

Suatu plat persegi mempunyai temperatur 95 °C mengalami gaya 10,5 N pada saat udara bertekanan pada 25 °C mengalir diatasnya pada kecepatan 30 m/s. Asumsi aliran turbulen dan gunakan analogi Colburn, hitunglah:

(i) Koefisien heat transfer.

(ii) Kerugian panas dari permukaan plat.

Properties of air are :

$\rho = 1.06\ kg/m^3$, $c_p = 1.005\ kJ/kg\ K$, $\nu = 18.97 \times 10^{-6}\ m^2/s$, $P_r = 0.696$.

Solution. *Given :* $F_D = 10.5$ N, $t_s = 95°C$, $t_\infty = 25°C$, U = 30 m/s

***(i)* The heat transfer coefficient, $\bar{h}$:**

For turbulent flow, the drag force is given by

$$F = \bar{C}_f \times \frac{1}{2} \rho A U^2$$

or,

$$10.5 = \frac{0.072}{(Re_L)^{0.2}} \times \frac{1}{2} \times 1.06 \times (L \times L) \times (30)^2$$

$$= 0.072 \left(\frac{\nu}{UL}\right)^{0.2} \times \frac{1}{2} \times 1.06 \times L^2 \times 900$$

$$= 0.072 \left(\frac{18.97 \times 10^{-6}}{25 \times L}\right)^{0.2} \times \frac{1}{2} \times 1.06 \times L^2 \times 900$$

$$= 1.424 \times \frac{L^2}{(L)^{0.2}} = 2.05\ (L)^{1.8}$$

or,

$$L = \left(\frac{10.5}{2.05}\right)^{1/1.8} = 2.478 \text{ m}$$

The Reynolds number at the end of the plate,

$$Re_L = \frac{UL}{\nu} = \frac{30 \times 2.478}{18.97 \times 10^{-6}} = 3.919 \times 10^6$$

Average skin friction coefficient; $\bar{C}_f = \dfrac{0.072}{(Re_L)^{0.2}}$

$$= \frac{0.072}{(3.919 \times 10^6)^{0.2}} = 3.457 \times 10^{-3}$$

From Colburn analogy, we have

$$\bar{St}\ (Pr)^{2/3} = \frac{\bar{Cf}}{2}$$

or,

$$\frac{\bar{h}}{\rho\ c_p U} (Pr)^{2/3} = \frac{\bar{C}_f}{2}$$

or,

$$\bar{h} = \frac{\rho\ c_p U}{(Pr)^{2/3}} \times \frac{\bar{C}_f}{2}$$

or,

$$\bar{h} = \frac{1.06 \times (1.005 \times 10^3) \times 30}{(0.696)^{0.666}} \times \left(\frac{3.457 \times 10^{-3}}{2}\right)$$

$$= \mathbf{70.32\ W/m^{2\circ}C} \quad \textbf{(Ans.)}$$

***(ii)* Heat loss from the plate surface, Q :**

$$Q = \bar{h}\ A\ \Delta t = 70.32 \times (2.478 \times 2.478) \times (95 - 25) = 30226 \text{ W}$$

$$= \mathbf{30.226\ kW} \quad \textbf{(Ans.)}$$

ALIRAN TURBULEN DALAM PIPA

Beberapa hubungan penting untuk aliran turbulen yang berkembang penuh dalam pipa:

(*i*) *The velocity distribution :* $\dfrac{u}{u_{max}} = \left(\dfrac{y}{R}\right)^{1/n}$

where, u = Local average velocity,

u_{max} = Velocity at centre line,

R = Radius of the pipe, and

y = Distance from the wall = $(R - r)$.

(*ii*) *The head loss :* $h_L = \dfrac{dp}{\rho} = \dfrac{f L \bar{u}^2}{2g}$

(where, f = friction factor, and $\bar{u}$ = average flow velocity)

The *friction factor* for turbulent flow, is well represented by the following empirical relations :

$f = 0.316\ (Re)^{-0.25}$ for $2 \times 10^4 < Re < 8 \times 10^4$

$f = 0.184\ (Re)^{-0.2}$ for $10^4 < Re < 10^5$ ...(7.147)

$f = 0.005 + 0.396\ (Re)^{0.3}$ for $2 \times 10^4 < Re < 2 \times 10^6$ ..(7.148) (*iii*) *The wall shear stress*, τ_w :

$$\tau_w = \frac{f}{8} \rho u_{max}^2$$

(*iv*) *From Colburn analogy* ($0.5 < P_r < 100$)

$$\bar{St}\,(Pr)^{2/3} = \frac{f}{8}$$

Substituting the value of f from eqn. (7.147) in eqn. (7.150), we get following equations for heat transfer coefficients :

$$\bar{St}\,(Pr)^{2/3} = \frac{0.184}{8}\,(Re)^{-0.2}$$

$$\frac{\bar{Nu}}{Pr\ Re}\,(Pr)^{2/3} = \frac{0.184}{8}\,(Re)^{-0.2}$$

or, $$\overline{Nu} = 0.023\,(Re)^{0.8}\,(Pr)^{1/3}$$

and, $$\bar{h} = \overline{Nu} \times \frac{k}{D} = 0.023\,\frac{k}{D}\,(Re)^{0.8}\,(Pr)^{1/3}$$

The above expressions are valid for

$$1 \times 10^4 < Re < 1 \times 10^5;\ 0.5 < Pr < 100;\ \frac{L}{D} > 60.$$

The properties of fluid are evaluated *at film temperature*.

