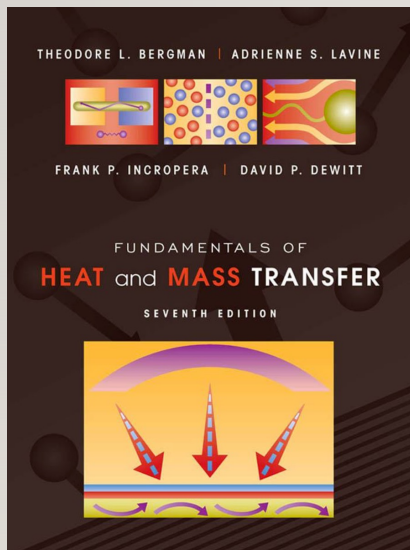# PERPINDAHAN KALOR KONVEKSI BEBAS

(FREE CONVECTION)
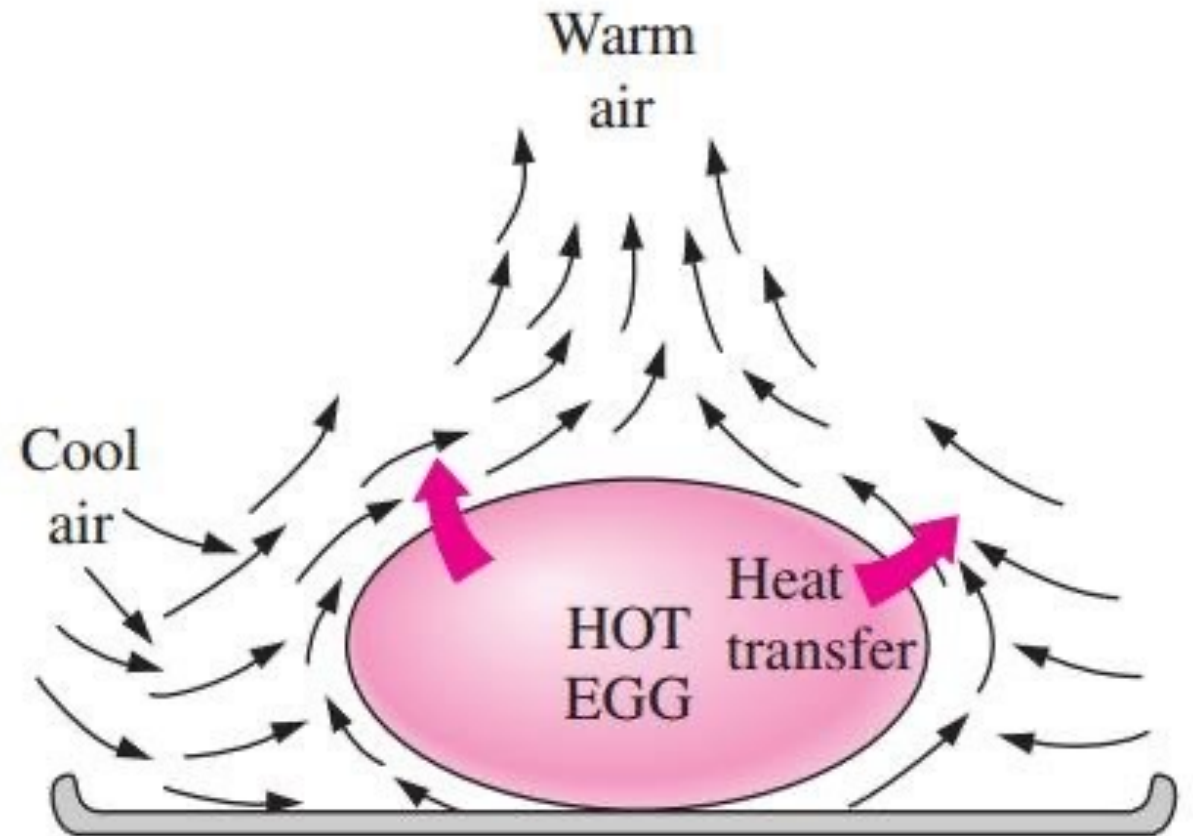
IR. J. VICTOR TUAPETEL, ST, MT, PHD, IPM, ASEAN ENG.

Pendahuluan

- Free Convection atau konveksi bebas/alami:
  - Tidak ada kecepatan fluida yang dipaksa.
  - Arus konveksi ada berasal dari *body force* yang berpengaruh terhadap gaya apung *(buoyancy force)* karena ada gradien densitas.

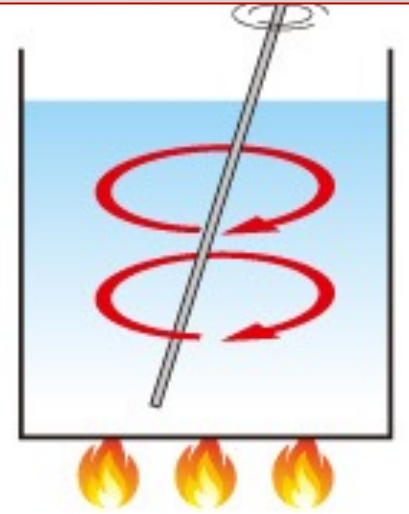- Gradien atau perbedaan densitas disebabkan oleh gradien suhu, dan *body force* yang disebabkan oleh medan gravitasi.

- Kecepatan aliran konveksi bebas lebih kecil dari konveksi paksa, yang sesuai juga lebih kecil.
- Konveksi bebas memberikan resistensi terbesar terhadap perpindahan kalor.
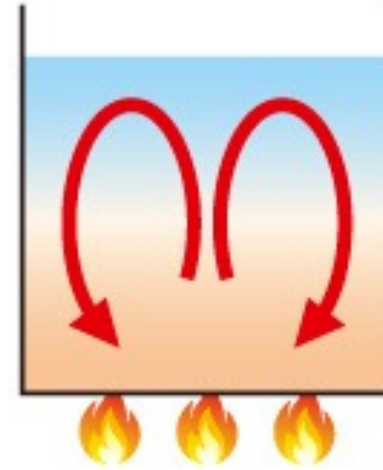
A flow is driven only by temperature difference

(a) Natural convection

A flow is driven by an external factor

(b) Forced convection

Physical Considerations

- Gaya apung disebabkan oleh adanya kombinasi gradien densitas fluida dan gaya benda atau *body force* yang sebanding dengan densitas.

- *Body force* biasanya gravitasi, atau mungkin gaya sentrifugal dari putaran mesin fluida.

- Ada juga beberapa cara dimana gradien densitas massa dapat muncul dalam fluida, tetapi umumnya disebabkan oleh adanya gradien suhu.

- Densitas gas dan cairan bergantung pada suhu, umumnya menurun (karena pemuaian fluida) dengan meningkatnya suhu $\left(\frac{\partial \rho}{\partial T} = 0 \right)$.

- Pembahasan mencakup masalah konveksi bebas dimana gradien densitas disebabkan oleh gradien suhu dan *body force* adalah gravitasi. Gradien densitas fluida dalam medan gravitasi tidak menjamin adanya arus konveksi bebas.
- Gambar 1. Fluida diapit oleh dua pelat horizontal besar yang temperaturnya berbeda $(T_1 \neq T_2)$.
- Densitas menurun ke arah gaya gravitasi. Jika perbedaan suhu melebihi nilai kritis, kondisi tidak stabil dan gaya apung mampu mengatasi pengaruh perlambatan gaya viskos.
- Gaya gravitasi pada fluida yang lebih padat di lapisan atas melebihi gaya gravitasi yang bekerja pada fluida yang lebih ringan di lapisan bawah, dan pola sirkulasi yang ditentukan akan ada.
- Cairan yang lebih berat akan turun, menjadi hangat dalam prosesnya, sedangkan cairan yang lebih ringan akan naik, mendingin saat bergerak. Namun, kondisi ini tidak mencirikan kasus *(b)*, di mana $T_1 > T_2$ dan densitasnya tidak lagi menurun ke arah gaya gravitasi.
- Perpindahan panas terjadi dari bawah ke permukaan atas dengan konveksi bebas; untuk kasus *(b)* perpindahan panas (dari atas ke bawah) terjadi secara konduksi.
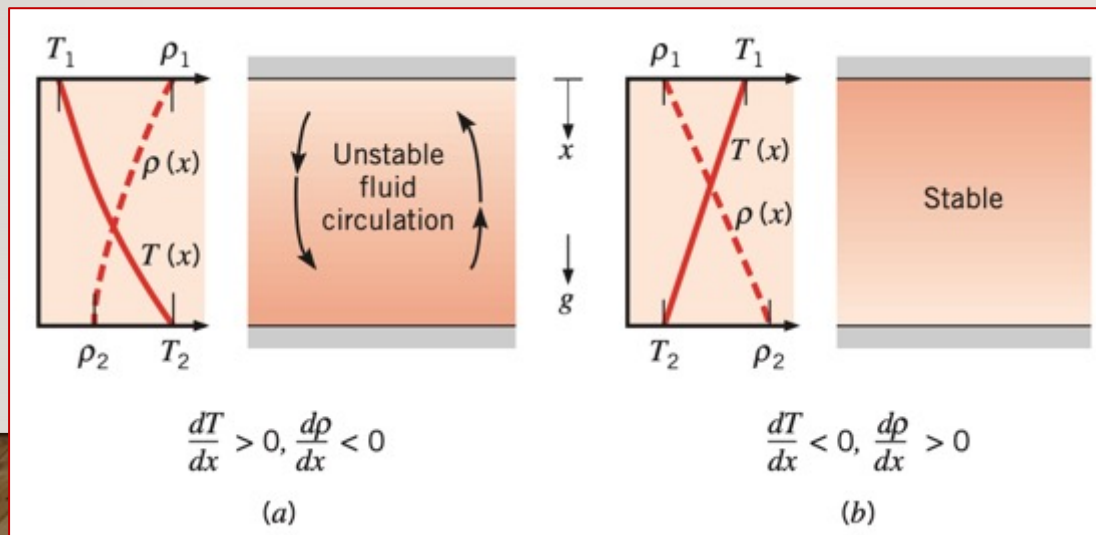

Gambar 1. Conditions in a fluid between large horizontal plates at different temperatures: (*a*) Unstable temperature gradient. (*b*) Stable temperature gradient.

- Aliran konveksi bebas dapat diklasifikasikan menurut apakah aliran dibatasi oleh suatu permukaan.

- Dengan tidak adanya permukaan yang berdampingan, batas bebas aliran dapat terjadi dalam bentuk *plume* atau jet apung (Gambar 2). Plume dikaitkan dengan cairan yang naik dari benda panas yang terendam.

- Pertimbangkan kawat yang dipanaskan pada Gambar 2a, yang direndam dalam cairan diam yang ekstensif. Cairan yang dipanaskan oleh kawat naik karena gaya apung, menarik cairan dari daerah diam. Meskipun lebar plume bertambah dengan jarak dari kawat, plume itu sendiri pada akhirnya akan menghilang sebagai akibat dari efek viskos dan pengurangan gaya apung yang disebabkan oleh pendinginan fluida di dalam plume.

- Perbedaan antara *plume* dan jet apung umumnya dibuat atas dasar kecepatan fluida awal. Kecepatan ini nol untuk *plume*, tetapi terbatas untuk jet apung.

- Gambar 2b menunjukkan fluida panas yang dikeluarkan sebagai pancaran horizontal ke dalam media diam dengan suhu lebih rendah. Gerakan vertikal yang mulai diasumsikan oleh jet disebabkan oleh gaya apung. Kondisi seperti itu terjadi ketika air hangat dari kondensor pembangkit listrik pusat dibuang ke reservoir air yang lebih dingin.
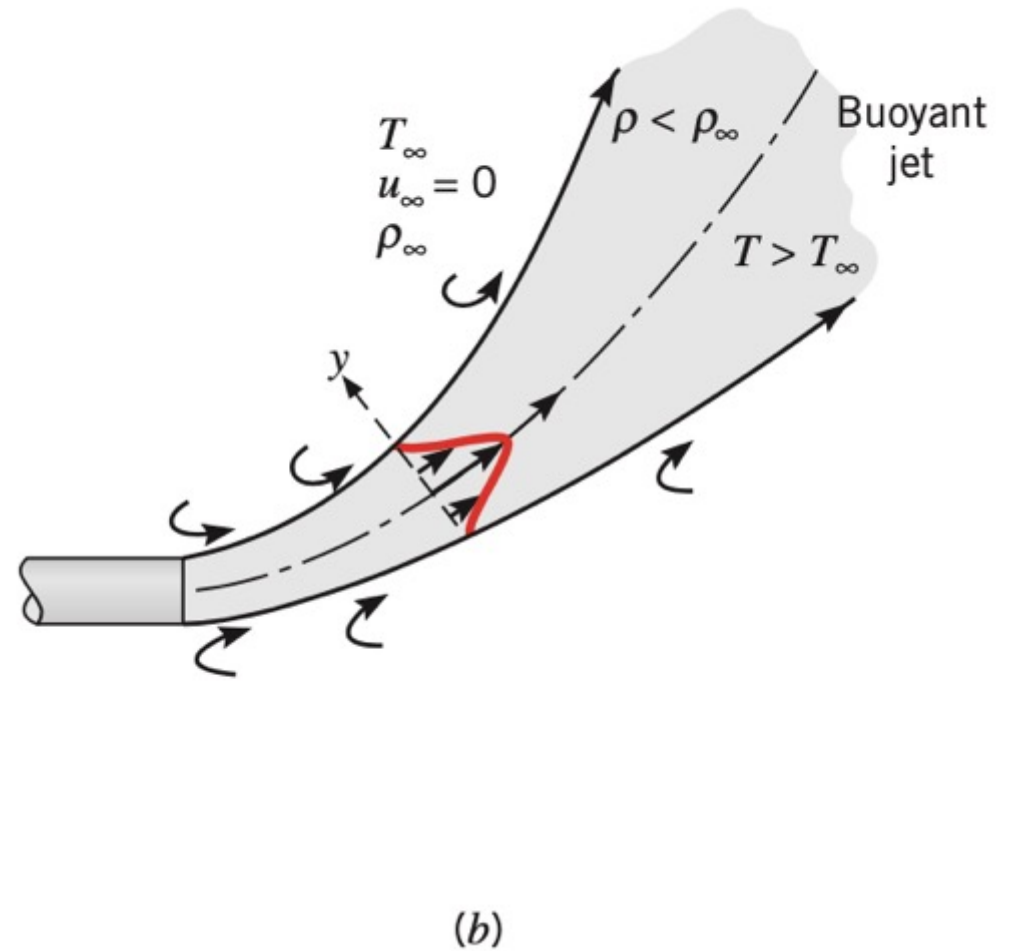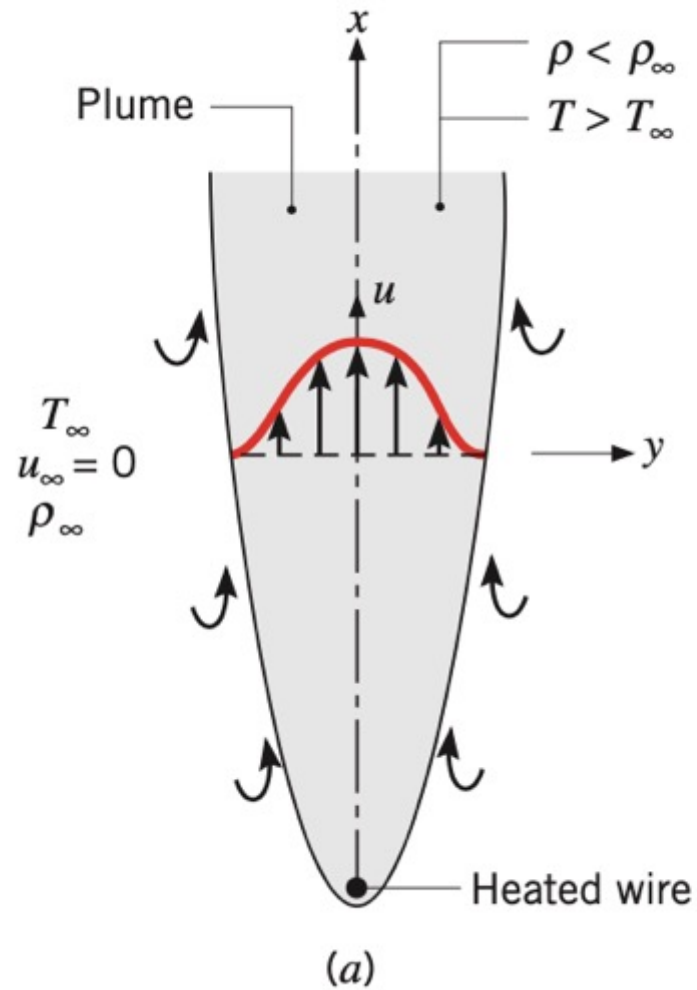
Gambar 2. Bouyancy–driven free boundary layer flow in an extensive quiescent medium
(*a*) Plume formation above a heated wire. (*b*) Buoyant jet associated with a heated discharge.

- Pembahasan fokus pada aliran konveksi bebas yang dibatasi oleh permukaan, dan contoh klasik berkaitan dengan pengembangan lapisan batas pada pelat vertikal yang dipanaskan (Gambar 3).

- Pelat direndam dalam cairan diam yang ekstensif, dan dengan $(T_s > T_\infty)$ cairan yang dekat dengan pelat kurang padat daripada cairan yang dipindahkan lebih jauh. Oleh karena itu, gaya apung menginduksi lapisan batas konveksi bebas di mana fluida yang dipanaskan naik secara vertikal, menarik fluida dari daerah diam.

- Distribusi kecepatan yang dihasilkan tidak seperti yang terkait dengan lapisan batas konveksi paksa. Secara khusus, kecepatannya nol pada $y \rightarrow l$, dan juga pada $y = 0$.

- Lapisan batas konveksi bebas juga berkembang jika $(T_s < T_\infty)$. Dalam hal ini, bagaimanapun, gerakan fluida ke bawah.
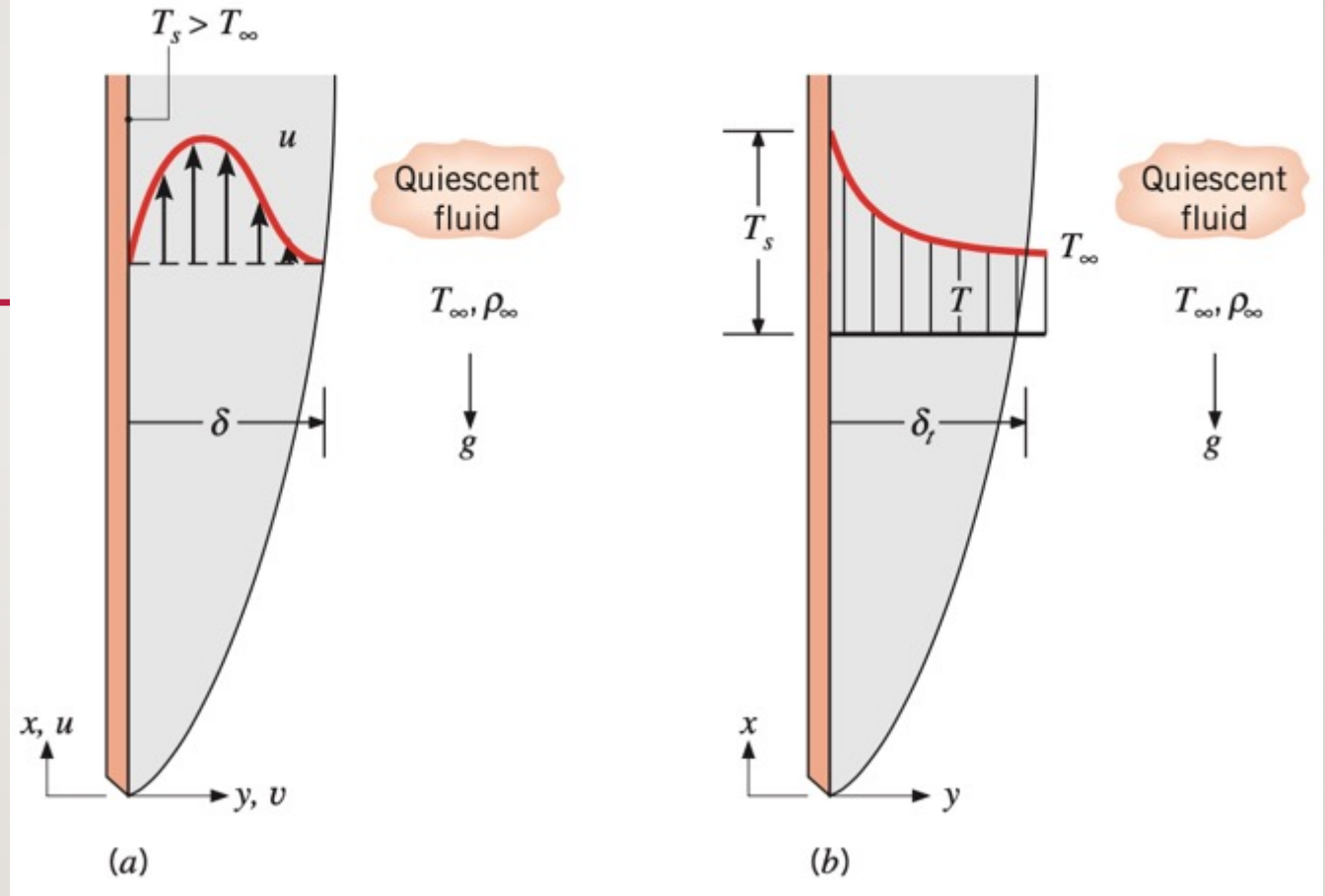
Gambar 3. Boundary layer development on a heated vertical plate: (a) Velocity boundary layer. (b) Thermal boundary layer.

- Dalam konveksi bebas, peran utama dimainkan oleh gaya apung.

- Gambar 3 memperlihatkan lapisan batas laminar yang didorong oleh gaya apung. Asumsikan kondisi sifat-sifat aliran konstan, dua dimensi, konstan di mana gaya gravitasi bekerja dalam arah x negatif. Juga, dengan satu pengecualian, anggap fluida tidak dapat dimampatkan. Pengecualian melibatkan penghitungan efek kepadatan variabel hanya dalam gaya apung, karena variasi inilah yang menginduksi gerakan fluida.

- Dengan penyederhanaan di atas, persamaan momentum x direduksi menjadi persamaan lapisan batas, kecuali bahwa suku *body force* x dipertahankan.

- Jika satu-satunya kontribusi untuk gaya ini akibat gravitasi, *body force* per satuan volume adalah $x = -\rho g$, dimana g adalah percepatan lokal karena gravitasi. Bentuk yang tepat dari persamaan *x-momentum* adalah:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp_\infty}{dx} - g + \nu\frac{\partial^2 u}{\partial y^2} \qquad \textbf{(1)}$$

  di mana dp_∞/dx adalah gradien tekanan aliran bebas di daerah diam di luar lapisan batas. Di daerah ini, $u = 0$ dan Persamaan (1) direduksi menjadi:

$$\frac{dp_\infty}{dx} = -\rho_\infty g \qquad \textbf{(2)}$$

- Substitusi Persamaan (2) ke (1) diperoleh:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g(\Delta\rho/\rho) + \nu\frac{\partial^2 u}{\partial y^2} \qquad \textbf{(3)}$$

- dimana $\Delta\rho = \rho_\infty - \rho$. Pernyataan ini harus berlaku di setiap titik di lapisan batas konveksi bebas.

- Suku pertama pada ruas kanan Persamaan (3) adalah gaya apung per satuan massa, dan aliran terjadi karena densitas ρ adalah suatu variabel. Jika variasi densitas hanya disebabkan oleh variasi suhu, hal ini berkaitan dengan sifat fluida yang dikenal sebagai koefisien ekspansi termal volumetric (*volumetric thermal expansion coefficient*):

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p \qquad (4)$$

- Sifat termodinamika fluida ini memberikan ukuran jumlah perubahan densitas sebagai respons terhadap perubahan suhu pada tekanan konstan. Jika dinyatakan dalam bentuk perkiraan berikut,

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T}$$

$$(\rho_\infty - \rho) \approx \rho\beta(T - T_\infty)$$

- Penyederhanaan ini dikenal sebagai aproksimasi Boussinesq, dan mensubstitusi ke Persamaan (3), persamaan *x*-momentum menjadi:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \qquad (5)$$

- Persamaan turunan adalah:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(6)}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \qquad \text{(7)}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad \text{(8)}$$

- Efek konveksi bebas jelas bergantung pada koefisien muai β dimana tergantung pada fluida. Untuk gas ideal, $\rho = p/RT$ dan

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p = \frac{1}{\rho}\frac{p}{RT^2} = \frac{1}{T} \qquad \text{(9)}$$

  dimana T adalah suhu mutlak. Untuk cairan dan gas nonideal, harus diperoleh dari tabel property.

- Grashof Number, Gr_L:

$$Gr_L \equiv \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

- Grashof Number digunakan untuk konveksi bebas menggantikan Reynold Number pada konveksi paksa.

- Korelasi heat transfer dari bentuk $Nu_L = f(Gr_L, Pr)$ dalam konveksi bebas.

- Bilangan Grashof adalah ukuran perbandingan gaya apung dengan gaya viskos yang bekerja pada fluida.

Persamaan semi-teoretis untuk konveksi alami menggunakan beberapa persamaan dari bilangan tak berdimensi berikut,

$$\mathrm{Gr} = \text{Grashof number} = \frac{L^3 g \beta \Delta T}{\nu^2} = \frac{L^3 \rho^3 g \beta \Delta T}{\mu^2}$$

$$\mathrm{Nu} = \text{Nusselt number} = hL/k$$

$$\mathrm{Ra} = \text{Rayleigh number} = (\mathrm{Gr})(\mathrm{Pr}) = \frac{L^3 g \beta \Delta T}{\nu \alpha} = \frac{L^3 g \beta \Delta T \rho^2 c_p}{\mu k}$$

$$\mathrm{Pr} = \text{Prandtl number} = \frac{\nu}{\alpha} = \frac{c_p \mu}{k}$$

where
ν = kinematic viscosity
μ = absolute viscosity
α = thermal diffusivity = $k/\rho c_p$
ρ = fluid density
c_p = fluid heat capacity
k = thermal conductivity
L = characteristic length of system
ΔT = temperature difference between the surface and the fluid = $|T_s - T_\infty|$

- Bilangan Rayleigh di atas digunakan untuk mengklasifikasikan konveksi alami sebagai laminar atau turbulen:

$$\mathrm{Ra} < 10^9 \quad \text{laminar free convection}$$

$$\mathrm{Ra} > 10^9 \quad \text{turbulent free convection}$$

Ukuran pengaruh setiap efek konveksi dinyatakan oleh rasio:

$$\frac{\mathrm{Gr}}{\mathrm{Re}^2} = \frac{\text{buoyancy force}}{\text{inertia force}} = \frac{\rho g \beta L (\Delta T)}{V^2}; \; V = \text{fluid velocity}$$

Laminar Free Convection on a Vertical Surface
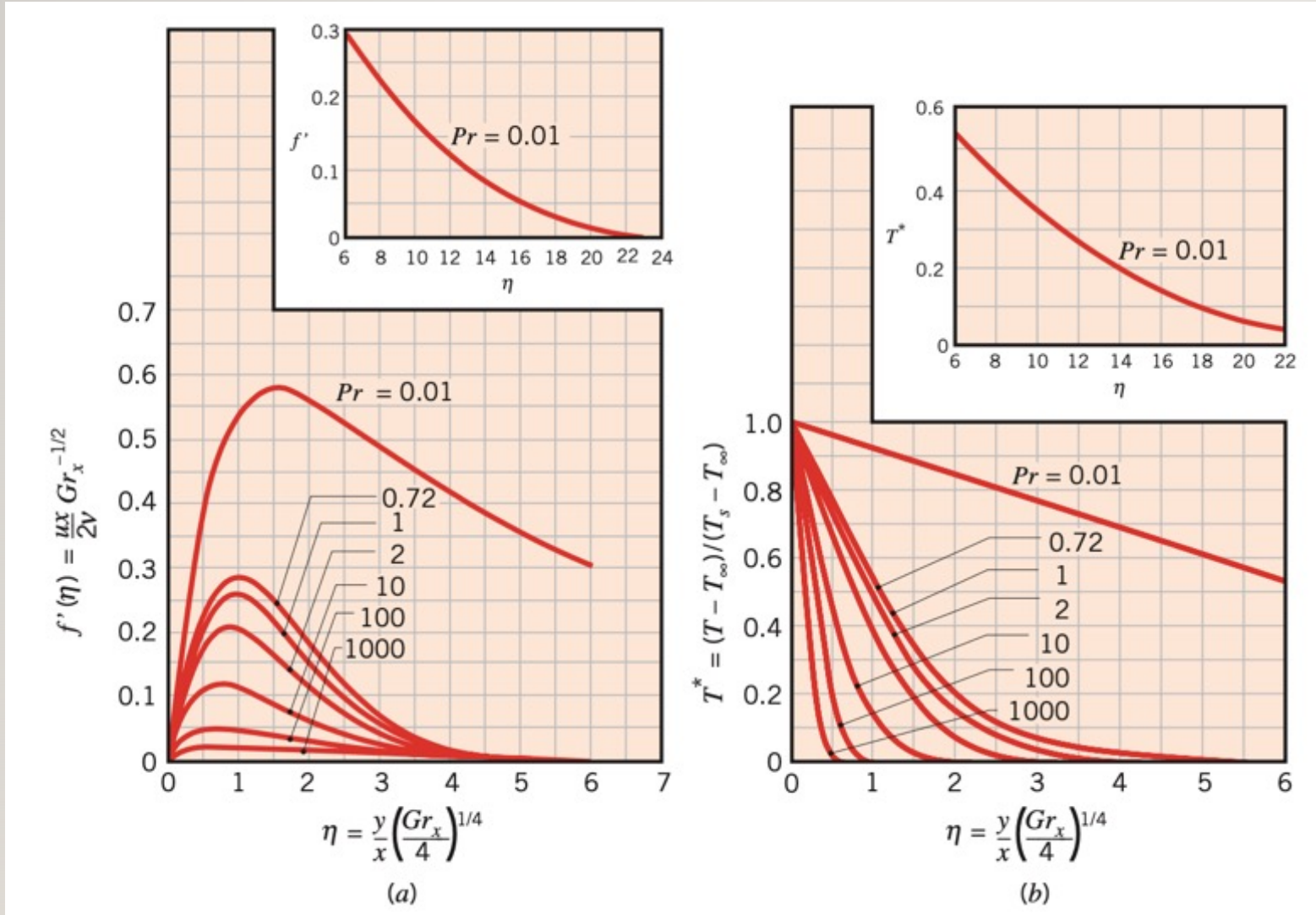
Gambar 4. Laminar, free convection boundary layer conditions on an isothermal, vertical surface. (*a*) Velocity profiles. (*b*) Temperature profiles

- Kondisi batas yang ditransformasikan yang diperlukan untuk menyelesaikan persamaan momentum dan energi berbentuk:

$$\eta = 0: \qquad f = f' = 0 \qquad T^* = 1$$
$$\eta \to \infty \qquad f' \to 0 \qquad T^* \to 0$$

- di mana f dan T^* adalah fungsi hanya dari dan bilangan prima ganda dan rangkap tiga, masing-masing, mengacu pada turunan kedua dan ketiga sehubungan dengan η.

$$\eta \equiv \frac{y}{x} \left(\frac{Gr_x}{4} \right)^{1/4}$$

- Local Nusselt number:

$$Nu_x = \frac{hx}{k} = \frac{[q_s''/(T_s - T_\infty)]\, x}{k}$$

$$Nu_x = \frac{hx}{k} = -\left(\frac{Gr_x}{4} \right)^{1/4} \frac{dT^*}{d\eta} \bigg|_{\eta=0} = \left(\frac{Gr_x}{4} \right)^{1/4} g(Pr)$$

- Local Grashof number:

$$Gr_x = \frac{g\beta(T_s - T_\infty)x^3}{\nu^2}$$

- Koefisien konveksi rata-rata untuk permukaan yang panjangnya *L* adalah:

$$\bar{h} = \frac{1}{L}\int_0^L h\,dx = \frac{k}{L}\left[\frac{g\beta(T_s - T_\infty)}{4\nu^2}\right]^{1/4} g(Pr)\int_0^L \frac{dx}{x^{1/4}}$$

$$\overline{Nu_L} = \frac{\bar{h}L}{k} = \frac{4}{3}\left(\frac{Gr_L}{4}\right)^{1/4} g(Pr)$$

The Effects of Turbulence

- Penting untuk dicatat bahwa lapisan batas konveksi bebas tidak terbatas pada aliran laminar. Seperti halnya konveksi paksa, ketidakstabilan hidrodinamik dapat muncul. Artinya, gangguan dalam aliran dapat diperkuat, menyebabkan transisi dari aliran laminar ke turbulen. Proses ini ditunjukkan secara skematis pada Gambar 5 untuk pelat vertikal yang dipanaskan.