Contoh 18:

Suatu pipa panjang 5 m, mempunyai temperatur 100 °C yang dipertahankan dengan uap disekitarnya. Fluida mengalir melalui pipa dengan massa lairan 2940 kg/h pada 30 °C. Diameter pipa 2 cm. Hitunglah koefisien perpindahan panas rata-rata.

Take the following properties of the fluid :

$\rho = 850$ kg/m^3, $c_p = 2000$ J/kg°C, $\nu = 5.1 \times 10^{-6}$ m^2/s and $k = 0.12$ W/m°C.

Solution. *Given :* $L = 5$ m, $D = 2$ cm $= 0.02$ m

Average heat transfer coefficient $\bar{h}$:

$$\overline{Nu} = \frac{\bar{h}D}{k} = 0.023\,(Re)^{0.8}\,(Pr)^{1/3} \text{ if } Re > 2300$$

where, $$Re = \frac{\rho U D}{\mu} = \frac{UD}{\nu}$$, where V is the average flow velocity.

The mass flow per second is given by

$$m = \frac{2940}{3600} = \frac{\pi}{4} D^2 U \rho = \frac{\pi}{4} \times (0.02)^2 \times 850 \times V$$

or, $$V = \frac{2940}{3600} \times \frac{4}{\pi \times (0.02)^2 \times 850} = 3.06 \text{ m/s}$$

$\therefore$ $$Re = \frac{3.06 \times 0.02}{5.1 \times 10^{-6}} = 1.2 \times 10^4$$

Since $Re > 2300$, the expression (i) holds good. Substituting the values, we get

$$\frac{\bar{h} \times 0.02}{0.12} = 0.023\,(1.2 \times 10^4)^{0.8} \times (72.3)^{0.333} \simeq 175.46$$

$$\left[\text{where } Pr = \frac{\mu\, c_p}{k} = \frac{\rho\nu\, c_p}{k} = \frac{850 \times 5.1 \times 10^{-6} \times 2000}{0.12} = 72.3\right]$$

$\therefore$ $$\bar{h} = \frac{175.46 \times 0.12}{0.02}$$

$= 1052.8$ W/m^2°C or **1.0528 kW/m²°C** **(Ans.)**

Contoh 19:

Air pada 25 ^oC mengalir dengan kecepatan 2 m/s melalui pipa tembaga horizontal yang mempunyai diameter luar 1,5 cm. Hitunglah laju perpindahan panas per satuan panjang jika temperatur dinding pipa 75 ^oC. Sifat sifat air:

$$\rho = 988 \ kg/m^3$$
$$k = 0.648 \ W/m \ K$$
$$\mu = 549.2 \times 10^{-6} \ N \ s/m^2$$
$$c_p = 4.174 \ kJ/kg \ K$$

$$\overline{N_{uD}} = 0.3 + \frac{0.62 \ R_{eD}^{1/2} \ Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{R_{eD}}{282000}\right)^{1/2}\right]$$

Solution. *Given:* $\Delta t = 75 - 25 = 50°C; D = 1.5 \text{ cm} = 0.015 \text{ m}, U = 2 \text{ m./s};$

$\rho = 988 \text{ kg/m}^3; k = 0.648 \text{ W/m K}; \mu = 549.2 \times 10^{-6} \text{ Ns/m}^2;$

$c_p = 4.174 \text{ kJ/kg K}.$

Heat transfer rate per unit length, $\frac{Q}{L}$:

Reynolds number, $Re = \frac{\rho U D}{\mu} = \frac{988 \times 2 \times 0.015}{549.2 \times 10^{-6}} = 53969.4$

Prandtl number, $Pr = \frac{\mu c_p}{k} = \frac{549.2 \times 10^{-6} \times 4.174 \times 10^3}{0.648} = 3.5376$

$$\bar{N}u = \frac{\bar{h}D}{k} = 0.3 + \frac{0.62 \ (Re)^{1/2} \ (Pr)^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re}{282000}\right)^{1/2}\right]$$

$$= 0.3 + \frac{0.62 \times (53969.4)^{1/2} \times (3.5376)^{1/3}}{\left[1 + \left(\frac{0.4}{3.5376}\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{53969.4}{282000}\right)^{1/2}\right]$$

$$= 0.3 + \frac{219.466}{1.054} \times 1.437 = 299.5$$

Terima Kasih