- Transisi dalam lapisan batas konveksi bebas tergantung pada besaran relatif gaya apung dan gaya viskos dalam fluida. Untuk mengkorelasikan dinyatakan dengan bilangan Rayleigh, yang hanya merupakan produk dari bilangan Grashof dan Prandtl.

- Untuk pelat vertikal bilangan Rayleigh kritis adalah:

$$Ra_{x,c} = Gr_{x,c}\, Pr = \frac{g\beta(T_s - T_\infty)x^3}{\nu\alpha} \approx 10^9$$

- Seperti pada konveksi paksa, transisi ke turbulensi memiliki efek yang kuat pada perpindahan panas.

- Oleh karena itu hasil dari bagian sebelumnya hanya berlaku jika $Ra_L \leq 10^9$.

- Untuk mendapatkan korelasi yang tepat untuk aliran turbulen, adalah pada hasil eksperimen.
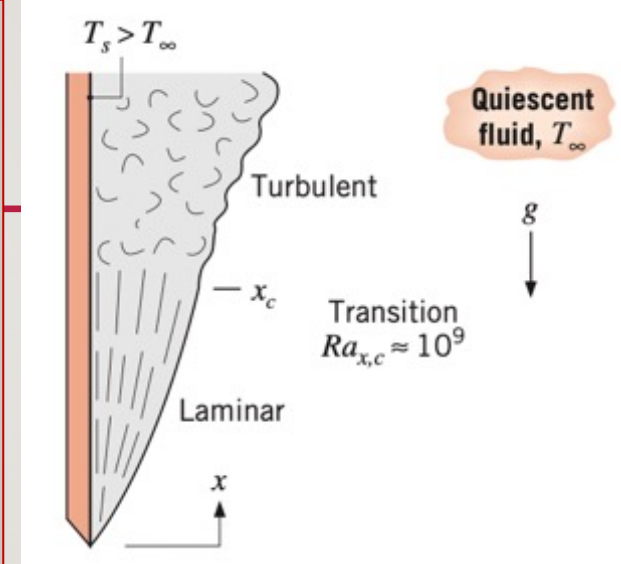
Gambar 5. Free convection boundary layer transition on a vertical plate

Contoh 1:

Perhatikan sebuah pelat vertikal dengan panjang 0,25 m yang bersuhu 70 °C. Pelat digantung pada keaadan udara yang diam dan bersuhu 25°C. Perkirakan ketebalan lapisan batas kecepatan dan kecepatan maksimum ke atas dari sisi masik *(trailing edge)* pelat. Bagaimana ketebalan lapisan batas dibandingkan dengan ketebalan yang akan ada jika udara mengalir di atas pelat dengan kecepatan aliran bebas 5 m/s?
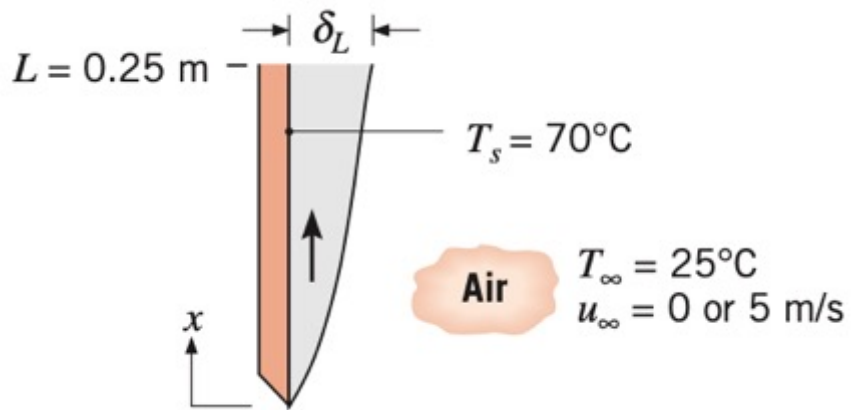
Assumptions:
1. Ideal gas.
2. Constant properties.
3. Buoyancy effects negligible when $u_\infty = 5$ m/s.

Properties: Table A.4, air ($T_f = 320.5$ K): $\nu = 17.95 \times 10^{-6}$ m²/s, $Pr = 0.7$, $\beta = T_f^{-1} = 3.12 \times 10^{-3}$ K⁻¹.

Analysis: For the quiescent air, Equation 9.12 gives

$$Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

$$= \frac{9.8 \text{ m/s}^2 \times (3.12 \times 10^{-3} \text{ K}^{-1})(70 - 25)°\text{C}(0.25 \text{ m})^3}{(17.95 \times 10^{-6} \text{ m}^2/\text{s})^2} = 6.69 \times 10^7$$

$$Ra_L = Gr_L \, Pr = 4.68 \times 10^7$$ Adalah laminar

Dari Gambar 4a, untuk Pr = 0,7 maka $\eta \approx 6{,}0$ pada sisi lapisan batas, yaitu pada $y \approx \delta$

$$\delta_L \approx \frac{6L}{(Gr_L/4)^{1/4}} = \frac{6(0.25 \text{ m})}{(1.67 \times 10^7)^{1/4}} = 0.024 \text{ m}$$

Dari gambar 4a terlihat bahwa kecepatan maksimum berhubungan dengan $f'(\eta) \approx 0{,}28$
Dan kecepatan adalah:

$$u = \frac{2\nu f'(\eta) Gr_L^{1/2}}{L} \approx \frac{2 \times 17.95 \times 10^{-6}\, \text{m}^2/\text{s} \times 0.28 \times (6.69 \times 10^7)^{1/2}}{0.25 \text{ m}} = 0.33 \text{ m/s}$$

Untuk aliran udara pada $u_\infty = 5 \; m/s$:

$$Re_L = \frac{u_\infty L}{\nu} = \frac{(5 \text{ m/s}) \times 0.25 \text{ m}}{17.95 \times 10^{-6}\, \text{m}^2/\text{s}} = 6.97 \times 10^4$$

Dan lapisan batas adalah lamiar. Maka:

$$\delta_L \approx \frac{5L}{Re_L^{1/2}} = \frac{5(0.25 \text{ m})}{(6.97 \times 10^4)^{1/2}} = 0.0047 \text{ m}$$

TABLE A.4 Thermophysical Properties of Gases at Atmospheric Pressure[a]

T (K)	ρ (kg/m³)	c_p (kJ/kg·K)	$\mu \cdot 10^7$ (N·s/m²)	$\nu \cdot 10^6$ (m²/s)	$k \cdot 10^3$ (W/m·K)	$\alpha \cdot 10^6$ (m²/s)	Pr
Air, $\mathcal{M}$ = 28.97 kg/kmol							
100	3.5562	1.032	71.1	2.00	9.34	2.54	0.786
150	2.3364	1.012	103.4	4.426	13.8	5.84	0.758
200	1.7458	1.007	132.5	7.590	18.1	10.3	0.737
250	1.3947	1.006	159.6	11.44	22.3	15.9	0.720
300	1.1614	1.007	184.6	15.89	26.3	22.5	0.707
350	0.9950	1.009	208.2	20.92	30.0	29.9	0.700
400	0.8711	1.014	230.1	26.41	33.8	38.3	0.690
450	0.7740	1.021	250.7	32.39	37.3	47.2	0.686
500	0.6964	1.030	270.1	38.79	40.7	56.7	0.684
550	0.6329	1.040	288.4	45.57	43.9	66.7	0.683
600	0.5804	1.051	305.8	52.69	46.9	76.9	0.685
650	0.5356	1.063	322.5	60.21	49.7	87.3	0.690
700	0.4975	1.075	338.8	68.10	52.4	98.0	0.695
750	0.4643	1.087	354.6	76.37	54.9	109	0.702
800	0.4354	1.099	369.8	84.93	57.3	120	0.709
850	0.4097	1.110	384.3	93.80	59.6	131	0.716
900	0.3868	1.121	398.1	102.9	62.0	143	0.720
950	0.3666	1.131	411.3	112.2	64.3	155	0.723
1000	0.3482	1.141	424.4	121.9	66.7	168	0.726
1100	0.3166	1.159	449.0	141.8	71.5	195	0.728
1200	0.2902	1.175	473.0	162.9	76.3	224	0.728
1300	0.2679	1.189	496.0	185.1	82	257	0.719
1400	0.2488	1.207	530	213	91	303	0.703
1500	0.2322	1.230	557	240	100	350	0.685
1600	0.2177	1.248	584	268	106	390	0.688
1700	0.2049	1.267	611	298	113	435	0.685
1800	0.1935	1.286	637	329	120	482	0.683
1900	0.1833	1.307	663	362	128	534	0.677
2000	0.1741	1.337	689	396	137	589	0.672
2100	0.1658	1.372	715	431	147	646	0.667
2200	0.1582	1.417	740	468	160	714	0.655
2300	0.1513	1.478	766	506	175	783	0.647
2400	0.1448	1.558	792	547	196	869	0.630
2500	0.1389	1.665	818	589	222	960	0.613
3000	0.1135	2.726	955	841	486	1570	0.536
Ammonia (NH₃), $\mathcal{M}$ = 17.03 kg/kmol							
300	0.6894	2.158	101.5	14.7	24.7	16.6	0.887
320	0.6448	2.170	109	16.9	27.2	19.4	0.870
340	0.6059	2.192	116.5	19.2	29.3	22.1	0.872
360	0.5716	2.221	124	21.7	31.6	24.9	0.872
380	0.5410	2.254	131	24.2	34.0	27.9	0.869

TABLE A.4 Continued

T (K)	ρ (kg/m³)	c_p (kJ/kg·K)	$\mu \cdot 10^7$ (N·s/m²)	$\nu \cdot 10^6$ (m²/s)	$k \cdot 10^3$ (W/m·K)	$\alpha \cdot 10^6$ (m²/s)	Pr
Ammonia (NH₃) (continued)							
400	0.5136	2.287	138	26.9	37.0	31.5	0.853
420	0.4888	2.322	145	29.7	40.4	35.6	0.833
440	0.4664	2.357	152.5	32.7	43.5	39.6	0.826
460	0.4460	2.393	159	35.7	46.3	43.4	0.822
480	0.4273	2.430	166.5	39.0	49.2	47.4	0.822
500	0.4101	2.467	173	42.2	52.5	51.9	0.813
520	0.3942	2.504	180	45.7	54.5	55.2	0.827
540	0.3795	2.540	186.5	49.1	57.5	59.7	0.824
560	0.3708	2.577	193	52.0	60.6	63.4	0.827
580	0.3533	2.613	199.5	56.5	63.8	69.1	0.817
Carbon Dioxide (CO₂), $\mathcal{M}$ = 44.01 kg/kmol							
280	1.9022	0.830	140	7.36	15.20	9.63	0.765
300	1.7730	0.851	149	8.40	16.55	11.0	0.766
320	1.6609	0.872	156	9.39	18.05	12.5	0.754
340	1.5618	0.891	165	10.6	19.70	14.2	0.746
360	1.4743	0.908	173	11.7	21.2	15.8	0.741
380	1.3961	0.926	181	13.0	22.75	17.6	0.737
400	1.3257	0.942	190	14.3	24.3	19.5	0.737
450	1.1782	0.981	210	17.8	28.3	24.5	0.728
500	1.0594	1.02	231	21.8	32.5	30.1	0.725
550	0.9625	1.05	251	26.1	36.6	36.2	0.721
600	0.8826	1.08	270	30.6	40.7	42.7	0.717
650	0.8143	1.10	288	35.4	44.5	49.7	0.712
700	0.7564	1.13	305	40.3	48.1	56.3	0.717
750	0.7057	1.15	321	45.5	51.7	63.7	0.714
800	0.6614	1.17	337	51.0	55.1	71.2	0.716
Carbon Monoxide (CO), $\mathcal{M}$ = 28.01 kg/kmol							
200	1.6888	1.045	127	7.52	17.0	9.63	0.781
220	1.5341	1.044	137	8.93	19.0	11.9	0.753
240	1.4055	1.043	147	10.5	20.6	14.1	0.744
260	1.2967	1.043	157	12.1	22.1	16.3	0.741
280	1.2038	1.042	166	13.8	23.6	18.8	0.733
300	1.1233	1.043	175	15.6	25.0	21.3	0.730
320	1.0529	1.043	184	17.5	26.3	23.9	0.730
340	0.9909	1.044	193	19.5	27.8	26.9	0.725
360	0.9357	1.045	202	21.6	29.1	29.8	0.725
380	0.8864	1.047	210	23.7	30.5	32.9	0.729
400	0.8421	1.049	218	25.9	31.8	36.0	0.719
450	0.7483	1.055	237	31.7	35.0	44.3	0.714
500	0.67352	1.065	254	37.7	38.1	53.1	0.710
550	0.61226	1.076	271	44.3	41.1	62.4	0.710
600	0.56126	1.088	286	51.0	44.0	72.1	0.707

T (K)	ρ (kg/m³)	c_p (kJ/kg·K)	μ·10⁷ (N·s/m²)	ν·10⁶ (m²/s)	k·10³ (W/m·K)	α·10⁶ (m²/s)	Pr
Carbon Monoxide (CO) *(continued)*							
650	0.51806	1.101	301	58.1	47.0	82.4	0.705
700	0.48102	1.114	315	65.5	50.0	93.3	0.702
750	0.44899	1.127	329	73.3	52.8	104	0.702
800	0.42095	1.140	343	81.5	55.5	116	0.705
Helium (He), M = 4.003 kg/kmol							
100	0.4871	5.193	96.3	19.8	73.0	28.9	0.686
120	0.4060	5.193	107	26.4	81.9	38.8	0.679
140	0.3481	5.193	118	33.9	90.7	50.2	0.676
160	—	5.193	129	—	99.2	—	—
180	0.2708	5.193	139	51.3	107.2	76.2	0.673
200	—	5.193	150	—	115.1	—	—
220	0.2216	5.193	160	72.2	123.1	107	0.675
240	—	5.193	170	—	130	—	—
260	0.1875	5.193	180	96.0	137	141	0.682
280	—	5.193	190	—	145	—	—
300	0.1625	5.193	199	122	152	180	0.680
350	—	5.193	221	—	170	—	—
400	0.1219	5.193	243	199	187	295	0.675
450	—	5.193	263	—	204	—	—
500	0.09754	5.193	283	290	220	434	0.668
550	—	5.193	—	—	—	—	—
600	—	5.193	320	—	252	—	—
650	—	5.193	332	—	264	—	—
700	0.06969	5.193	350	502	278	768	0.654
750	—	5.193	364	—	291	—	—
800	—	5.193	382	—	304	—	—
900	—	5.193	414	—	330	—	—
1000	0.04879	5.193	446	914	354	1400	0.654
Hydrogen (H₂), M = 2.016 kg/kmol							
100	0.24255	11.23	42.1	17.4	67.0	24.6	0.707
150	0.16156	12.60	56.0	34.7	101	49.6	0.699
200	0.12115	13.54	68.1	56.2	131	79.9	0.704
250	0.09693	14.06	78.9	81.4	157	115	0.707
300	0.08078	14.31	89.6	111	183	158	0.701
350	0.06924	14.43	98.8	143	204	204	0.700
400	0.06059	14.48	108.2	179	226	258	0.695
450	0.05386	14.50	117.2	218	247	316	0.689
500	0.04848	14.52	126.4	261	266	378	0.691
550	0.04407	14.53	134.3	305	285	445	0.685

T (K)	ρ (kg/m³)	c_p (kJ/kg·K)	μ·10⁷ (N·s/m²)	ν·10⁶ (m²/s)	k·10³ (W/m·K)	α·10⁶ (m²/s)	Pr
Hydrogen (H₂) *(continued)*							
600	0.04040	14.55	142.4	352	305	519	0.678
700	0.03463	14.61	157.8	456	342	676	0.675
800	0.03030	14.70	172.4	569	378	849	0.670
900	0.02694	14.83	186.5	692	412	1030	0.671
1000	0.02424	14.99	201.3	830	448	1230	0.673
1100	0.02204	15.17	213.0	966	488	1460	0.662
1200	0.02020	15.37	226.2	1120	528	1700	0.659
1300	0.01865	15.59	238.5	1279	568	1955	0.655
1400	0.01732	15.81	250.7	1447	610	2230	0.650
1500	0.01616	16.02	262.7	1626	655	2530	0.643
1600	0.0152	16.28	273.7	1801	697	2815	0.639
1700	0.0143	16.58	284.9	1992	742	3130	0.637
1800	0.0135	16.96	296.1	2193	786	3435	0.639
1900	0.0128	17.49	307.2	2400	835	3730	0.643
2000	0.0121	18.25	318.2	2630	878	3975	0.661
Nitrogen (N₂), M = 28.01 kg/kmol							
100	3.4388	1.070	68.8	2.00	9.58	2.60	0.768
150	2.2594	1.050	100.6	4.45	13.9	5.86	0.759
200	1.6883	1.043	129.2	7.65	18.3	10.4	0.736
250	1.3488	1.042	154.9	11.48	22.2	15.8	0.727
300	1.1233	1.041	178.2	15.86	25.9	22.1	0.716
350	0.9625	1.042	200.0	20.78	29.3	29.2	0.711
400	0.8425	1.045	220.4	26.16	32.7	37.1	0.704
450	0.7485	1.050	239.6	32.01	35.8	45.6	0.703
500	0.6739	1.056	257.7	38.24	38.9	54.7	0.700
550	0.6124	1.065	274.7	44.86	41.7	63.9	0.702
600	0.5615	1.075	290.8	51.79	44.6	73.9	0.701
700	0.4812	1.098	321.0	66.71	49.9	94.4	0.706
800	0.4211	1.122	349.1	82.90	54.8	116	0.715
900	0.3743	1.146	375.3	100.3	59.7	139	0.721
1000	0.3368	1.167	399.9	118.7	64.7	165	0.721
1100	0.3062	1.187	423.2	138.2	70.0	193	0.718
1200	0.2807	1.204	445.3	158.6	75.8	224	0.707
1300	0.2591	1.219	466.2	179.9	81.0	256	0.701
Oxygen (O₂), M = 32.00 kg/kmol							
100	3.945	0.962	76.4	1.94	9.25	2.44	0.796
150	2.585	0.921	114.8	4.44	13.8	5.80	0.766
200	1.930	0.915	147.5	7.64	18.3	10.4	0.737
250	1.542	0.915	178.6	11.58	22.6	16.0	0.723
300	1.284	0.920	207.2	16.14	26.8	22.7	0.711

TABLE A.4 Continued

T (K)	ρ (kg/m³)	c_p (kJ/kg·K)	$\mu \cdot 10^7$ (N·s/m²)	$\nu \cdot 10^6$ (m²/s)	$k \cdot 10^3$ (W/m·K)	$\alpha \cdot 10^6$ (m²/s)	Pr
Oxygen (O₂) *(continued)*							
350	1.100	0.929	233.5	21.23	29.6	29.0	0.733
400	0.9620	0.942	258.2	26.84	33.0	36.4	0.737
450	0.8554	0.956	281.4	32.90	36.3	44.4	0.741
500	0.7698	0.972	303.3	39.40	41.2	55.1	0.716
550	0.6998	0.988	324.0	46.30	44.1	63.8	0.726
600	0.6414	1.003	343.7	53.59	47.3	73.5	0.729
700	0.5498	1.031	380.8	69.26	52.8	93.1	0.744
800	0.4810	1.054	415.2	86.32	58.9	116	0.743
900	0.4275	1.074	447.2	104.6	64.9	141	0.740
1000	0.3848	1.090	477.0	124.0	71.0	169	0.733
1100	0.3498	1.103	505.5	144.5	75.8	196	0.736
1200	0.3206	1.115	532.5	166.1	81.9	229	0.725
1300	0.2960	1.125	588.4	188.6	87.1	262	0.721
Water Vapor (Steam), $\mathcal{M} = 18.02$ kg/kmol							
380	0.5863	2.060	127.1	21.68	24.6	20.4	1.06
400	0.5542	2.014	134.4	24.25	26.1	23.4	1.04
450	0.4902	1.980	152.5	31.11	29.9	30.8	1.01
500	0.4405	1.985	170.4	38.68	33.9	38.8	0.998
550	0.4005	1.997	188.4	47.04	37.9	47.4	0.993
600	0.3652	2.026	206.7	56.60	42.2	57.0	0.993
650	0.3380	2.056	224.7	66.48	46.4	66.8	0.996
700	0.3140	2.085	242.6	77.26	50.5	77.1	1.00
750	0.2931	2.119	260.4	88.84	54.9	88.4	1.00
800	0.2739	2.152	278.6	101.7	59.2	100	1.01
850	0.2579	2.186	296.9	115.1	63.7	113	1.02

Empirical Correlations: External Free Convection Flows

- Pada bagian sebelumnya, dibahas konveksi bebas yang terkait dengan pengembangan lapisan batas laminar yang berdekatan dengan pelat vertikal yang dipanaskan dan transisi aliran laminar ke keadaan turbulen.

- Ada dua parameter tak berdimensi, bilangan Grashof *Gr* dan bilangan Rayleigh *Ra*, yang muncul dalam korelasi empiris untuk konveksi bebas yang melibatkan kondisi aliran laminar dan turbulen dan dalam geometri selain pelat datar.

- Selanjutnya ditunjukkan korelasi empiris yang telah dikembangkan untuk geometri aliran eksternal).

- Korelasi yang cocok untuk banyak perhitungan teknik dan sering berbentuk:

$$\overline{Nu_L} = \frac{\bar{h}L}{k} = C\,Ra_L^n$$

- Dimana Bilangan Rayleigh:

$$Ra_L = Gr_L\,Pr = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}$$

- didasarkan pada panjang karakteristik L dari geometri. Biasanya, *n=1/4* untuk laminar dan *n=1/3* untuk aliran turbulen, Untuk aliran turbulen hL tidak tegantung pada L.

- Semua sifat-sifat fluida dievaluasi pada suhu film, $T_f \equiv (Ts + T_\infty)/2$.

The Vertical Plate

Untuk aliran laminar pada plat vertikal: $(10^4 \lesssim Ra_L \lesssim 10^9)$, $C = 0.59$ and $n = 1/4$,

Untuk aliran turbulen: $(10^9 \lesssim Ra_L \lesssim 10^{13})$, $C = 0.10$ and $\bar{n} = 1/3$.

Bilangan Nusselt:

$$\overline{Nu_L} = \left\{ 0.825 + \frac{0.387 \, Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \right\}^2$$

Atau yang lebih akurat:

$$\overline{Nu_L} = 0.68 + \frac{0.670 \, Ra_L^{1/4}}{[1 + (0.492/Pr)^{9/16}]^{4/9}} \qquad Ra_L \lesssim 10^9$$

Contoh 2:
Sebuah *firescreen* pintu kaca yang digunakan untuk mengurangi pembuangan udara dari ruangan melalui cerobong asap yang memiliki tinggi 0,71 m dan lebar 1,02 m dan mencapai suhu 232 ℃. Jika suhu ruangan 23 ℃, perkirakan laju perpindahan panas konveksi dari perapian ke ruangan.

SOLUTION

Known: Glass screen situated in fireplace opening.

Find: Heat transfer by convection between screen and room air.

Schematic:

Glass panel
Height, $L = 0.71$ m
Width, $w = 1.02$ m
Quiescent air
q_{conv}
$T_\infty = 23℃$
Fire
$T_s = 232℃$
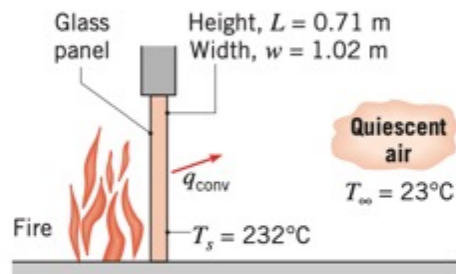
Assumptions:
1. Screen is at a uniform temperature T_s.
2. Room air is quiescent.
3. Ideal gas.
4. Constant properties.

Properties: Table A.4, air ($T_f = 400$ K): $k = 33.8 \times 10^{-3}$ W/m·K, $\nu = 26.4 \times 10^{-6}$ m²/s, $\alpha = 38.3 \times 10^{-6}$ m²/s, $Pr = 0.690$, $\beta = (1/T_f) = 0.0025$ K^{-1}.

$$T_f = [(232+273) + (23+273)] / 2 = 400 \text{ K}$$

$$q = \bar{h}A_s(T_s - T_\infty)$$

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\alpha\nu}$$

$$= \frac{9.8 \text{ m/s}^2 \times 0.0025 \text{ K}^{-1} \times (232 - 23)℃ \times (0.71 \text{ m})^3}{38.3 \times 10^{-6} \text{ m}^2/\text{s} \times 26.4 \times 10^{-6} \text{ m}^2/\text{s}} = 1.813 \times 10^9$$

$$q = \bar{h}A_s(T_s - T_\infty)$$

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\alpha\nu}$$

$$= \frac{9.8 \text{ m/s}^2 \times 0.0025 \text{ K}^{-1} \times (232 - 23)°\text{C} \times (0.71 \text{ m})^3}{38.3 \times 10^{-6} \text{ m}^2/\text{s} \times 26.4 \times 10^{-6} \text{ m}^2/\text{s}} = 1.813 \times 10^9$$

$$\overline{Nu_L} = \left\{0.825 + \frac{0.387 \, Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$$

$$\overline{Nu_L} = \left\{0.825 + \frac{0.387(1.813 \times 10^9)^{1/6}}{[1 + (0.492/0.690)^{9/16}]^{8/27}}\right\}^2 = 147$$

$$\bar{h} = \frac{\overline{Nu_L} \cdot k}{L} = \frac{147 \times 33.8 \times 10^{-3} \text{ W/m} \cdot \text{K}}{0.71 \text{ m}} = 7.0 \text{ W/m}^2 \cdot \text{K}$$

$$q = 7.0 \text{ W/m}^2 \cdot \text{K} \, (1.02 \times 0.71) \text{ m}^2 \, (232 - 23)°\text{C} = 1060 \text{ W}$$

Inclined and Horizontal Plates
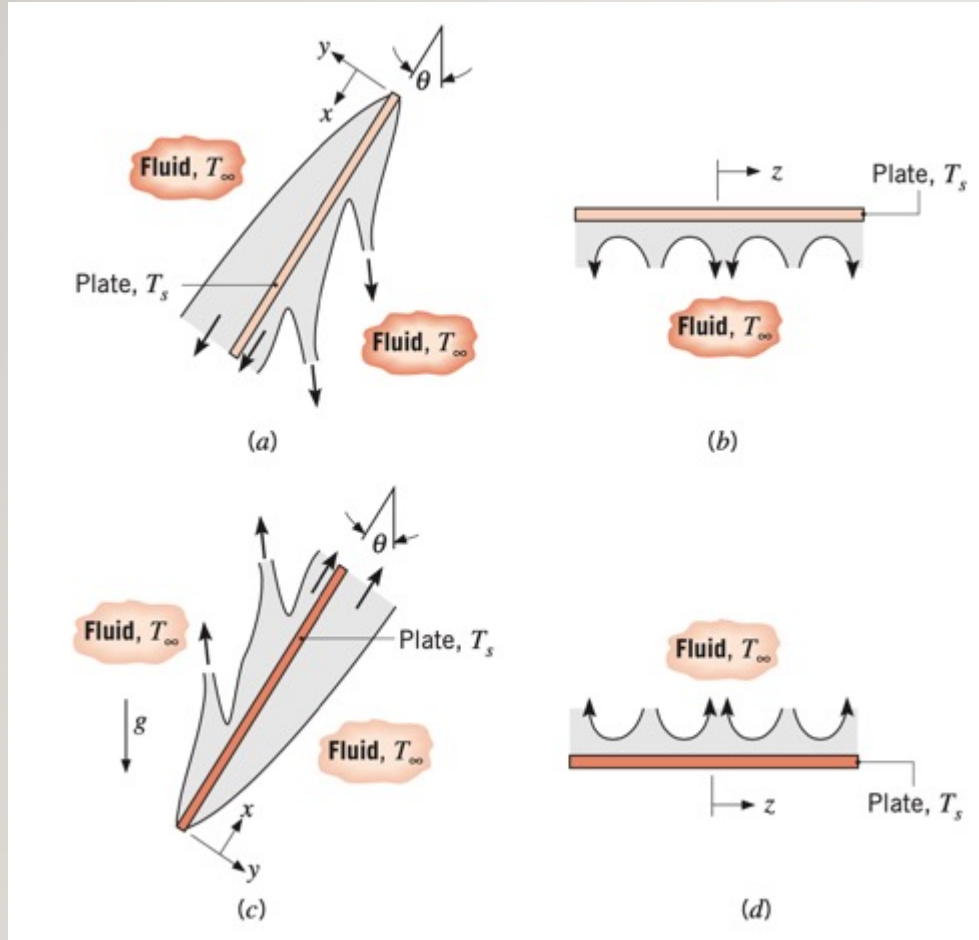
Gambar 6. Buoyancy-driven flows on an inclined plate: (a) Side view of flows at top and bottom surfaces of a cold plate ($T_s < T_\infty$). (b) End view of flow at bottom surface of cold plate. (c) Side view of flows at top and bottom surfaces of a hot plate ($T_s > T_\infty$). (d) End view of flow at top surface of hot plate.

- Gambar 6a, jika pelat dingin, komponen y dari gaya apung mempunyai arah normal terhadap pelat.

- Karena komponen x dari percepatan gravitasi direduksi menjadi $g \cos \theta$, kecepatan fluida di sepanjang pelat berkurang dan terjadi pengurangan perpindahan panas konveksi ke permukaan atas.

- Pada permukaan bawah, komponen y dari gaya apung bertindak untuk memindahkan fluida dari permukaan, dan pengembangan lapisan batas terganggu.

- Aliran yang dihasilkan adalah tiga dimensi, dan, seperti yang ditunjukkan oleh variasi bentang (arah-z) pada Gambar 6b, fluida dingin dari permukaan bawah digantikan oleh fluida sekitar yang lebih hangat.

- Perpindahan fluida lapisan batas dingin oleh lingkungan yang lebih hangat dan pengurangan yang menyertai dalam ketebalan lapisan batas termal bertindak untuk meningkatkan perpindahan panas konveksi ke permukaan bawah.

- Tren serupa mencirikan pelat panas (Gambar 6c,d), dan aliran tiga dimensi sekarang dikaitkan dengan permukaan atas, dari mana cairan hangat dilepaskan.

- Untuk permukaan pelat dingin yang menghadap ke atas (Gambar 7a) dan permukaan panas yang menghadap ke bawah (Gambar 7d), kecenderungan fluida untuk turun dan naik, berturut-turut, dihalangi oleh pelat.

- Aliran harus bergerak secara horizontal sebelum dapat turun atau naik dari tepi pelat, dan perpindahan panas konveksi agak tidak efektif.

- Sebaliknya, untuk permukaan dingin yang menghadap ke bawah (Gambar 7b) dan permukaan panas yang menghadap ke atas (Gambar 7c), aliran didorong oleh fluida yang turun dan naik.

- Kekekalan massa menyatakan bahwa fluida dingin (hangat) yang turun (naik) dari suatu permukaan digantikan oleh fluida yang lebih hangat (lebih dingin) yang naik (turun) dari lingkungan, dan perpindahan panas jauh lebih efektif.
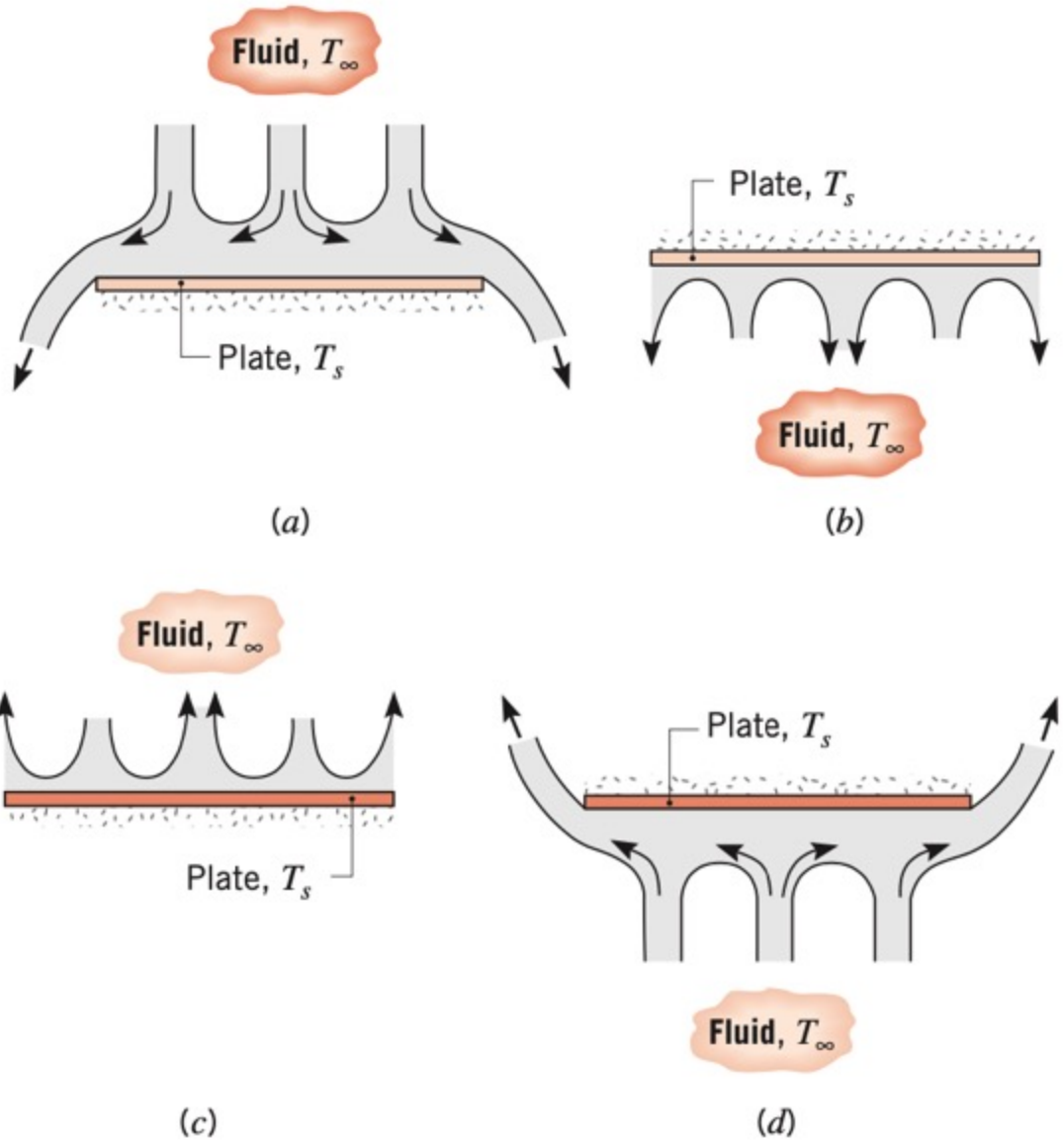
Gambar 7. Buoyancy-driven flows on horizontal cold $(T_s < T_\infty)$ and hot $(T_s > T_\infty)$ plates: (a) Top surface of cold plate. (b) Bottom surface of cold plate. (c) Top surface of hot plate. (d) Bottom surface of hot plate.

Upper Surface of Hot Plate or Lower Surface of Cold Plate [19]:

$$\overline{Nu_L} = 0.54\, Ra_L^{1/4} \quad (10^4 \lesssim Ra_L \lesssim 10^7, Pr \gtrsim 0.7)$$

$$\overline{Nu_L} = 0.15\, Ra_L^{1/3} \quad (10^7 \lesssim Ra_L \lesssim 10^{11}, \text{all } Pr)$$

Lower Surface of Hot Plate or Upper Surface of Cold Plate [20]:

$$\overline{Nu_L} = 0.52\, Ra_L^{1/5} \quad (10^4 \lesssim Ra_L \lesssim 10^9, Pr \gtrsim 0.7)$$

Contoh 3:

Aliran udara melalui saluran pemanas persegi panjang dengan lebar 0,75 m dan tinggi 0,3 m dan permukaan saluran luar bertemperatur 45 °C. Jika saluran tidak diisolasi dan terpapar udara pada 15 °C di ruang bagian bawah rumah, berapa kehilangan panas dari saluran per meter panjangnya?

Schematic:

Quiescent air
$T_\infty = 15°C$
Long duct
Airflow
$w = 0.75$ m
$H = 0.3$ m
$T_s = 45°C$
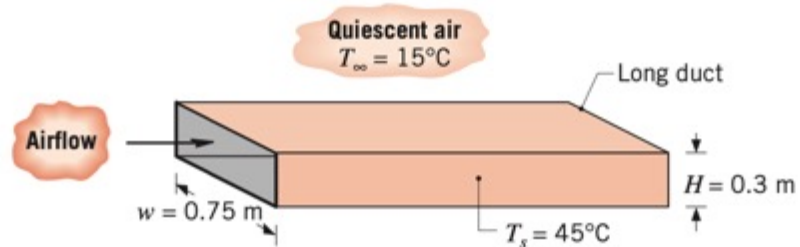
Assumptions:

1. Ambient air is quiescent.
2. Surface radiation effects are negligible.
3. Ideal gas.
4. Constant properties.

Properties: Table A.4, air ($T_f = 303$ K): $\nu = 16.2 \times 10^{-6}$ m²/s, $\alpha = 22.9 \times 10^{-6}$ m²/s, $k = 0.0265$ W/m·K, $\beta = 0.0033$ K⁻¹, $Pr = 0.71$.

Analysis: Surface heat loss is by free convection from the vertical sides and the horizontal top and bottom. From Equation 9.25

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} = \frac{(9.8 \text{ m/s}^2)(0.0033 \text{ K}^{-1})(30 \text{ K}) L^3 \text{ (m}^3)}{(16.2 \times 10^{-6} \text{ m}^2/\text{s})(22.9 \times 10^{-6} \text{ m}^2/\text{s})}$$

$$Ra_L = 2.62 \times 10^9 L^3$$

For the two sides, $L = H = 0.3$ m. Hence $Ra_L = 7.07 \times 10^7$. The free convection boundary layer is therefore laminar, and from Equation 9.27

$$\overline{Nu}_L = 0.68 + \frac{0.670\, Ra_L^{1/4}}{[1 + (0.492/Pr)^{9/16}]^{4/9}}$$

The convection coefficient associated with the sides is then

$$\overline{h}_s = \frac{k}{H}\, \overline{Nu}_L$$

$$\overline{h}_s = \frac{0.0265\ \text{W/m} \cdot \text{K}}{0.3\ \text{m}} \left\{ 0.68 + \frac{0.670(7.07 \times 10^7)^{1/4}}{[1 + (0.492/0.71)^{9/16}]^{4/9}} \right\} = 4.23\ \text{W/m}^2 \cdot \text{K}$$

For the top and bottom, $L = (A_s/P) \approx (w/2) = 0.375$ m. Hence $Ra_L = 1.38 \times 10^8$, and from Equations 9.31 and 9.32, respectively,

$$\overline{h}_t = [k/(w/2)] \times 0.15\, Ra_L^{1/3} = \frac{0.0265\ \text{W/m} \cdot \text{K}}{0.375\ \text{m}} \times 0.15(1.38 \times 10^8)^{1/3}$$

$$= 5.47\ \text{W/m}^2 \cdot \text{K}$$

$$\overline{h}_b = [k/(w/2)] \times 0.52\, Ra_L^{1/5} = \frac{0.0265\ \text{W/m} \cdot \text{K}}{0.375\ \text{m}} \times 0.52(1.38 \times 10^8)^{1/5}$$

$$= 1.56\ \text{W/m}^2 \cdot \text{K}$$

The rate of heat loss per unit length of duct is then

$$q' = 2q'_s + q'_t + q'_b = (2\bar{h}_s \cdot H + \bar{h}_t \cdot w + \bar{h}_b \cdot w)(T_s - T_\infty)$$

$$q' = (2 \times 4.23 \times 0.3 + 5.47 \times 0.75 + 1.56 \times 0.75)(45 - 15) \text{ W/m}$$

$$q' = 234 \text{ W/m}$$

Contoh 4:

Hitung koefisien film perpindahan panas untuk udara pada pipa horizontal berdiameter 6 inci yang temperatur permukaannya 200 °F dalam ruangan yang berisi udara pada 70 °F.

SOLUTION:

$$T_{av} = (200 + 70)/2 = 135°F$$

The fluid volume density is obtained directly from the ideal gas law:

$$v_f = RT/P = (0.73)(135 + 460)/(1.0) = 434 \text{ ft}^3/\text{lbmol}$$

The mass density of the fluid is

$$\rho_f(\text{air, } MW = 29) = 29/434 = 0.0668 \text{ lb/ft}^3$$

At 70°F (from the Appendix)

$$\mu = 0.019 \text{ cP} = 1.28 \times 10^{-5} \text{ lb/ ft} \cdot \text{s}$$
$$k = 0.016 \text{ Btu/h} \cdot \text{ft} \cdot °F$$

Calculate the Grashof number; employ Equation (10.2). Since

$$\beta = 1/T_{av} = 1/(135 + 460) = 0.0017\,°R^{-1}$$

$$Gr = \frac{D^3 \rho^2 g \beta \Delta T}{\mu^2} = \frac{(0.5)^3(0.0668)^2(32.174)(0.017)(660 - 530)}{(1.28 \times 10^{-5})^2} = 24.2 \times 10^7$$

The Prandtl number may also be calculated. At 135°F, $c_p = 0.25$ Btu/lb · °F. Substituting into Equation (10.5),

$$Pr = \frac{c_p \mu}{k} = \frac{(0.25)(1.28 \times 10^{-5})}{0.016/3600} = 0.72$$

Note that for air, Pr is generally taken as 0.7 (see earlier examples). The following term is calculated:

$$\log_{10}[(Gr)(Pr)] = 8.24$$

From Holman,[3] at this value

$$\log_{10}(Nu) \cong 1.5$$

so that

$$Nu = 31.6$$

Since

$$Nu = \frac{hD}{k} \qquad (10.13)$$

$$h = Nu\left(\frac{k}{D}\right) = 31.6\left(\frac{0.016}{0.5}\right) = 1.01\ \text{Btu/h} \cdot \text{ft} \cdot °F \qquad \blacksquare$$

Contoh 5:

Hitung koefisien perpindahan panas konveksi bebas untuk pelat setinggi 6 kaki dan lebar 8 kaki pada 120 ^{o}F yang terkena nitrogen pada 60 ^{o}F.

SOLUTION: The mean film temperature is

$$T_f = (120 + 60)/2 = 90°F = 550°R$$

From the Appendix,

$$\rho = 0.0713 \, lb/ft^3$$
$$k = 0.01514 \, Btu/h \cdot ft \cdot °F$$
$$\nu = 16.82 \times 10^{-5} \, ft^2/s$$
$$Pr = 0.713$$

In addition,

$$\beta = 1/T = 1/550 = 1.818 \times 10^{-3} \, °R^{-1}$$

Therefore

$$Gr = \frac{g\beta(T_S - T_\infty)L^3}{\nu^2} \qquad (10.2)$$

Substituting,

$$Gr = \frac{(32.2 \text{ ft/s}^2)(1.818 \times 10^{-3} \text{ °R}^{-1})[(120 - 40)\text{°R}](6 \text{ ft})^3}{(16.82 \times 10^{-5})^2 \text{ ft}^4/\text{s}^2} = 3.576 \times 10^{10}$$

In addition, from Equation (10.4),

$$Ra = (Gr)(Pr) = (3.576 \times 10^{10})(0.713) = 2.549 \times 10^{10}$$

The flow is therefore turbulent (see Table 10.2). Equation (10.13) applies, with appropriate constants from Table 10.1, to give

$$\frac{\bar{h}L}{k} = (0.10)(Ra)^{1/3}$$

$$\bar{h} = \frac{0.01514 \text{ Btu/h} \cdot \text{ft} \cdot \text{°F}}{6 \text{ ft}}(0.10)(2.549 \times 10^{10})^{1/3}$$

$$= 0.743 \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{°F}$$

■

Contoh 6:

Hitung laju perpindahan kalor dari contoh 5.

$$\dot{Q} = \bar{h}A(T_S - T_\infty)$$

$$= (0.743\,\text{Btu/h} \cdot \text{ft}^2 \cdot °\text{F})[(6 \times 8)\,\text{ft}^2][(120 - 60)°\text{F}]$$

$$= 2140\,\text{Btu/h}$$

Contoh 7:

Hitung perpindahan panas dari bola lampu 100 W yang bertemperatur 113°C ke udara lingkungan 31°C. Perkirakan bola lampu sebagai bola berdiameter 120 mm.

From the Appendix,

$$\nu = (22.38 \times 10^{-5})(0.0929) = 2.079 \times 10^{-5}\,\text{m}^2/\text{s}$$

$$\text{Pr} = 0.70$$

$$k = (0.01735)(1.729) = 0.0300\,\text{W/m} \cdot \text{K}$$

$$\beta = 1/T = 1/345 = 2.899 \times 10^{-3}\,\text{K}^{-1}$$

SOLUTION: For this example,

$$T_f = (T_S + T_\infty)/2 = 72°C$$

Employ the characteristic length as the diameter of the sphere, D, in Equation (10.2).

$$\text{Gr} = \frac{g\beta(T_S - T_\infty)D^3}{\nu^2}$$

$$= \frac{(9.80\,\text{m/s}^2)(2.899 \times 10^{-3}\,\text{K}^{-1})(113 - 31)(\text{K})(0.060\,\text{m})^3}{(2.079 \times 10^{-5})^2\,\text{m}^4/\text{s}^2}$$

$$= 1.16 \times 10^6$$

Apply Equation (10.13) with constants drawn from Table 10.1, i.e.,

$$\frac{\bar{h}D}{k} = (0.60)(\text{Ra})^{1/4}$$

$$\bar{h} = \frac{0.0300\,\text{W/m} \cdot \text{K}}{0.060\,\text{m}}(0.60)[(1.16 \times 10^6)(0.7)]^{1/4}$$

$$= 9.01\,\text{W/m}^2 \cdot \text{K}$$

Laju perpindahan kalor:

$$\dot{Q} = \bar{h}A(T_S - T_\infty)$$

$$= (9.01\,\text{W/m}^2 \cdot \text{K})\,\pi(0.060\,\text{m})^2(82\,\text{K})$$

$$= 8.36\,\text{W}$$

LANJUTAN CONTOH SOAL
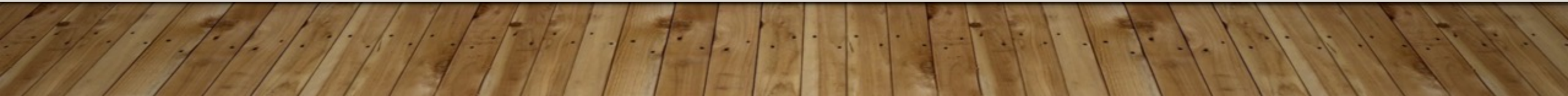
CONTOH SOAL-SOAL KONVEKSI BEBAS

(REFERENSI: HEAT & MASS TRANSFER BY R. K. RAJPUT)

Ir. J. Victor Tuapetel, ST, MT, PhD, IPM
Prodi Teknik Mesin ITI

Example 8.1. *A vertical cylinder 1.5 m high and 180 mm in diameter is maintained at 100°C in an atmosphere environment of 20°C. Calculate heat loss by free convection from the surface of the cylinder. Assume properties of air at mean temperature as, $\rho = 1.06$ kg/m³, $v = 18.97 \times 10^{-6}$ m²/s, $c_p = 1.004$ kJ/kg°C and $k = 0.1042$ kJ/mh°C.* **(AMIE Summer, 2000)**

Solution. *Given :* $L = 1.5$ m; $D = 180$ mm $= 0.18$ m, $t_s = 100°C$;

$$t_\infty = 20°C; \rho = 1.06 \text{ kg/m}^3; v = 18.97 \times 10^{-6} \text{ m}^2/\text{s};$$

$$c_p = 1.004 \text{ kJ/kg°C}; k = 0.1042 \text{ kJ/m h°C}.$$

Heat loss by free convection, Q :

$$\mu = \rho v = 1.06 \times (18.97 \times 10^{-6} \times 3600) = 0.07239 \text{ kg/mh}$$

$$\beta = \frac{1}{T} = \frac{1}{273 + t_f} = \frac{1}{273 + \left(\dfrac{100 + 20}{2}\right)} = 0.003 \text{ K}^{-1}$$

$$Gr = \frac{L^3 g \beta \Delta t}{v^2} \qquad \qquad \text{...[Eqn. (8.3)]}$$

$$= \frac{(1.5)^3 \times 9.81 \times 0.003 \times (100 - 20)}{(18.97 \times 10^{-6})^2} = 2.208 \times 10^{10}$$

$$Pr = \frac{\mu c_p}{k} = \frac{0.07239 \times 1.004}{0.1042} = 0.6975$$

$$Gr \, Pr = 2.208 \times 10^{10} \times 0.6975 = 1.54 \times 10^{10}$$

For this value of *Gr Pr* (tubulent range),

$$\overline{Nu}_L = \frac{\overline{h}L}{k} = 0.10 \, (Gr.Pr)^{1/3} \quad (\text{for } 10^9 < Gr.Pr < 10^{12})$$

$$= 0.10 \, (1.54 \times 10^{10})^{1/3} = 248.79$$

$$\therefore \qquad \overline{h} = \frac{k}{L} \times 248.79 = \frac{0.1042}{1.5} \times 248.79 = 17.283 \text{ kJ/h m}^2\,°C$$

∴ Rate of heat loss, $Q = \overline{h}A \, (t_s - t_\infty)$

$$= 17.283 \times (\pi \times 0.18 \times 1.5) \times (100 - 20)$$

$$= \textbf{1172.8 kJ/h} \qquad \textbf{(Ans.)}$$

Example 8.2. *A cylindrical body of 300 mm diameter and 1.6 m height is maintained at a constant temperature of 36.5°C. The surrounding temperature is 13.5°C. Find out the amount of heat to be generated by the body per hour if ρ = 1.025 kg/m³; c_p = 0.96 kJ/kg°C; v = 15.06 × 10⁻⁶ m²/s; k = 0.0892 kJ/m-h-°C and $\beta = \dfrac{1}{298} K^{-1}$. Assume Nu = 0.12 (Gr . Pr)^(1/3) (the symbols have their usual meanings).*

(AMIE Winter, 1997)

Solution. *Given :* D = 300 mm = 0.3 m; L = 1.6 m; t_s = 36.5°C; t_∞ = 13.5°C;

$$\rho = 1.025 \text{ kg/m}^3; \, c_p = 0.96 \text{ kJ/kg°C}; \, v = 15.06 \times 10^{-6} \text{ m}^2/\text{s};$$

$$k = 0.0892 \text{ kJ/m-h-°C}; \, \beta = \frac{1}{298} K^{-1}; \, Nu = 0.12 \, (Gr \, . \, Pr)^{1/3}$$

The amount of heat to be generated :

Grashoff number, $Gr = \dfrac{L^3 g \beta \, (t_s - t_\infty)}{v^2}$

$$= \frac{(1.6)^3 \times 9.81 \times \left(\dfrac{1}{298}\right) \times (36.5 - 13.5)}{(15.06 \times 10^{-6})^2} = 1.3674 \times 10^{10}$$

Prandtl number, $Pr = \dfrac{\mu c_p}{k} = \dfrac{\rho v c_p}{k} = \dfrac{1.025 \times (15.06 \times 10^{-6} \times 3600) \times 0.96}{0.0892}$ $\left(\because v = \dfrac{\mu}{\rho} \right)$

Nusselt number, $Nu = \dfrac{hL}{k} = 0.12 \, (Gr.Pr)^{1/3}$

$\qquad = 0.12 \, (1.3674 \times 10^{10} \times 0.598)^{1/3} = 241.75$

$\therefore \qquad h = \dfrac{k}{L} \times 241.75 = \dfrac{0.0892}{1.6} \times 241.75 = 13.478 \text{ kJ/h m}^2\,^\circ\text{C}$

Heat lost from the surface by natural convection,

$\qquad Q = hA \, (t_s - t_\infty)$

$\qquad = 13.478 \times (\pi \times 0.3 \times 1.6) \, (36.5 - 13.5) = \textbf{*467.5 kJ/h}$ **(Ans.)**

* This is the amount of heat to be generated.

Example 8.3. *A hot plate 1.2 m wide, 0.35 m high and at 115°C is exposed to the ambient still air at 25°C. Calculate the following:*

(i) *Maximum velocity at 180 mm from the leading edge of the plate;*

(ii) *The boundary layer thickness at 180 mm from the leading edge of the plate;*

(iii) *Local heat transfer coefficient at 180 mm from the leading edge of the plate;*

(iv) *Average heat transfer coefficient over the surface of the plate;*

(v) *Total mass flow through the boundary;*

(vi) *Heat loss from the plate;*

(vii) *Rise in temperature of the air passing through the boundary.*

Use the approximate solution.

Solution. *Given* : $t_s = 115°C$, $t_\infty = 25°C$, $x = 180$ mm $= 0.18$ m

The mean film temperature, $\qquad t_f = \dfrac{t_s + t_\infty}{2} = \dfrac{115 + 25}{2} = 70°C$

The thermo-physical properties of air at 70°C are :

$$\rho = 1.029 \text{ kg/m}^3; \quad k = 0.02964 \text{ w/m°C}; \quad v = 20.02 \times 10^{-6} \text{ m}^2/\text{s}$$

$$Pr = 0.694, \quad \beta = \dfrac{1}{T} = \dfrac{1}{(273 + t_f)} = \dfrac{1}{(273 + 70)} = \dfrac{1}{343} = 0.002915$$

Grashoff number $Gr_x = \dfrac{x^3 \, g\beta \, (t_s - t_\infty)}{v^2}$

$$= \dfrac{(0.18)^3 \times 9.81 \times 0.002915 \, (115 - 25)}{(20.02 \times 10^{-6})^2} = 3.745 \times 10^7$$

and, $\qquad Gr_L = 3.745 \times 10^7 \times \left(\dfrac{0.35}{0.18}\right)^3 = 27.532 \times 10^7$

(i) Maximum velocity at 180 mm from the leading edge of the plate, u_{max} :

$$u_{max} = 0.766 \, v \left(0.952 + \dfrac{v}{\alpha}\right)^{-1/2} \left[\dfrac{g\beta \, (t_s - t_\infty)}{v^2}\right]^{1/2} (x)^{1/2} \qquad \text{...[Eqn. (8.25)]}$$

$$= 0.766 \times 20.02 \times 10^{-6} \, (0.952 + 0.694)^{-1/2} \times \left[\dfrac{9.81 \times 0.002915 \, (115 - 25)}{(20.02 \times 10^{-6})^2}\right]^{1/2} (0.18)^{1/2}$$

$$= \textbf{0.406 m/s} \qquad \textbf{(Ans.)} \qquad\qquad \left(\because Pr = \dfrac{v}{\alpha} = 0.694 \text{ ...as given above}\right)$$

(ii) The boundary layer thickness at 180 mm from the leading edge of the plate, δ:

$$\frac{\delta}{x} = 3.93 \, (0.952 + Pr)^{1/4} \, (Gr_x)^{-1/4} \, (Pr)^{-1/2} \qquad \text{...[Eqn. (8.27)]}$$

$$= 3.93 \, (0.952 + 0.694)^{1/4} \, (3.745 \times 10^7)^{-1/4} \, (0.694)^{-1/2} = 0.0683$$

or,
$$\delta = 0.0683 \times 0.18 = 0.01229 \text{ m} = \textbf{12.29 mm} \quad \textbf{(Ans.)}$$

(iii) Local heat transfer coefficient at 180 mm from the leading edge of the plate, h_x:

$$Nu_x = \frac{h_x \, x}{k} = 0.508 \, (Pr)^{1/2} \, (0.952 + Pr)^{-1/4} \, (Gr_x)^{1/4} = \frac{2x}{\delta} \qquad \text{...[Eqn. (8.31)]}$$

$$h_x = \frac{k}{x} \times \frac{2x}{\delta} = \frac{2k}{\delta} = \frac{2 \times 0.02964}{0.01229} = \textbf{4.823 W/m}^2\textbf{°C} \qquad \textbf{(Ans.)}$$

(iv) Average heat transfer coefficient over the surface of the plate, $\overline{h}$:

$$\overline{h} = \frac{k}{L} \, [0.677 \, (Pr)^{1/2} \, (0.952 + Pr)^{-1/4} \, (Gr_L)^{1/4}] \qquad \text{...[Eqn. (8.33)]}$$

$$= \frac{0.02964}{0.35} \, [0.677 \, (0.694)^{1/2} \, (0.952 + 0.694)^{-1/4} \, (27.532 \times 10^7)^{1/4}]$$

$$= \textbf{5.43 W/m}^2\textbf{°C} \quad \textbf{(Ans.)}$$

(v) Total mass flow through the boundary, m_f:

Total mass flow through the boundary can be calculated from the following formula :

$$m_t = 1.7 \, \rho.v \left[\frac{Gr_L}{(P_r^2) \, (Pr + 0.952)} \right]^{1/4}$$

$$= 1.7 \times 1.029 \times 20.02 \times 10^{-6} \left[\frac{27.532 \times 10^7}{(0.694)^2 \, (0.694 + 0.952)} \right]$$

or, $\quad\quad\quad\quad m_t = \textbf{0.00478 kg/s} \quad$ **(Ans.)**

(vi) Heat loss from the plate, Q :

Heat lost from the plate will be from both the sides, hence

$$Q = 2 \, \bar{h} \, A_s \, (t_s - t_\infty)$$

$$= 2 \times 5.43 \times (0.35 \times 1.2) \, (115 - 25) = \textbf{410.5 W} \quad\quad \textbf{(Ans.)}$$

(vii) Rise in temperature of the air passing through the boundary, Δt:

Heat lost, $Q = m_t \, c_p \, \Delta t$

or, $\quad\quad\quad\quad 410.5 = 0.00478 \times (1.005 \times 10^3) \times \Delta t$

or, $\quad\quad\quad\quad \Delta t = \dfrac{410.5}{0.00478 \times (1.005 \times 10^3)} = \textbf{85.45°C} \quad\quad \textbf{(Ans.)}$

Example 8.4. *A 350 mm long glass plate is hung vertically in the air at 24°C while its temperature is maintained at 80°C. Calculate the boundary layer thickness at the trailing edge of the plate. If a similar plate is placed in a wind tunnel and air is blown over it at a velocity of 5 m/s, find the boundary layer thickness at its trailing edge.*

Also determine the average heat transfer coefficient, for natural and forced convection for the above mentioned data.

Solution. *Given:* $L = 350$ mm $= 0.35$ m, $t_\infty = 24°C$, $t_s = 80°C$, $U = 5$ m/s

Film temperature, $t_f = \dfrac{t_s + t_\infty}{2} = \dfrac{80 + 24}{2} = 52°C$

The properties of air at 52°C are:

$k = 28.15 \times 10^{-3}$ W/m°C; $v = 18.41 \times 10^{-6}$ m²/s; $Pr = 0.7$; $\beta = \left(\dfrac{1}{52 + 273}\right) = 3.07 \times 10^{-3}$ K^{-1}

Boundary layer thickness, δ:

Free convection:

Grashoff number, $Gr_L = \dfrac{L^3\, g\beta\,(t_s - t_\infty)}{v^2} = \dfrac{(0.35)^3 \times 9.81 \times 3.07 \times 10^{-3}\,(80-24)}{(18.41 \times 10^{-6})^2} = 2.133 \times 10^8$

∴   Rayleigh number, $Ra_L = Gr_L \cdot Pr = 2.133 \times 10^8 \times 0.7 = 1.493 \times 10^8$

This value of the Rayleigh number, according to eqn. (8.36), indicates a laminar boundary layer. Using the eqn. (8.27), we get

$$\dfrac{\delta}{x} = 3.93\,(0.952 + Pr)^{1/4}\,(Gr)^{-1/4}\,(Pr)^{-1/2}$$

or, $\delta = 0.35\,[\,3.93\,(0.952 + 0.7\,)^{1/4}\,(2.133 \times 10^8)^{-1/4}\,(0.7)^{-1/2}\,] = 0.0154$ m

$= 15.4$ mm   **(Ans.)**

Forced convection:

Reynolds number, $Re = \dfrac{UL}{v} = \dfrac{5 \times 0.35}{18.41 \times 10^{-6}} = 9.505 \times 10^4$

So the boundary layer is *laminar*.

$$\delta = \dfrac{5L}{\sqrt{Re}} = \dfrac{5 \times 0.35}{\sqrt{9.505 \times 10^4}} = 0.00567 \text{ m} = \textbf{5.67 mm} \qquad \textbf{(Ans.)}$$

Thus the boundary layer thickness in forced convection is *less* than that in free convection.

Heat transfer coefficient $\bar{h}$:

Free convection :

$$Nu_L = \dfrac{\bar{h}L}{k} = 0.677 \, (Pr)^{1/2} \, (0.952 + Pr)^{-1/4} \, (Gr)^{1/4} \qquad \text{...[Eqn. (8.33)]}$$

$$= 0.677 \, (0.7)^{1/2} \, (0.952 + 0.7)^{-1/4} \, (2.133 \times 10^8)^{1/4} = 60.378$$

or, $\qquad \bar{h} = \dfrac{k}{L} \times 60.378 = \dfrac{28.15 \times 10^{-3}}{0.35} \times 60.378 = \textbf{4.856 W/m}^2\textbf{°C} \qquad \textbf{(Ans.)}$

Forced convection :

$$\overline{Nu}_L = \dfrac{\bar{h}L}{k} = 0.664 \, (Re_L)^{1/2} \, (Pr)^{1/3}$$

$$= 0.664 \, (9.505 \times 10^4)^{1/2} \, (0.7)^{0.333} = 181.78$$

or, $\qquad \bar{h} = \dfrac{k}{L} \times 181.78 = \dfrac{28.15 \times 10^{-3}}{0.35} \times 181.78 = \textbf{14.62 W/m}^2\textbf{°C} \qquad \textbf{(Ans.)}$

Thus it is seen that heat transfer coefficient in forced convection is *much larger* than that in free convection.

Example 8.5. *A sheet metal air duct carries air-conditioned air at an average temperature of 10°C. The duct size is 320 mm × 200 mm and length of the duct exposed to the surrounding air at 30°C is 15 m long. Find the heat gain by the air in the duct. Assume 200 mm side is vertical and top surface of the duct is insulated. Use the following properties :*

$Nu = 0.6 (Gr.Pr)^{0.25}$ *for vertical surface.*

$Nu = 0.27 (Gr.Pr)^{0.25}$ *for horizontal surface.*

Take the properties of the air at mean temperature of $(30 + 10)/2 = 20°C$ as given below :

$c_p = 100$ *J/kg K;* $\rho = 1.204$ *kg/m³;* $\mu = 18.2 \times 10^{-6}$ *N-s/m²;*

$v = 15.1 \times 10^{-6}$ *m²/s; k = 0.256 W/m K and Pr = 0.71.*

Solution. Refer Fig. 8.4.

There will be two vertical surfaces and one horizontal surface from which the heat will be gained by *natural convection*. There is no heat transfer from the top surface as it is insulated.

Air-conditioned air

Insulated

$H = 200$ mm

$B = 320$ mm

Fig. 8.4.

Now, $(Gr)_v = \dfrac{g\beta H^3 (\Delta t)}{v^2}$

$$= \dfrac{9.81 \times \left(\dfrac{1}{273 + 20}\right) \times 0.2^3 \times (30 - 10)}{(15.1 \times 10^{-6})^2}$$

$= 2.35 \times 10^7$

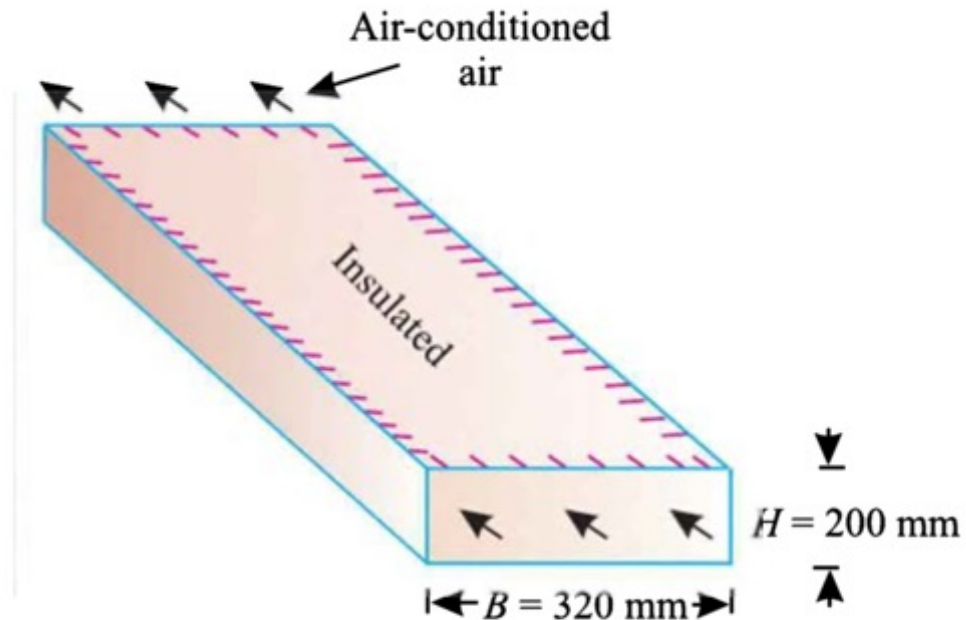$(Gr.Pr)_v = 2.35 \times 10^7 \times 0.71 = 16.68 \times 10^6$

$$(Nu)_v = \frac{h_v \cdot H}{k} = 0.6\,(16.68 \times 10^6)^{0.25} = 38.34$$

$$\therefore \qquad h_v = \frac{k}{H} \times 38.34 = \frac{0.256}{0.2} \times 38.34 = 49.07 \ \text{W/m}^2\,{}^\circ\text{C}$$

$$Q_v = h_v \times A_v \times (\Delta t) = 49.07 \times (2 \times 15 \times 0.2) \times (30 - 10) = 5888.4 \ \text{W}$$

$$(Gr)_h = \frac{9.81 \times \dfrac{1}{(273 + 20)} \times (0.32)^3 \times (30 - 10)}{(15.1 \times 10^{-6})^2} = 9.62 \times 10^7$$

$$(Gr.Pr)_h = 9.62 \times 10^7 \times 0.71 = 6.83 \times 10^7$$

$$(Nu)_h = \frac{h_h \times B}{k} = 0.27\,(6.83 \times 10^7)^{0.25} = 24.54$$

$$h_h = \frac{k}{\beta} = 24.54 = \frac{0.256}{0.32} = 24.54 = 19.63 \ \text{W/m}^2\,{}^\circ\text{C}$$

$$Q_h = h_h \times A_h \times (\Delta t) = 19.63 \times (15 \times 0.32) \times (30 - 10) = 1884.5 \ \text{W}$$

$\therefore$ Total heat gain, $Q = Q_v + Q_h = 5888.4 + 1884.5 = \textbf{7772.9 W}$ **(Ans.)**

Example 8.6. *Air flows through long rectangular heating duct of width and height of 0.75 m and 0.3 m respectively. The outer surface temperature of the duct is maintained at 45°C. If the duct is exposed to air at 15°C in a cramp-space beneath a home, what is the heat loss from the duct per metre length ?*

Use the following correlations :

(i) Top surface :

Take $\qquad L_c = \dfrac{B}{2} = 0.375 \, m$.

$$\overline{Nu}_L = 0.54 \, (Ra_L)^{0.25} \text{ if } 10^4 \le Ra_L \le 10^7$$
$$= 0.15 \, (Ra_L)^{0.33} \text{ if } 10^7 \le Ra_L \le 10^{11}$$

(ii) Bottom surface :

Take $\qquad L_c = \dfrac{B}{2} = 0.375 \, m$

$$\overline{Nu}_L = 0.27 \, (Ra_L)^{0.25}, \ 10^5 \le Ra_L \le 10^{10}$$

(iii) Sides of the duct :

$$\overline{Nu}_L = 0.68 + \dfrac{0.67 \, (Ra_L)^{0.25}}{\left[1 + \left(\dfrac{0.492}{Pr}\right)^{9/16}\right]^{4/9}}$$

Use the following properties of air :

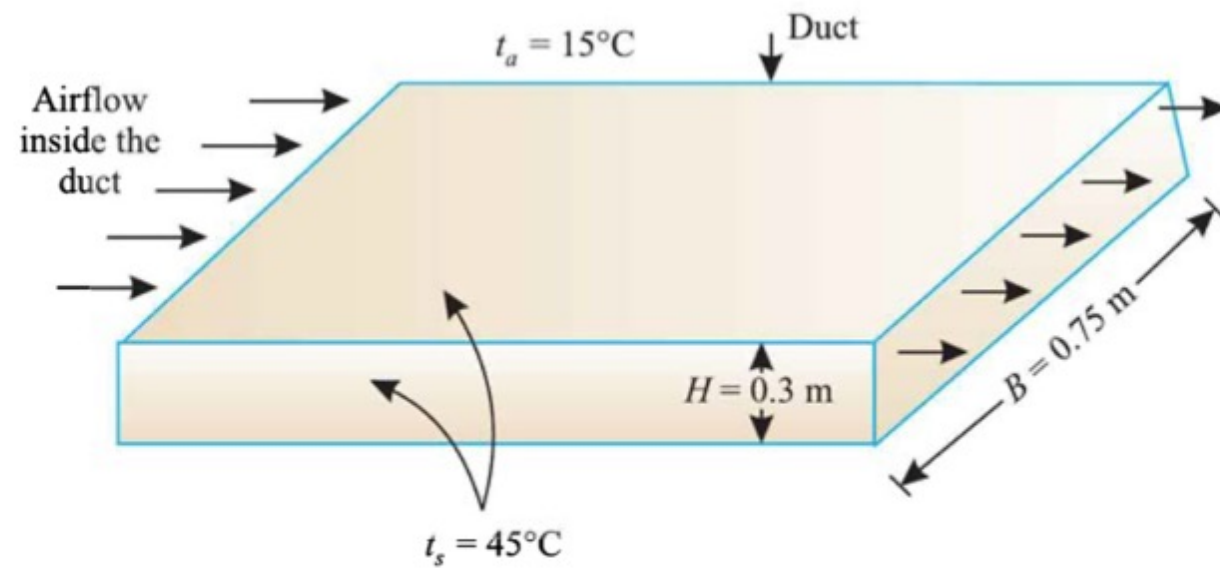$$v = 16.2 \times 10^{-6} \, m^2/s, \ \alpha = 22.9 \times 10^{-6} \, m^2/s$$
$$k = 0.0265 \, W/m \, K, \ \beta = 0.0033 \, K^{-1} \text{ and } Pr = 0.71.$$

(N.M.U. Winter, 1995)

Solution. The configuration of the duct is shown in Fig. 8.5.

$$Ra_L = \dfrac{g\beta \, (t_s - t_a) \, L_c^3}{\alpha v} \qquad\qquad ...[Eqn. \, (8.36)]$$

Fig. 8.5

$$= \frac{9.81 \times 0.0033 \times (45 - 15)\ L_c^3}{(22.9 \times 10^{-6}) \times (16.2 \times 10^{-6})} = 2.62 \times 10^9\ (L_c^3)$$

For two sides :

$$L_c = H = 0.3\ \text{m}$$

$\therefore$
$$Ra_L = 2.62 \times 10^9\ (0.3)^3 = 7.07 \times 10^7$$

$\therefore$
$$Nu_L = 0.68 + \frac{0.67\ (7.07 \times 10^7)^{0.25}}{\left[1 + \left(\dfrac{0.492}{0.71}\right)^{9/16}\right]^{4/9}} = 0.68 + \frac{61.44}{1.3} = 47.94$$

$\therefore$
$$\frac{h_s\ L_c}{k} = 47.94$$

or,
$$h_s = \frac{k}{L_c} \times 47.94 = \frac{0.0265 \times 47.94}{0.3} = 4.23\ \text{W/m}^2{}^\circ\text{C}$$

For top surface :

$$\overline{Nu}_L = \frac{h_t L_c}{k} = 0.15 \, (Ra_L)^{0.33} = 0.15 \, (7.07 \times 10^7)^{0.33} = 58.4$$

$$\therefore \quad h_t = \frac{k}{L_c} \times 58.4 = \frac{0.0265 \times 58.4}{0.375} = 4.127 \text{ W/m}^2{}^\circ\text{C}$$

$$\left(\because L_c = \frac{B}{2} = \frac{0.75}{2} = 0.375 \text{ m} \right)$$

For bottom surface :

$$\overline{Nu}_L = \frac{h_b L_c}{k} = 0.27 \, (Ra_L)^{0.25} = 0.27 \, (7.07 \times 10^7)^{0.25} = 24.76$$

$$\therefore \quad h_b = \frac{k}{L_c} \times 24.76 = \frac{0.0265}{0.375} \times 24.76 = 1.75 \text{ W/m}^2{}^\circ\text{C}$$

The rate of heat loss per unit length of the duct is given by,

$$Q = Q_s + Q_t + Q_b$$

$$= [2 \times h_s \times (0.3 \times 1) + h_t \times (0.75 \times 1) + h_b \times (0.75 \times 1)] \, (t_s - t_a)$$

$$= [2 \times 4.23 \times 0.3 + 4.127 \times 0.75 \times 1 + 1.75 \times 0.75 \times 1] \, (45 - 15)$$

$$= \mathbf{208.4 \text{ W/m}} \quad \textbf{(Ans.)}$$

Example 8.7. *A vertical plate measuring 180 mm × 180 mm and at 50°C is exposed to atmosphere at 10°C. Compare the free convection heat transfer from this plate with that which would result due to forced convection over the plate at a velocity equal to twice the maximum velocity which would occur in free convection boundary layer.*

Solution. Free convection :

The film temperature, $t_f = \dfrac{50 + 10}{2} = 30°C$.

The thermo-physical properties at 30°C are :

$k = 0.02673$ W/m°C; $v = 16 \times 10^{-6}$ m²/s; $Pr = 0.701$

$$\beta = \frac{1}{(t_f + 273)} = \frac{1}{303} = 0.0033 \ K^{-1}$$

$$Gr_L = \frac{L^3 \, g\beta \, \Delta t}{v^2}$$

$$= \frac{(0.18)^3 \times 9.81 \times 0.0033 \times (50 - 10)}{(16 \times 10^{-6})^2}$$

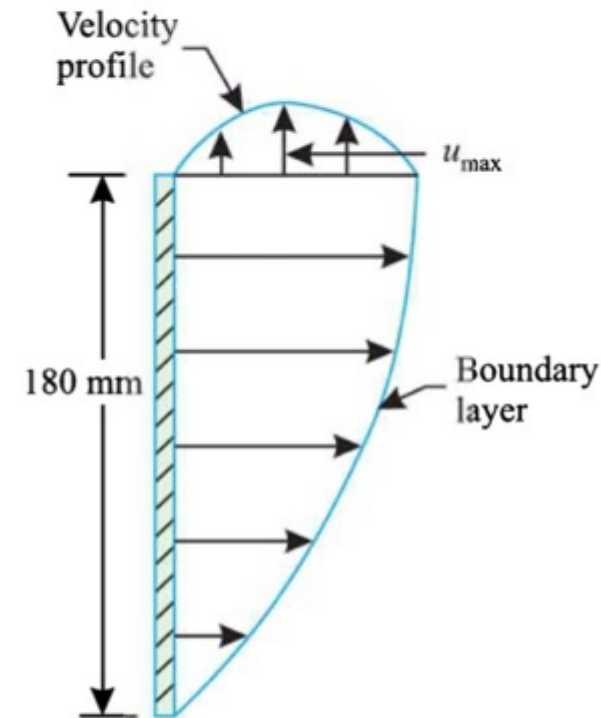$$= 2.95 \times 10^7$$

Since $Gr < 10^9$, hence the flow is laminar.

For laminar flow, using the following relation, we get,

$$\overline{Nu}_L = \frac{\overline{h}L}{k} = 0.677 \ (Pr)^{1/2} \ (0.952 + Pr)^{-1/4} \ (Gr_L)^{1/4}$$

$$= 0.677 \ (0.701)^{1/2} \ (0.952 + 0.701)^{-1/4} \ (2.95 \times 10^7)^{1/4} = 36.84$$

or, $\overline{h} = \dfrac{k}{L} \times 36.84 = \dfrac{0.02673}{0.18} \times 36.84 = 5.47$ W/m²°C

The heat lost from one side of the plate,

$$Q_{free} = \overline{h} \ A_s \ \Delta t = 5.47 \times (0.18 \times 0.18) \times (50 - 10) = 7.089 \ W$$

Velocity profile

u_{max}

180 mm

Boundary layer

Fig. 8.6.

Forced convection :

$$u_{max} = 0.766 \, v \left(0.952 + \frac{v}{\alpha}\right)^{-1/2} \left[\frac{g\beta \, (t_s - t_\infty)}{v^2}\right]^{1/2} (x)^{1/2} \qquad \text{...[Eqn. (8.25)]}$$

$$= 0.766 \times 16 \times 10^{-6} \, (0.952 + 0.701)^{-1/2} \left[\frac{9.81 \times 0.0033 \, (50 - 10)}{(16 \times 10^{-6})^2}\right]^{1/2} (0.18)^{1/2}$$

$$= 0.287 \text{ m/s} \qquad\qquad \left(\because \frac{v}{\alpha} = Pr = 0.701\right)$$

The average heat transfer coefficient with forced convection if velocity is assumed equal to $2u_{max}$;

$$\overline{Nu}_L = 0.664 \, (Re)^{1/2} \, (Pr)^{1/3}$$

$$Re = \frac{LU}{v} = \frac{0.18 \times (2 \times 0.287)}{16 \times 10^{-6}} = 6457.5$$

$$\overline{Nu} = \frac{\overline{h}L}{k} = 0.664 \, (6457.5)^{1/2} \, (0.701)^{0.333} = 47.4$$

or, $\qquad \overline{h} = \frac{k}{L} \times 47.4 = \frac{0.02673}{0.18} \times 47.4 = 7.039 \text{ W/m}^2\text{°C}$

Heat lost due to forced convection,

$$Q_{forced} = \overline{h} \, A_s \, \Delta t = 7.039 \times (0.18 \times 0.18) \, (50 - 10) = 9.12 \text{ W}$$

Hence, $\qquad \dfrac{Q_{free}}{Q_{forced}} = \dfrac{7.089}{9.12} = \mathbf{0.777}$ **(Ans.)**

Example 8.8. *Two vertical plates, each 120 mm high and at 85°C are placed in a tank of water at 15°C. Calculate the minimum spacing which will prevent interference of the free convection boundary layers.*

Solution. Let, δ = Boundary layer thickness at the trailing edge of the plate.

$$L = 2\delta = \text{The minimum spacing required.}$$

The film temperature, $t_f = \dfrac{85 + 15}{2} = 50°C$
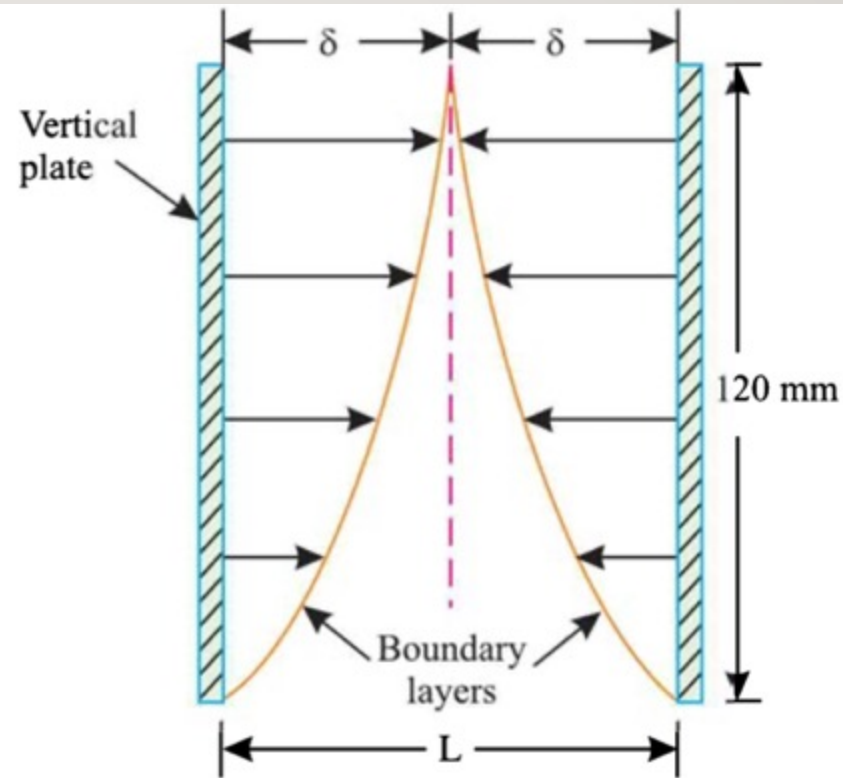
The thermo-physical properties of water at 50°C are:

$$k = 0.674 \text{ W/m°C}; \quad v = 0.556 \times 10^{-6} \text{ m}^2/\text{s}$$

$$Pr = 3.54; \quad \beta = \frac{1}{(t_f + 273)} = \frac{1}{323}$$
$$= 0.003095 \text{ K}^{-1}$$

$$Gr_x = \frac{x^3 \, g\beta\Delta t}{v^2} = \frac{(0.12)^3 \times 9.81 \times 0.003095 \times (85 - 15)}{(0.556 \times 10^{-6})^2} = 11.88 \times 10^9$$

$$Gr \cdot Pr = 11.88 \times 10^9 \times 3.54 = 42.055 \times 10^9$$

Since $Gr \cdot Pr > 10^9$, hence the character of the flow is *turbulent*.

Vertical plate

120 mm

Boundary layers

L

Fig. 8.7.

For turbulent flow, using the following relation, we get

$$\frac{\delta}{x} = 0.565 \, (Pr)^{-8/15} \left[\frac{1 + 0.494 \, (Pr)^{2/3}}{Gr_x} \right]^{0.1}$$

$$= 0.565 \, (3.54)^{-8/15} \left[\frac{1 + 0.494 \, (3.54)^{2/3}}{11.88 \times 10^9} \right]^{0.1} = 0.0306$$

$\delta = 0.0306 \times 0.12 = 0.003672 \text{ m} = 3.672 \text{ mm}$

$L = 2\delta = 2 \times 3.672 = \textbf{7.344 mm}$ **(Ans.)**

Example 8.9. *A hot plate 1 m × 0.5 m at 130°C is kept vertically in still air at 20°C. Find :*

(i) *Heat transfer coefficient,*

(ii) *Initial rate of cooling the plate in °C/min.*

(iii) *Time required for cooling plate from 180°C if the heat transfer is due to convection only.*
Mass of the plate is 20 kg and c_p = 400 J/kg K.

Assume 0.5 m side is vertical and that the heat transfer coefficient calculated in (i) above remains constant and convection takes place from both sides of the plate.

Take properties of air at $\dfrac{130 + 20}{2}$ = 75°C as :

c_p = 1007 J/kg°C, ρ = 1.07 m²/s; k = 0.029 J/kg K; v = 19.1 × 10⁻⁶ m²/s   **(N.M.U., 1998)**

Solution. *Given* : $A = 1 \times 0.5 = 0.5 \text{ m}^2$; $t_p \, (= t_s) = 130°C$; $t_a = 20°C$; $m = 20$ kg; $c_p = 400$ J/kg K

$$Pr = \frac{\mu c_p}{k} = \frac{\rho v c_p}{k} = \frac{1.07 \times 19.1 \times 10^{-6} \times 1007}{0.029} = 0.709$$

$$Gr = \frac{L^3 g \beta \, (t_s - t_\infty)}{v^2} = \frac{(0.5)^3 \times 9.81 \times \left(\dfrac{1}{273 + 75}\right) \times (130 - 20)}{(19.1 \times 10^{-6})^2}$$

$$= 1.06 \times 10^9$$

∴ $$Gr.Pr = 1.06 \times 10^9 \times 0.709 = 0.75 \times 10^9$$

The heat transfer coefficient for the vertical plate is given by

$$\overline{Nu_L} = 0.59 \, (Gr.Pr)^{1/4} \text{ for } (10^4 < Gr.Pr < 10^9)$$
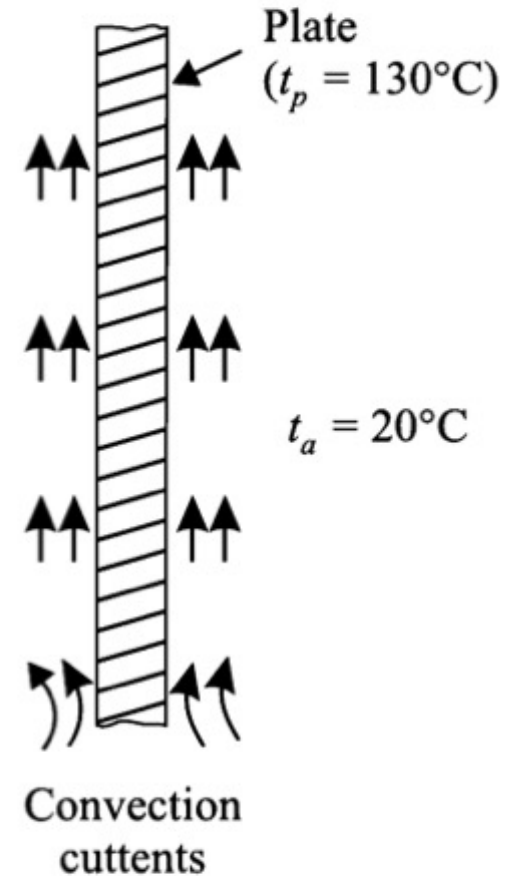
...[Eqn. (8.37)]

Substituting the values in the above eqn., we get

$$\overline{Nu}_L = \frac{\overline{h}L}{k} = 0.59 \, (0.75 \times 10^9)^{1/4}$$

or, $$\overline{h} = \frac{k}{L} \times 0.59 \, (0.75 \times 10^9)^{1/4}$$

$$= \frac{0.029}{0.5} \times 0.59 \, (0.75 \times 10^9)^{1/4} = \mathbf{5.66 \text{ W/m}^2 \text{°C}} \quad \textbf{(Ans.)}$$

Plate
$(t_p = 130°C)$

$t_a = 20°C$

Convection
cuttents

Fig. 8.8

(ii) Initial rate of cooling the plate in °C/min. :

Heat lost from *both sides* of the plate is given by

$$Q = 2[h \times A(t_s - t_\infty)] = 2[5.66 \times (1 \times 0.5) \times (130 - 20)] = 622.6 \text{ W}$$

Also, $Q = mc_p(\Delta t)$ where Δt is the rate of cooling.

$$\therefore \quad \Delta t = \frac{622.6}{20 \times 400} = 0.0778°C/s = \textbf{4.668°C/min} \quad \textbf{(Ans.)}$$

(iii) Time required for cooling plate from 180°C to 80°C, τ :

The instantaneous heat lost by the plate is given by

$$2 \times h \times A(t - t_\infty) = -mc_p \frac{dt}{d\tau}$$

or,

$$-\frac{mc_p}{2hA} \int_{t_1}^{t_2} \frac{dt}{(t - t_\infty)} = \int_0^\tau d\tau$$

or,

$$-\frac{mc_p}{2hA} \ln\left(\frac{t_2 - t_\infty}{t_1 - t_\infty}\right) = \tau$$

$$\therefore \quad \tau = \frac{-20 \times 400}{2 \times 5.66 \times (1 \times 0.5)} \ln\left(\frac{80 - 20}{180 - 20}\right)$$

$$= -1413.43 \times (-0.9808) = 1386 \text{ s or } \textbf{0.385h (Ans.)}$$

Example 8.10. *A square plate 40 cm × 40 cm maintained at 400 K is suspended vertically in atmospheric air at 300 K.*

(i) *Determine the boundary layer thickness at trailing edge of the plate.*

(ii) *Calculate the average heat transfer coefficient using a relation,*

$$Nu = 0.516 \, (Gr_L \cdot Pr)^{0.25}$$

Take the following properties of air :

$v = 20.75 \times 10^{-6} \, m^2/s; \; k = 0.03 \, W/m°C; \; \beta = 2.86 \times 10^{-3} \, K^{-1}; \; Pr = 0.7$ **(P.U.)**

Solution. *Given :* $t_s = 400$ K, $t_\infty = 300$ K

(i) Boundary layer thickness, δ:

The boundary layer thickness is given by

$$\delta = x \, [3.93 \, (0.952 + Pr)^{1/4} \, (Gr)^{-1/4} \, (Pr)^{-1/2}] \qquad \qquad \text{...[Eqn. (8.27)]}$$

$$Gr_x = \frac{x^3 \, g\beta \, (\Delta t)}{v^2} = \frac{(0.4)^3 \times 9.81 \times (2.86 \times 10^{-3}) \times (400 - 300)}{(20.75 \times 10^{-6})^2} = 4.17 \times 10^8$$

Substituting the values in the above equation, we get

$$\delta = 0.4 \, [3.93 \, (0.952 + 0.7)^{1/4} \, (4.17 \times 10^8)^{-1/4} \, (0.7)^{-1/2}] = 0.0149 \text{ m}$$

$$= \textbf{14.9 mm} \qquad \textbf{(Ans.)}$$

(ii) Average heat transfer coefficient, $\overline{h}$:

$$Gr \,.\, Pr = (4.17 \times 10^8) \, (0.7) = 2.919 \times 10^8$$

$$\overline{Nu} = \frac{\overline{h} \, L}{k} = 0.516 \, (Gr.Pr)^{0.25}$$

or,

$$\overline{h} = \frac{k}{L} \times 0.516 \, (Gr.Pr)^{0.25}$$

$$= \frac{0.03}{0.4} \times 0.516 \, (2.919 \times 10^8)^{0.25} = \textbf{5.058 W/m}^2 \textbf{°C} \quad \textbf{(Ans.)}$$

Example 8.11. *A nuclear reactor with its core constructed of parallel vertical plates 2.2 m high and 1.4 m wide has been designed on free convection heating of liquid bismuth. The maximum temperature of the plate surfaces is limited to 960°C while the lowest allowable temperature of bismuth is 340°C. Calculate the maximum possible heat dissipation from both sides of each plate.*

For the convection coefficient, the appropriate correlation is

$$Nu = 0.13 \, (Gr.Pr)^{0.333}$$

where different parameters are evaluated at the mean film temperature.

Solution. The mean film temperature, $t_f = \dfrac{960 + 340}{2} = 650°C$

The thermo-physical properties of bismuth are:

$\rho = 10^4 \ \text{kg/m}^3$; $\qquad \mu = 3.12 \ \text{kg/m-h}; \ c_p = 150.7 \ \text{J/kg°C}; \ k = 13.02 \ \text{W/m°C}$

$$\beta = \frac{1}{650 + 273} = 1.08 \times 10^{-3} \ \text{K}^{-1}$$

$\therefore \qquad\qquad Pr = \dfrac{\mu c_p}{k} = \dfrac{(3.12/3600) \times 150.7}{13.02} = 0.01$

$$Gr = \frac{L^3 \rho^2 g \beta \Delta t}{\mu^2} = \frac{(2.2)^3 \times (10^4)^2 \ 9.81 \times 1.08 \times 10^{-3} \times (960 - 340)}{(3.12/3600)^2}$$

$$= 9.312 \times 10^{15}$$

$$Gr.Pr = 9.312 \times 10^{15} \times 0.01 = 93.12 \times 10^{12}$$

Using the given correlation, we get

$$Nu = \frac{hL}{k} = 0.13 \, (Gr.Pr)^{0.333}$$

or,

$$h = \frac{k}{L} \times 0.13 \, (Gr.Pr)^{0.333}$$

$$= \frac{13.02}{2.2} \times 0.13 \, (93.12 \times 10^{12})^{0.33} = 34500 \text{ W/m}^2\text{°C}$$

∴ Heat dissipation from both sides of each plate,

$$Q = 2 \, h \, A_s \, \Delta t$$

$$= 2 \times 34500 \times (2.2 \times 1.45) \times (960 - 340)$$

$$= 136.47 \times 10^6 \text{ W} = \textbf{136.47 MW} \quad \textbf{(Ans.)}$$

Example 8.12. *Find the convective heat loss from a radiator 0.6 m wide and 1.2 m high maintained at a temperature of 90°C in a room at 14°C. Consider the radiator as a vertical plate.*

Solution. *Given :* $L = 1.2$ m, $B = 0.6$ m, $t_s = 90°C$, $t_\infty = 14°C$.

Convective heat loss, Q :

Film temperature $t_f = \dfrac{t_s + t_\infty}{2} = \dfrac{90 + 14}{2} = 52°C$

The thermo-physical properties of air at 52°C are:

$k = 28.15 \times 10^{-3}$ W/m°C; $v = 18.41 \times 10^{-6}$ m²/s, $Pr = 0.7$;

$$\beta = \frac{1}{(52 + 273)} = 3.077 \times 10^{-3} \ K^{-1}$$

Rayleigh number $= Ra_L = Gr_L.Pr = \dfrac{L^3 g \beta \ (t_s - t_\infty)}{v^2} \cdot Pr$

$$= \frac{(1.2)^3 \times 9.81 \times 3.077 \times 10^{-3} \times (90 - 14)}{(18.41 \times 10^{-6})^2} \times 0.7$$

$= 11.69 \times 10^9$

From eqn. (8.36) it follows that the flow becomes turbulent on the radiator plate. Using eqn. (8.40), we have

$$\overline{Nu_L} = \left[0.825 + \frac{0.387 \, (Gr.Pr)^{1/6}}{\left[1 + \left(\dfrac{0.492}{Pr} \right)^{9/16} \right]^{8/27}} \right]^2$$

$$= \left[0.825 + \frac{0.387 \, (11.69 \times 10^9)^{1/6}}{\left[1 + \left(\dfrac{0.492}{0.7} \right)^{9/16} \right]^{8/27}} \right]^2 = 263.74$$

$$\therefore \quad \overline{h} = \frac{k}{L} \times \overline{Nu_L} = \frac{28.15 \times 10^{-3}}{1.2} \times 263.74 = 6.187 \; W/m^2{}^\circ C$$

The convective heat loss,

$$Q = h \, A_s \, \Delta t = 6.187 \times (1.2 \times 0.6)(90 - 14) = \mathbf{338.55 \; W \; (Ans.)}$$

Example 8.13. *A gas at 195°C is flowing through a thin walled vertical duct which is in the form of circular cross-section having diameter of 450 mm. The ambient air at 15°C, which may be considered still, surrounds the duct. Find the rate of heat transfer using the following simplified relation for air for laminar flow.*

$$\bar{h} = 1.38 \left(\frac{\Delta t}{L} \right)^{0.25}$$

where, L is the length of the duct in metres.

Solution. *Given :* $\quad \Delta t = 195 - 15 = 180°C, D = 450 \text{ mm} = 0.45 \text{ m}$

$$\bar{h} = 1.38 \left(\frac{180}{1} \right)^{0.25} = 5.055 \text{ W/m}^2°C$$

∴   The rate of heat transfer,

$$Q = \bar{h} \, A_s \, \Delta t = 5.055 \times (\pi \times 0.45 \times 1) \times 180 = \textbf{1286.34 W/m length.} \qquad \textbf{(Ans.)}$$

Example 8.14. *A horizontal heated plate measuring 1.5 m × 1.1 m and at 215°C, facing upwards, is placed in still air at 25°C. Calculated the heat loss by natural convection. The convective film coefficient for free convection is given by the following empirical relation:*

$$h = 3.05 \, (T_f)^{1/4} \text{ W/m}^2°C$$

where, T_f is the mean film temperature in degrees kelvin.

Solution. *Given* : $A_s = 1.5 \times 1.1 = 1.65 \text{ m}^2$, $t_s = 215°C$, $t_\infty = 25°C$.

Mean film temperature,

$$T_f = 273 + \left(\frac{215 + 25}{2}\right) = 393 \text{ K}$$

∴ $h = 3.05 \, (393)^{1/4} = 13.58 \text{ W/m}^2°C$

Rate of heat loss, by natural convection,

$$Q = h \, A_s \, (t_s - t_\infty) = 13.58 \times 1.65 \times (215 - 25) = \textbf{4257.33 W} \qquad \textbf{(Ans.)}$$

Example 8.15. *A transformer is cooled by immersing in an oil bath which is housed in a cylindrical tank which is 0.8 m in diameter and 1.3 m long. If the electrical loss is 1.3 kW, calculate the surface temperature of the tank; the entire loss of electrical energy may be assumed to be due to free convection from the bottom of the tank. For this case the following simplified relations for the boundary layer are applicable :*

$$h = 1.35 \left(\frac{\Delta t}{L} \right)^{0.25} \qquad \text{... For a cylindrical plane;}$$

$$h = 1.45 \left(\frac{\Delta t}{L} \right)^{0.25} \qquad \text{... For a vertical plane.}$$

Assume the ambient air temperature as 20°C.

Solution. *Given* : $D = 0.8$ m, $L = 1.3$ m.

The total rate of heat transfer from the tank,

$$Q_{total} = Q_{side} + Q_{top} \qquad \qquad \qquad \text{...(i)}$$

$$Q_{side} = h_{side} \times A_{side} \times \Delta t$$

$$= 1.35 \left(\frac{t - 20}{1.3} \right)^{0.25} \times (\pi \times 0.8 \times 1.3) \times (t - 20) = 4.13 \, (t - 20)^{1.25}$$

$$Q_{top} = h_{top} \cdot A_{top} \, \Delta t$$

$$= 1.45 \left(\frac{t - 20}{0.8} \right)^{0.25} \times \left(\frac{\pi}{2} \times 0.8^2 \right) \times (t - 20) = 0.77 \, (t - 20)^{1.25}$$

Substituting the values in eqn. (*i*), we get

$$1.3 \times 10^3 = 4.13 \, (t - 20)^{1.25} + 0.77 \, (t - 20)^{1.25} = 4.9 \, (t - 20)^{1.25}$$

or, $\qquad \qquad (t - 20) = \left[\dfrac{1.3 \times 10^3}{4.9} \right]^{1/1.25} = 86.89$

or, $\qquad \qquad t = 86.89 + 20 = \mathbf{106.89°C}$ **(Ans.)**

Latihan Soal. No. 1:

Suatu silinder vertical: tinggi 2,5 m, diameter 200 mm, mempunyai temperatur 120 °C berada di lingkungan udara yang bertemperatur 20 °C. Hitunglah panas yang dilepaskan dari permukaan silinder secara konveksi bebas. Asumsikan sifat udara pada temperatur rata-rata: $\rho = 1,06 \frac{kg}{m^3}, \nu =$ 18,97 x 10^{-6} m^2/s, c$_p$ = 1,004 kJ/kg °C dan k = 0,1042 kJ/mh°C

TERIMA KASIH